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AKR'S QUEST FOR EXCELLENCE : SEEDS OF PHYSICS NOBEL 2020

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The lifelong solitary struggle of eminent theoretical physicist Amal Kumar Raychaudhuri, towards ex-cellence in research in the field of general relativistic cosmology, is chronicled.

Introduction

If one is to recall outstanding original contributions by theoretical physicists in India in the post-independence era, that have withstood the test of time and changed the course of physics forever, the Raychaudhuri equation comes readily to mind. Despite the increasingly greater availability of funds for research and greater efforts at organizing it nationally, little else of a similar quality has been accomplished so far. Yet the beginnings were unbelievably humble, and the road to professional accomplishment was strewn with thorns. It is a story of a lifelong struggle compounded by a resolve that could break, but never did bend. But it remains an inspiration for those who find it hard to follow the herd.

Fig. 1 : Amal Kumar Raychaudhuri

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Education

Amal Kumar Raychaudhuri (AKR) was born in 1923 in Barisal district in eastern Bengal, now in Bangladesh[1]. His father Sureshchandra was a teacher of mathematics in a high school, who had a bright academic record, but was not successful professionally in the same measure. His mother Surabala was a housewife, but clearly one who could imbue in her children a sense of uprightness and deep-rooted ideals. The family migrated to Kolkata while AKR was still a child, and his early education was in Tirthapati Institution. Later he completed his Matriculation (equivalent to today's Secondary level) from Hindu School, and joined City College for the Intermediate in science (ISc) programme (equivalent to today's Higher Secondary), then run by Calcutta University. Emerging with very good grades in the ISc Examination, AKR wanted to pursue an undergraduate programme (BSc Honours) in mathematics, presumably in his father's footsteps. But Sureshchandra was not encouraging in this plan at all, thinking perhaps of his own inadequacy in professional achievement as a mathematician[2]. The other option was to do physics. This is what led AKR to join the BSc Honours programme of Calcutta University at Presidency College.

Presidency College those days was already at the helm of its reputation, especially in physics, boasting of the legacy of eminent scientists like Acharya Jagadis Chandra Bose as a Professor of physics at the College in earlier times, and alumni including the legendary Satyendra Nath Bose, Meghnad Saha and Prashanta Chandra Mahalanobis. The physics faculty had some quality teachers, like Professor Kulesh Kar, whom AKR found inspiring (to a degree, as we shall see later). As a student, AKR appears to have been of a quiet disposition, not vocal with numerous queries in class, as some were and others still are. But if a professor's lectures seemed to have logical or mathematical gaps, AKR would make mental notes of disagreement, but would rarely challenge the teacher in class. Most of the honours studying was done solo, it seems. The end result of this painstaking self-study was the top position in Calcutta University in physics in the BSc (Honours) Examination.

AKR then followed the canonical path to the University College of Science in Rajabazar, a short distance away from Presidency College, for his Masters degree in physics. At that time, the University of Calcutta had the option of either two special or elective papers, or a dissertation (thesis) equivalent, in the final year of the MSc programme. During AKR's time, the dissertation option turned out to be more popular than the special elective paper option among the students. Since this meant very few students for the special elective option, teachers offering those elective subjects complained to the-then Departmental Chair, the reputed Professor Meghnad Saha. Prof Saha concurred with these faculty colleagues, and decided that the dissertation option should be only available to those who pass an elimination test, which he himself was to administer. However, on the day of the test, AKR came down with high fever and missed the test. He had been very keen on a theoretical dissertation involving physics and mathematics, but was not fated to get this option, despite having very good grades. Prof Saha sternly and summarily refused a re-examination. Disappointed that he was denied the dissertation option for no fault of his, AKR decided to choose Xray-Crystallography as the elective subject, ostensibly because of rumours that it fetched good grades. He despaired at having missed an opportunity to do a dissertation project in an area close to his heart. But he overcame his despair, and completed his MSc with flying colors.

Uncertain Entry into Research

Pursuing an academic career had always been a top priority. So, the next course of action was to pursue doctoral research, preferably in some area of theoretical and mathematical physics, an area that has always been close to AKR's heart. With his good MSc grades, AKR felt confident that a research fellowship ought not to pose a problem, even though they were few and far between compared to opportunities available today. He recalled that he had enjoyed Professor Kulesh Kar's lectures in theory in Presidency College, for their thoroughness and clarity, compared to those who had taught him at the University. AKR approached Prof Kar and was soon accepted as an apprentice. But not for very long. Prof Kar was a nice and caring person, and the two got along well. However, a discussion between them on a mathematical issue led to disillusionment and AKR's eventual withdrawal from this apprenticeship. This was regarding the use of Bessel functions in solving a problem in quantum mechanics. Prof Kar was trying to use the asymptotic expansion (i.e., the behaviour at large values of the argument) of the Bessel function, to solve a problem where the value of the function close to the origin was required. Rightly, AKR felt that this was just no-go, untenable. He complained to Prof Kar about this 'illegitimate' mathematical step, and wanted to try something else. But Prof Kar felt that the results that this step yielded were rather attractive, and 'one should not be too finicky about the mathematics while doing physics'. The young AKR was stung by this opacity from a person he revered greatly as a teacher and PhD advisor, and decided to quietly quit.

A research fellowship became available at the Indian Association for the Cultivation of Science (IACS) in Experimental Solid State Physics (nowadays called Condensed Matter Physics). AKR used his Xray-Crystallography elective background from his MSc days to demonstrate his competence, and was selected for the fellowship. But this choice turned out to be a big blunder. The research problem that was given to him was not interesting. The laboratory he was assigned to was not very well-equipped apparently. For any serious work in condensed matter physics, a high degree of vacuum is a necessity. So the young research fellow had to spend his first years trying to produce a decent vacuum in a chamber where various samples were to be processed and tested. This turned out to be an impossible task, not to mention the drudgery it meant for one whose heart lay in theoretical physics. So, more often than not, AKR would escape the boredom of the laboratory and spend hours in the well-equipped library at IACS, immersed deeply in studying various classics on theoretical and mathematical physics. This was where the honing of his immense mathematical skills actually occurred, skills which would hold AKR in good stead in later life. The downside was the research fellowship project - it continued to suffer a chronic lack of progress. Eventually, after slightly longer than four years of inactivity where a good part had been spent chasing an elusive vacuum, the fellowship was terminated in 1949 by the authorities led by the-then Director who was none other than the stern authority from AKR's student days - Prof Meghnad Saha.

This was a period of a 'great depression' - more than four years spent without anything to show in research, the termination of a research fellowship to add insult to injury, what was AKR to do?

Freedom at last, but ...

Fortunately for him, a temporary lecturer's position soon fell vacant at Asuthosh College, thanks to a regular professor in the physics department going on leave. AKR's superlative grades at the University proved adequate to obtain that position with a three to four year tenure. The temporary nature of the job meant that he did not have any of the monetary benefits of a regular academic appointment, but it also meant less responsibilities and more time to follow his own fancies. AKR found that across the city at the University College of Science, Professor Nikhil Ranjan Sen, an illustrious contemporary of Profs Satyendranath Bose and Meghnad Saha, who at the time headed the Department of Applied Mathematics, conducted a periodic seminar on various aspects of general relativity. Now, during his periods of self-study at IACS while his furtive experiment was running aground, AKR had developed a great love for this subject, to the extent that this is what he wanted to do. So he did not lose time to approach Prof Sen and to seek his permission to attend the seminar. The youthful eagerness swayed Prof Sen, and permission was granted. This was like a breath of fresh air from the dreary days at the IACS. It also marked the beginning of AKR's remarkable series of contributions to general relativity.

Listening to the seminars, AKR realized that in one of the topics he could do the mathematics somewhat better, with greater clarity and ease. He approached Prof Sen who was very forthcoming and encouraged AKR to go ahead and complete the task. This was AKR's first foray into research, and youthful vigour and creativity soon led to the final result which received Prof Sen's almost immediate approval. But it received more than that. The very generous Prof Sen offered to write the paper on AKR's behalf, *without any claim of an authorship!* This is almost unthinkable nowadays. The work eventually appeared in the well-known journal of the time, Bulletin of the Calcutta Mathematical Society, in 1951. There is no question that AKR was thrilled with this 'success' after years of frustration. He was spurred on to work on more ambitious and non-trivial issues of general relativity, not content with providing 'also ran' type of mathematical footnotes to extant research, even if publishable in good journals. He wanted to 'fly', because his soaring confidence made him feel he could do something far better than just follow pedestrian ideas. However, this is where Prof Sen did not seem very forthcoming. The illustrious elder scientist perhaps felt that AKR's ideas were a tad bizarre, and would most likely not lead to any substantive result - a conservative attitude typical of a front-ranking senior academic in India. Unlike the first interaction, Prof Sen did not encourage AKR to work on these very 'difficult' core issues of general relativity, which were topics for debate among stalwarts abroad, and hence not 'safe' for green horns trying to enter the field. There was the fear of turning out something ridiculous for the top professionals to laugh about, and this would hurt a novice's future. Sensible advice no doubt, but not for the brash youthful AKR rearing to have a go at these problems. The relation between the two cooled down to the extent that AKR felt that Prof Sen's seminars were no longer useful. He quietly withdrew from them, and was solitary once more. But he had less than a year left of his tenure as temporary lecturer at Asutosh College, so there was pressure to do something to show for himself.

Entering the Professional Fray

The inspiration came from a paper by the founding father of general relativity, Albert Einstein, which had appeared thirteen years ago in 1939. The paper dealt with a nagging enigma in general relativity theory, what was then called the 'Schwarzschild singularity'. Karl Schwarzschild was a German theoretical physicist who, on his deathbed from an incurable infection when still quite young, had turned out the first exact solution of the enormously difficult Einstein equation of general relativity. Such was the simplicity and elegance of this solution, that Einstein was overjoyed when it was communicated to him, because he did not believe that anything similar could even exist. However, the Schwarzschild solution had some disagreeable features, among which is the conundrum called the 'Schwarzschild singularity'. To get an idea of what this is all about, a short background of general relativity is in order.

Aside : Short Excursion in General Relativity : In special relativity, Einstein had established that propagation of light (and indeed all electromagetic waves) necessitated the concept of an amalgamation of space and time into a four dimensional smooth continuum known as *spacetime*. The points of spacetime were in reality *events*, with a time coordinate indicating the time of occurrence and three coordinates indicating the location of the event in space. Just like graphs in space, one could designate events in spacetime through spacetime diagrams (Fig. 2) simplified to one space and one time dimension. The reference axes

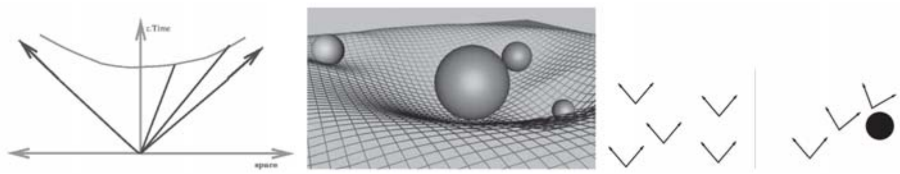

Fig. 2. Space-time diagram

are the directions of light propagation, the time (multiplied here by c, the velocity of light) and space axes are drawn in-between them. On the axes of light propagation, the space and time coordinates are always equal, while for all other events, connected by signals travelling slower than light - the so-called causal events, the value of the space coordinate is always smaller than the time coordinate, indicating that everything else travels slower than light. The hyperbola is the graph of a single event as observed by different inertial observers. The picture shows 'future' events compared to the origin, there is also a 'past' diagram below the *x* axis identical to the one shown, which has not been drawn.

The diagram in the middle shows curvature in space, created by heavy objects placed on a smooth planar sheet under gravity. However, when generalizing special relativity to general relativity, Einstein introduced the notion of *spacetime curvature* which is always a difficult idea to visualize. The diagram on the right is an attempt to do this, using the special relativistic 'light cone' diagrams. In absence of gravity, the spacetime is depicted by identical light cones of special relativity, as shown in the left part of the diagram on the right. However, in presence of a gravitating mass, these light cones undergo a tilt towards the mass, implying that light from any point may now take longer to reach external distant observers, than compared to the 'flat' spacetime of special relativity. This happens because the gravitating mass creates spacetime curvature which makes the light cones tilt. So the tilting of the light cones is a signal of spacetime curvature.

What causes spacetime to curve? The answer to this question is the Einstein equation

$$\text{sptm curvature} = \frac{8\pi G}{c^4}\ (\text{energy-momentum density}) \quad (1)$$

Thus, anything with energy and momentum will cause the spacetime around it to curve, not necessarily mass. Thus very strong laser light in vacuum, which has no mass but certainly possesses energy and momentum, will curve the space around it. This is what gravity means now - spacetime curvature. Any freely moving object, or even light rays, in such a spacetime terrain is now constrained to follow a curved trajectory. Einstein equation has dramatic consequences : the change of the color of light emitted by gravitating objects, the bending of light rays under gravity, and the perihelion shift of elliptic orbits of objects moving around gravitating bodies, like the planets around the sun. However, the equation is notoriously difficult to solve, being a nonlinearly coupled partial differential equation in 10 variables. Einstein himself could only consider simple approximations - the Newtonian limit and the *linearized* gravitational wave solution - such is the degree of difficulty. No wonder he was ecstatic to receive Schwarzschild's exact solution.

Schwarzschild's Solution : The key to the solution is the assumption of spherical symmetry of the spacetime - this cuts down the number of unknown variables drastically, in fact to two functions from ten. The equations themselves also simplify to become exactly solvable for the two unknowns, yielding a most elegant-looking metric which depicts the spacetime. The spacetime is characterized by an inherent length scale as the Schwarzschild radius r_S (Fig. 3) of an abstract *geometri*c sphere which separates two regions of disparate nature. This sphere is called a *horizon*. Outside this surface, the spacetime is *identical* to that outside a mass $M = c^2 r_S / 2G$, where G is Newton's gravitational constant, as shown in the second figure from the left. In the third figure from the left, a strange character of the horizon is depicted : this sphere is like a one-way membrane; it allows particles and light rays to enter the *inside* of the region, but does not allow anything to escape outside from inside. This clearly has to do with the 'black' of the black hole.

The fourth and fifth diagrams from the left depict what used to be called the 'Schwarzschild singularity' the topic of our discussion in this section. What is shown is that the distance of the horizon from any fixed, external observer is actually *infinite* according to the solution. Thus, if anyone or anything is allowed to fall freely into

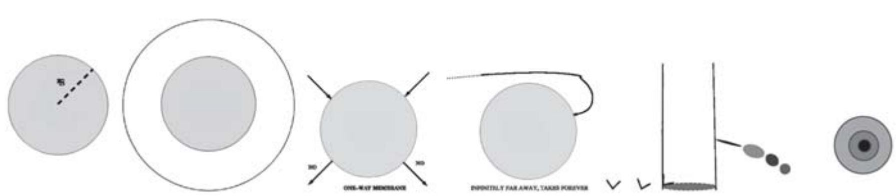

Fig. 3. Schwarzschild spacetime', because the parts of this figure are explained in the text.

the horizon, it will take an infinite time to reach it, getting tidally stretched by the curvature as it nears the horizon, while suffering a change of colour to larger wavelengths (redshift). But it will never be seen to actually fall through the horizon by an external fixed observer. These infinities in time and distance are an anathema to general relativists, they simply do not make sense within the established mathematical framework.

Einstein's 'No-go' Result (1939), and An 'Unreasonable Doubt' : Returning to Einstein's paper which AKR was inspired by in 1952, the essential content was an analysis of the motion of a cluster of particles moving in random concentric circular orbits around the horizon which was thought to suffer from the Schwarzschild 'singularity'. Einstein's claim was to have shown 'beyond any reasonable doubt' that nothing, no particle or light ray, could ever get close to the horizon. This, the Master felt, would render the Schwarzschild singularity physically innocuous. What he did not count on was a young man on the verge of unemployment with an 'unreasonable doubt'.

AKR's solo foray into the professional fray thus started with such an unreasonable doubt of a claim by none other than the founder of general relativity! This is courage beyond measure from a person with an uncertain future, verging perhaps on recklessness. His argument was simple[3] : a contracting spherical cluster of particles would produce a contracting ('cosmological') spacetime with no singularity at any finite radius $R \neq 0$. Now if such a spherical spacetime is cut out and placed in empty space, with $R > r_S$, outside the sphere is a singularity-free Schwarzschild exterior spacetime. According to the Schwarzschild solution, as R decreases to r_S, the so-called Schwarzschild singularity appears. But according to the geometry of the collapsing cluster, no singularity exists for R approaching 0. So there is a clear contradiction with the original Schwarzschild formulation. Thus, AKR concludes '...the Schwarzschild singularity is only a property of some particular coordinate systems and would not appear in other properly chosen coordinate systems'. A proper 'double null' coordinate system in which the metric is smooth across the horizon $r = r_S$ was finally obtained in 1967 by Kruskal and Szekeres, thus confirming AKR's surmise.

But he had done a bit more than that in that paper. In the extreme right figure above, is shown the *interior* Schwarzschild spacetime, i.e., the region *inside* the horizon. This, unlike the static spacetime outside, is violently time dependent, and continuously contracting. This, AKR felt is exactly like the cosmological evolution of a spatially isotropic and homogeneous spacetime, running in reverse, i.e., from expansion to contraction. The final state of the contraction is clearly analogous to the so-called Big Bang - a region of spacetime which has infinite curvature, and hence according to Einstein's equation, infinite energy-momentum density. This, AKR concluded, is a real *physical* singularity, which, just like the Big Bang, defied understanding. Just like one cannot say anything about spacetime 'before' the Big Bang, one cannot say anything about spacetime 'after' the Schwarzschild spacetime shrinks to zero volume, when the sphere radius shrinks to zero. This is the real conundrum of the Schwarzschild spacetime, since it violates one of the basic premises of general relativity - that of spacetime being a *smooth* continuum. In doing so, a spacetime singularity *invalidates* all laws of physics, because all of them involve the assumption of a smooth continuum of spacetime, and would dissolve where this continuum itself ceases to exist.

Suspicions were raised that the heightened symmetry of the spacetime assumed by Schwarzschild (spherical symmetry) may be the reason for the appearance of the geometrical and physical pathologies at the singularity alluded to in the previous paragraph. Similarly, the isotropy and homogeniety of cosmological spacetime, embodied in the very popular cosmological formulation of Friedmann, Lemaitre, Robertson and Walker, could be responsible for the Big Bang singularity. Perhaps spacetimes with less symmetry might evade this dreadful fate. This was a serious charge that demanded a totally new approach to the problem, since finding solutions to Einstein's equations without assumptions of symmetries was an impossible task. What was one to do?

The Raychaudhuri Equation

In his own words[4], AKR's aim was 'an attempt ...to study the temporal behavior of a gravitating cloud on the basis of the Einstein gravitational equations under very general conditions' (i.e., without assuming any symmetries of the spacetime). The method was[4] '...to investigate the temporal behavior of a gravitating system as observed by a member of the system itself in its neighborhood'. In

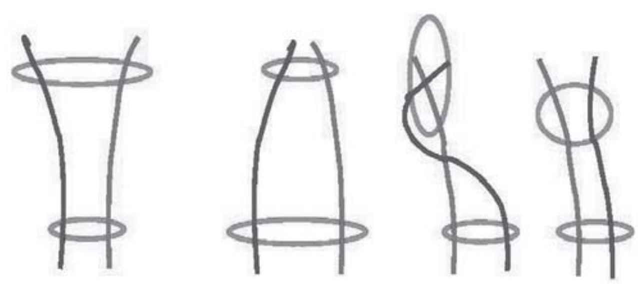

Fig. 4. Geodesic congruences : expansion, contraction, spin, shear.

other words, study the relative convergence or divergence of a bunch of geodesics of particles with mass moving in an arbitrary curved spacetime.

What could these 'timelike' geodesics do relative to each other, as they evolved in time? They could either converge towards each other, or diverge away from each other. They could *rotate or spin* around one another. Or, like fluid particles, there is the possibility of some moving faster relative to others, known in fluid mechanics as shear.

These four situations are crudely depicted in the figure 4, the leftmost diagram showing divergence of geodesics with the cross-sectional area increasing with time, the second from left showing the reverse - convergence of the geodesics with the area decreasing, the third from the left depicting rotation of geodesics around each other, with the cross-section rotating to capture this effect while the rightmost showing a deformation of the cross-section because of shear. The great novelty in the derivation of the Raychaudhuri equation is the discovery of a relation between the rate of expansion (with time) to the expansion itself, the shear, the rotation and the spacetime curvature at each instant of the evolution:

$$\textit{Rate of expansion} = -(\textit{expansion})^2 - (\textit{shear})^2 - \textit{curvature} + (\textit{spin})^2$$

This is a relation in (pseudo)-Riemannian geometry, and does not yet involve the use of the Einstein equation of general relativity. Its applicability thus *goes beyond* general relativity, to any curved space wherever geodesics evolve with respect to their intrinsic (affine) parameter.

Within general relativity, the curvature is replaced, via Einstein's equaion, by the energy-momentum density of the geodesics, and this is in general positive. Thus, the equation above has three negative terms on the right hand side, and only one term - the spin term, has a positive coefficient. This has deep implications for the expansion, if it starts off as negative (contraction), and the spin term is small or absent, it ends up being negative. In other words, an initially converging geodesic congruence ends up being convergent. Such a convergence of geodesics forms a *caustic* in spacetime. Now caustics in space are fairly common in geometrical optics, and despite being cusp singularities, diffractive effects of wave optics smoothen out the singularity. There is also the possibility of cutting out the portion of space in which the caustic resides, an operation called excision. However, for timelike geodesics, the formation of a caustic in spacetime implies that the evolution of the geodesics is no longer unambiguous, because at the caustic, no unique time exists for future evolution. As AKR was able to show, for the Schwarzschild solution, as also for the Big Bang in homogeneous and isotropic cosmology, the spacetime caustic formed by converging geodesics is the harbinger of 'doomsday', because in these particular cases physical curvature singularities are inevitable. However, in general, there is no one-to-one correspondence between appearance of physical curvature singularities and those of spacetime caustics[7]. For AKR, a physical singularity was always the blowing up of spacetime curvature, and he realized that in the case of the Schwarzschild spacetime and the cosmological Big Bang singularity, the formation of caustics in a geodesic congruence is a unique signal. His conclusion in his celebrated paper[4] was that, apart from spin, the path to spacetime singularity formation was *perhaps* inevitable in general relativity, even when no symmetries are assumed.

The original Raychuadhuri equation involved the focusing of *timelike* geodesics. M. Sachs generalized the equation for *lightlike* or null geodesics in 1961, and an independent proof of the same appeared in Penrose's 1965 paper, cited recently by the Nobel Committee. A comprehensive review, on technical issues of the Raychaudhuri equation, appears in ref.[5].

Equipped with his Equation, why not Prove Singularity Theorems?

It became clear to AKR, soon after his discovery of the Raychaudhuri equation, that he has created for himself a powerful tool to address general conceptual issues pertaining to spacetime singularities, without any assumptions about the symmetries of those spacetimes. Should he not have used this equation, as several famous physicists did more than a decade later, to derive *theorems* about spacetime singularities in general?

There are several reasons that one can garner in favour of AKR for not having chosen the path to proving singularity theorems. First of all, AKR was not convinced and correctly so, that caustic formation for a converging congruence of geodesics, inevitably implied curvature singularities. Unlike what Penrose[6], Hawking[7] and a host of others did a decade later, AKR did not wish to change the original notion of singularities in terms of unbounded growth of spacetime curvature, to *geodesic incompleteness* which was the idea that spacetime caustics would form in a geodesic congruence. Secondly, AKR was motivated by physics issues of relativistic cosmology, and not by mathematical theorems. For him, the Raychaudhuri equation was a tool to explore possible existence of cosmological spacetimes which are singularity-free. This was his primary professional aim after his great discovery, and not to merely point out the existence of a whole class of spacetimes which did not make any geometrical or physical sense.

But in all fairness to later mathematical relativists focused on proving singularity theorems, it is also true that the application of the mathematical apparatus of differential *topology* pioneered by Penrose in 1965 and perfected by him and later physicists, was not known to AKR a decade earlier. While he was quite familiar with the 19th century mathematical techniques of *local analysis* (calculus) on curved spaces, introduced by Levi Civita and Ricci, global methods of topology which Penrose brought into general relativity later were beyond AKR's mathematical repertoire at that time. So, the task of constructing mathematical proofs of existence of spacetime singularities, with his rigid adherence to the old definition of singularity, would have proved a very daunting task for AKR to pursue in the late 1950s, not to mention the dreary 'desert' of a lack of physics motivation.

Did he regret missing this opportunity of great professional accomplishment and accompanying fame, later in life, after the work of Penrose, Hawking, Geroch and others stole the limelight, and sealed the fate of general relativity, so to speak? Perhaps not. In his very last paper which appeared in 2004, less than a year before his demise, he quotes Earman[13], in suggesting that modifying the definition of a spacetime singularity to geodesic incompleteness was 'a piece of opportunism as it allows many theorems to be proved'. Not surprisingly, in this very last paper too, AKR categorically states that for him the definition of a singularity is always the blowing up of spacetime curvature. Rather than interpreting this as an obscurantist rigidity brought perhaps by senility, as some might do, I think it shows a fiercely *independent* theoretical physicist who is not swayed by extant notions and fashions, even if these were started by far more 'famous' physicists than himself. This uncompromising quest for excellence *on his own terms* is the hallmark of true greatness. Afterall, singularity theorems are a *tour de force* in the structure of general relativity as a theory of spacetime geometry, because they delineate the *limitations* of the entire approach. But these great theorems do *not* give a clue as to what might replace general relativity, or how its conundra can be ameliorated. Perhaps this negative aspect displeased AKR to quite an extent.

Aftermath : After the Discovery

There are adequate accounts of AKR's tribulations in building a professional career in Kolkata, even after his momentous discovery[8,9]. After a second stint at the Indian Association, during which AKR was in a sense compelled to work on condensed matter physics, and he did that with great elan, in 1961 he quit the IACS to join his alma mater - Presidency College - as a full professor. This marked the beginning of a legendary teaching career, spanning more than twenty years, which trained and inspired most of the high-calibre physicists from West Bengal. Not being among those fortunate to have been taught by AKR, I have little to add to the very good extant accounts of AKR's teaching excellence, except to add that perhaps in teaching too, AKR followed his own instincts closely. As far as I know, not everybody in class benifitted from his lectures, because AKR usually did not follow any standard textbook or other source too closely. He was maximally open to questions and discussions, within and outside the class, but would not go out of his way to make things easier for everybody, than what they truly are. To paraphrase Einstein, he would 'make things simple, but not simpler'. Students who wanted to get the most, needed to take some initiative which he appreciated very much. But this clearly would not please all who attended his lectures, or could not handle the slightly non-trivial problems which he was fond of assigning. But even the not-so-good students in class appreciated his passion for fundamental physics, and his great integrity in communicating it. He was a pure academic, who took his teaching very seriously, but his administrative duties received slightly attenuated attention, and there was no compunction in his mind for this asymmetry.

But unlike many for whom the beginning of a teaching career signals an end to research, AKR continued to pursue research at his highest levels, throughout his life. Many years were spent in an unsuccessful search for singularity-free universes, using the role of the spin term in the Raychaudhuri equation to its maximum possible benefit, but this turned out to be inadequate. Nature did not oblige him. Generalization from Einstein's general relativity to Einstein-Cartan theories which included torsion of spacetime as an additional geometrical attribute, over and above curvature, also did not fit the bill. These are not mere mathematical exercises; in each of these works, the evidence of a masterly perspective and technical competence is ample. Towards the end of his days, AKR was inspired by the work of Senovilla and collaborators in Valencia, Spain,[10] who had derived a class of singularity-free spacetimes based on some assumptions. In a Physical Review Letters paper[11], AKR established that the spacetime *average* of curvature scalars actually vanished for such spacetimes. However, since the average of the matter energy density also vanishes by assumption in these spacetimes, these spacetimes are not realistic, since our universe certainly has matter in it. As a continuation, in his very last paper[12], AKR established very generally that the class of cosmological spacetimes identified by Senovilla is the *only* singularity-free class of cosmological spacetimes possible. There, however, appears this note of disappointment from a physical perspective : 'Application to the real universe, where observations seem to rule out such an empty universe, suggests that the hope of a reasonable realistic singularity-free cosmological model has to be abandoned.' The quest came to an end, but uncovering a wealth of knowledge and unanswered questions enroute, for future generations of relativity theorists to grapple with.

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