DOI: https://doi.org/10.36094/sc.v86.2020.N_R_Sen_and_his_Contribution.Singh_Roy.363

N.R. SEN AND HIS CONTRIBUTION TO TURBULENCE RESEARCH

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Nikhil Ranjan Sen (also written as Nikhilranjan Sen), the founder of the Calcutta School of Theory of Relativity, is well-known for his contribution to the theory of relativity and stellar structure. Going by the chronological order of his publications, it appears that he became interested in the research on turbulence quite late in his life. As a result, his research work in the field of fluid dynamics, in particular, theory of turbulence is less known. This area of research of N.R. Sen will be explored in the present communication.

Introduction

Turbulence is a common experience which we face during airplane flights, cyclone, tsunami etc. It also occurs in the case of moving cars, ships, submarines, gas ejected from chimney and swimming animals. Their study is not trivial. Nikhil Ranjan Sen (1894-1963) was one of the pioneers of this field in India.

To the best of our knowledge, very few articles give a scant impression of his life and science.[1] George Keith Batchelor in his well known book 'The Theory of Homogeneous Turbulence' has referred Sen's article.[2] However, there is not much literature, which explores his contribution to fluid dynamics. The present article intends to fill this gap.

To start with a short biography of Nikhilranjan Sen is presented below.

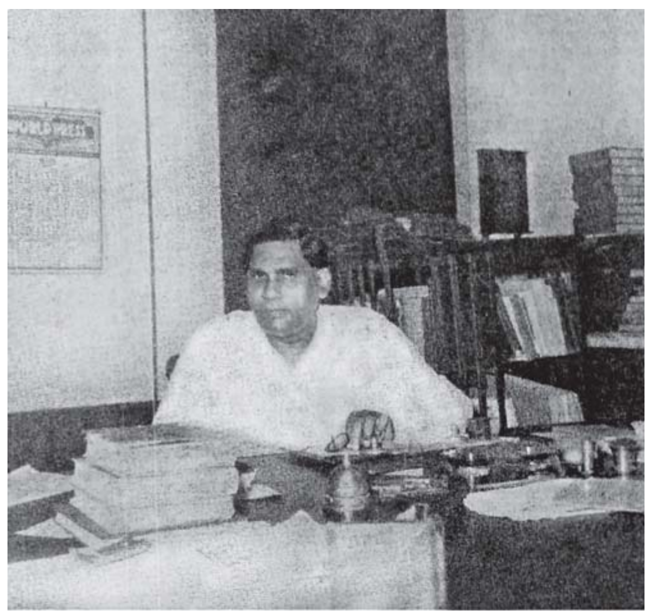

Figure 1: N.R. Sen. Credit: "Science and Culture".

Nikhil Ranjan Sen – A short biography

Nikhil Ranjan Sen (NRS) was born on May 23, 1894 in Dhaka (now in Bangladesh) of undivided India. NRS was the youngest child of his parents Kalimohan Sen and Bidhumukhi Devi who had four sons and four daughters. NRS started his primary education in a school in Dhaka where Meghnad Saha was his fellow student. NRS, however, left Dhaka and moved to Rajshahi (also in Bangladesh) to complete his school education at Rajshahi Collegiate School. In 1909, he passed the Entrance Examination of Calcutta University from Rajshahi College securing third position in order of merit. After passing Intermediate Examination in 1911, he joined Presidency

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College in Calcutta (now Kolkata) and did B.Sc. and M.Sc., in 1913 and 1916, respectively. In Presidency College, he found Meghnad Saha and Satyendranath Bose in his class.

In Sept. 1917, NRS joined the department of applied mathematics as a lecturer, in the newly founded University College of Science, University of Calcutta. His contemporaries were Meghnad Saha and Satyendranath Bose who joined Calcutta University almost around the same time. Saha joined in the department of Applied Mathematics and Bose joined in the department of Physics. After a brief stint Meghnad Saha joined the Physics department while Nikhil Ranjan continued in the department of Applied Mathematics till his retirement.

Nikhil Ranjan Sen started his research career in the newly formed department and in recognition of his remarkable research work, he was awarded the Doctorate (D.Sc.) degree by the Calcutta University in the year 1921.[3] Immediately thereafter, he proceeded to Germany and his investigations in the General Theory of Relativity under Prof. Max von Laue earned him the Ph.D. degree of the Berlin University in 1923.[4] He returned to India in 1924 to resume his teaching at the Calcutta University as Ghosh Professor of Applied Mathematics.

Nikhil Ranjan was an active scientist and was intimately associated with different scientific organizations and institutions. He was associated with Calcutta Mathematical Society established by Sir Asutosh Mukherjee, from the beginning of his research career. He served the society in different responsible positions like Council member, Vice-President and President. He was a founder member of the Indian Statistical Institute (ISI) Council; he was a founder member of the council and first Treasurer of the Indian Science News Association (ISNA) established by Acharya Prafulla Chandra Ray and Meghnad Saha. He was a founder Fellow of the National Institute of Sciences of India, Member of the Council and Vice President of INSA (1959-1960). NRS was the President of the Mathematics Section, Indian Science Congress Association (1936), Member of Mathematics and Statistics Sectional Committee, ISCA. He was a Life Member of the Indian Association for the Cultivation of Science (IACS).[5] In a meeting on July 28, 1952, he was elected as a Member of the Council of IACS.[6] Also, he was appointed as Rippon Professor for the year 1951.[7,8] He delivered lectures, which were published as a monograph "The modern theory of turbulence"[9] by Indian Association for the Cultivation of Science. That was the only monograph on theory of turbulence by Indian researchers working in the field since late eighties of last decade and among the very few worldwide.

In 1936, NRS made an improvement in his department of Calcutta University by the addition of a mathematical laboratory: Computational Laboratory and Fluid Dynamics Laboratory.[10] His memoir published by INSA reads :

> "The computation laboratory and also the Hydrodynamic laboratory in the Department of Applied Mathematics of Calcutta University are monuments of his constructive vision. Considering his notable contribution in different fields of Applied Mathematics perhaps it will be no exaggeration to say that Prof. Sen is the 'Father of Applied Mathematics in this country.' "[3]

Professor Nikhil Ranjan Sen did research over a wide range of subjects and topics. He did pioneering work in the fields of turbulence and cosmology. Apart from his own brilliant research contributions, he introduced new subjects in the Post-Graduate curriculum and inspired his young colleagues and research associates to take up original and challenging problems and solve them in modern areas like Relativity, Astrophysics, Quantum Mechanics, Geophysics, Statistical Mechanics, Fluid Dynamics, Magneto-Hydrodynamics, Elasticity and Ballistics. He was the fountainhead of inspiration to research workers in Calcutta University and under his dynamic leadership the Department of Applied Mathematics became a vibrant centre of teaching and research and earned a reputation throughout the country, which would be hard to match.[11]

It is said that the first Fluid Dynamics Laboratory (FDL) in India was established by NRS. This laboratory could be turned into a full-fledged N.R. Sen Institute of Fluid dynamics if sincere and adequate attention were given to the research that NRS started. Multiple stories were built over the laboratory and as a mark of respect to NRS, the building has been named as 'N.R. Sen Building' (Figure 2). The laboratory was once used as a godown for storing building materials during the construction of the building.[12] It is most unfortunate that FDL, which could be a pride to Calcutta, has been turned into a relic of the historical past standing as a witness to a laboratory which is totally shutdown now (Figure 1).

Figure 1: Left: N.R. Sen Fluid Dynamics Laboratory. Right: N.R. Sen Building built over the laboratory. Credit: Mrs. Rinku Debnath, Indian Science News Association, Kolkata.

Nikhil Ranjan Sen's Work on Hydrodynamics:

N.R. Sen's work on hydrodynamics can be divided into two phases :

1. Initial work done in the 1920s in which he scantly touched upon the topic.
2. About two decades later, he extensively worked on the theory of turbulence and wrote a monograph.

Phase I - Theory of seiches and N.R. Sen's Contribution

NRS as a young scientist taught Theory of Elasticity, Hydrodynamics and Solid Geometry at Calcutta University (Figure 2).[13] During teaching, he was greatly interested in unsolved problems. One such example is discussed in his paper "On the equation of long waves in canals of varying sections."[14] In the paper he referred to "Lamb, 'Hydrodynamics', 3rd edition, p. 259", "Green's Mathematical Papers, I), 225", and two more papers from the years 1904 and 1914.

Historically, Nicolas Fatiode Duillier (1664-1753) of Switzerland, systematically studied seiches of the Lake Léman, Geneva. According to 'Britannica", seiche is a rythmic oscillation of water in a lake or a partially enclosed coastal inlet such as bay, gulf or harbor. A seiche may last for a few minutes to as long as two days.

He found that owing to the peculiar configuration of the end of the lake, the variation in waves occasionally reached a magnitude of 5 to 6 feet.[16] In the first decade of the 20th century, various aspects of seiches were experimentally studied by some of the British scientists.[17] For instance, G. Chrystal gave a theory to explain seiches for a lake, whose depth, cross section and surface breadth did not vary rapidly from point to point.[18] His countryman, Albert E. Green (1912-1999) had shown that for the progressive waves in a canal with slowly varying sections, the elevation of water was inversely proportional to the square root of the breadth and to the fourth root of the average depth at the section.[15] In 1924, NRS studied propagation of long waves in canals of varying sections. Since it is well known that breadth and average depth of a section play significant role in the formation of seiches, NRS limited his investigations to "certain types of solutions of the general equation with suitable values for the two arbitrary functions which correspond to the breadth and depth at any section of the canal"; and obtained some simple formulae "which automatically adjust the two arbitrary functions in such a way as to lead directly to the solutions under consideration."

NRS came back to hydrodynamics again at the beginning of the 1950s (details below).

Phase II - Sen's contribution to the Modern Theory of Turbulence

Credit is given to Leonhard Euler (1707-1783) for mathematically describing the flow of fluids. He applied the law of conservation of mass and Newton's second law of motion to arrive at two nonlinear partial differential

MIXED MATHEMATICS

1. Advanced Statics, including Theory of Potential—
   Minchin ... ... Statics
   Routh ... ... Statics
2. Dynamics of a Particle—
   Besant ... ... Dynamics
   Routh ... ... Dynamics of a Particle
3. Rigid Dynamics—
   Routh ... ... Elementary Rigid Dynamics
4. Hydrostatics, including Capillarity—
   Besant and Ramsey ... Hydromechanics, Part I (Hydrostatics)
5. Hydrodynamics—
   Houstoun ... ... Mathematical Physics, Chapter II
   Besant and Ramsey ... Hydromechanics, Part II
   Lamb ... ... Hydrodynamics
6. Spherical Astronomy—
   Ball ... ... Spherical Astronomy
7. (a) Theory of Elasticity—
   Love ... ... Theory of Elasticity, 2nd edition

Figure 2: Subjects taught in Post-Graduate classes of "Mixed Mathematics" (Applied Mathematics).[15] Credit: University of Calcutta.

equations. By including viscosity, Claude-Louis Navier (in 1822) and George Stokes (in 1845) improved upon the old theories and formulated equations, which we know today as the Navier-Stokes equations.[19] According to old mathematical theory of turbulence, a liquid flowing between two walls, the laminar flow is always streamed. As a consequence, the observed turbulence has a finite amplitude.

In 1921, Ludwig Prandtl (1875-1953) of Germany, performed experiments with a water channel. He made the turbulence visible with lycopodium spores and photographed it. From his experiments, he came to the conclusion that the previous mathematical theories were incorrect since the observed turbulences were not laminar.[20]

In 1925, L. Prandlt presented 'momentum transfer'or 'mixing length' theory, according to which, in the case of turbulent motion, masses of liquids move out of the path of mean motion through some distance conserving their momenta for a while. He named the mean distance, for which the momentum remained conserved, as the mixing length.[21] As the "mixing length" was based on several assumptions, the theory had limited success.

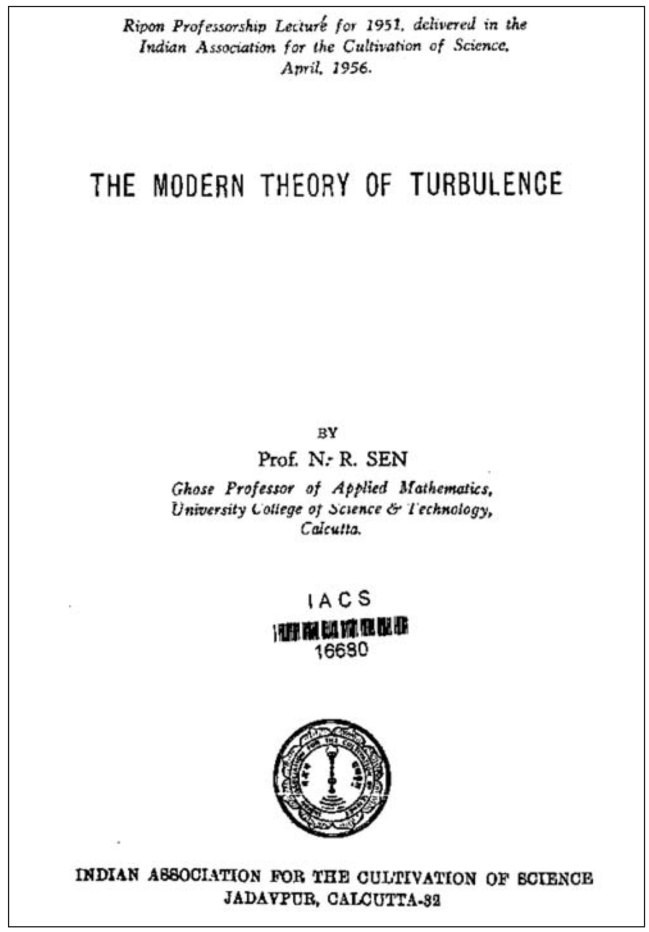
According to the annual report of the IACS for the year 1954-55, NRS was appointed as the Rippon Professor for the year 1951. He delivered three lectures, which were published in the form of a booklet under the title "The modern theory of turbulence" (Figure 3). Sen's motivation to choose this topic was partly due to his own interest in the subject and also from the conviction that "the subject of Turbulence would be playing a very important role in the development of many branches of science in future"[22], wrote Sen.

Sen's conviction was reflected when at starting of 21st century international scientific community have declared study of turbulence as a thrust area of research. Perhaps it is worthwhile to note from N.C. Ghosh[11], that turbulence is one of the oldest and most difficult open problems in physics. In 1922, Lewis F. Richardson in his book "Weather prediction by numerical process" wrote:

> "Thus C.K.M. Douglas writing on observations from aeroplanes remarks: 'The upward currents of large cumuli give rise to much turbulence within, below, and around the clouds, and the structure of the clouds is often very complex.' One gets a similar impression when making drawing of a rising cumulus from a fixed point; the details change before the sketch can be completed. We realize thus: big whirls have little whirls that feed on their velocity, and little whirls have lesser whirls and so on to viscosity – in the molecular sense."[23]

Perhaps considering complexities of analysing turbulence one scientist wrote: If there is a god and I can meet him, I will ask him something about willy-lilly behaviours of turbulent phenomena.

> "The story is told of many giants of modern physics, but most plausibly of Heisenberg, on his death-bed, he remarked that the two great unsolved problems were reconciling quantum mechanics and general relativity, and turbulence. 'Now, I'm optimistic about gravity...'"[24]


Ripon Professorship Lecture for 1951, delivered in the Indian Association for the Cultivation of Science, April, 1956.

THE MODERN THEORY OF TURBULENCE

BY

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IACS

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Figure 3: Title page of N.R. Sen's book. Credit: IACS.

In his first lecture, NRS began with the recapitulation of the history of the subject and started with Osborne Reynolds (1842-1912), who had defined turbulent motion as a simple superposition of mean and fluctuating motions.[25] NRS stated that in hydrodynamics, streamline motions of fluid is governed by the Navier-Stokes equations. The Reynolds number, which characterizes the types of motion, is a function of velocity, length, density, coefficient of viscosity and the kinematic viscosity of the fluid. For low Reynolds number the motion is laminar. At the end of his first lecture, NRS concluded that there was another statistical theory of turbulence in which the fluctuating velocities were not even considered to be continuous, and they need not satisfy the equations of motion.

Other lectures were devoted exclusively to the statistical theory of turbulence which was put forth by Geoffrey Ingram Taylor(1886-1975) and extended by others, such as Werner Heisenberg (1901-1976), Subramanyam Chandrasekhar (1910-1995), Theodore von Kármán (1881-1963) and Leslie Howarth (1911-2001) etc. (details below).

In Chapter II, NRS wrote on different ways to construct mean values of entities like velocity, pressure and density, which affect the fluctuating parts in turbulent motion.[26] He began with isotropic and homogeneous turbulences. He stated that the turbulence in windtunnel behind the grid, though not exactly homogeneous, could be taken to be statistically homogeneous to a great extent.

In 1932, Geoffrey Ingram Taylor, U.K., parallel to L. Prandlt's idea, suggested a theory in which the vorticity of the turbulent fluid elements was considered conserved in the course of their motion.[27] According to NRS, G.I. Taylor's diffusion or vorticity transfer theory could explain some observations in engineering and meteorology. However, it had limited success. The main problem was Taylor's assumption of the "bodies of fluid", which had no experimental justification.

In 1935, G.I. Taylor wrote on the "Statistical theory of turbulence".[28] In order to facilitate the study of fundamental properties of turbulent flows, he introduced an idealized concept of isotropic turbulence, i.e. he proposed a suitable scale of turbulence which would be an experimentally determined quantity. According to it, the turbulent fluctuations were statistically uniform in all directions, such as the turbulent flow far away from boundaries. He applied the concept of isotropic turbulence to the problem of the decay of turbulence in a wind-stream. He also showed the way to measure the scale of turbulence by hot wire anemometers. NRS in his lecture extensively talked on these measurements.

G.I. Taylor's idea was extended by Theodore von Karman and Leslie Howarth. They gave a general theory of isotropic turbulence. They wrote that the mean products of the derivatives of the velocity fluctuations could be expressed by the derivatives of the tensor components. They discussed the correlation between three components, i.e. triple correlation. They developed the kinematics of isotropic and also considered the dynamical problem of variation of the different mean values with time. They proved that by using the equations of motion, a partial differential equation connecting the double and triple correlation functions could be established. They also deduced equations for the dissipation of energy and velocity.[29] At the end of their article, they gave a possible solution for large Reynolds number and applied it to Taylor's problem of the decay of turbulence behind a grid.[30] To sum up, the authors showed that "the mean values of the products of two and three components of the velocity fluctuations at two points in the turbulent field could each be specified by a single scalar function of the distance between the points."[31]

The statistical theory satisfactorily explained the spectral distribution of energy among the turbulent eddies. Andrei Nikolaevich Kolmogorov[32] (1903–1987), Lars Onsager[33] and Carl F. von Weizsaecker (1912-2007) introduced a geometric similarity of fluid flow boundaries hypothesis to determine the spectrum of eddies with large Reynolds number. The latter (C.F. von Weizsäcker) mathematically dealt with turbulence with the Fourier-decomposition method. He derived the spectrum of turbulent motion up to the smallest wavelength, i.e.in the laminar range. He calculated the mean pressure-variation and the correlation function. He derived the constant characteristic of energy dissipation in statistical turbulent motion from the hydrodynamic equations.[34]

In 1948, W. Heisenberg was of the opinion that distribution of energy in the large eddies is not statistical but a geometrical problem. He pointed out the importance of viscosity which leads to the rotational motions near the walls, damps all motions in the small eddies and enables laminar motion. He saw turbulence as a statistical mechanical problem with a very large number of degrees of freedom. "Just as in Maxwell theory this problem can be solved without going into any details of the mechanical motion, so it can be solved here by simple considerations of similarity", wrote Heisenberg.[35] He developed an elementary theory for determining the spectrum of turbulent medium. He considered the rate of dissipation of energy by eddies with wave numbers less than a particular k (k = wavenumber = spatial angular frequency). He distinguished between the energy directly dissipated in the form of molecular motion and thermal energy and the energy communicated in the form of kinetic energy to all eddies with wave numbers exceeding the specified wavenumber.[36]

Based on Heisenberg's theory, S. Chandrasekhar discussed the spectrum of turbulence. He obtained explicit solutions for the spectrum, when the conditions were stationary, non-stationary, and the turbulence was decaying.[37] S. Chandrasekhar wrote "In the former case the problem admits of an explicit solution. In the latter case the problem reduces to determining a one-parametric family of solutions of a certain second-order differential equation."

From the U.K., George Keith Batchelor (1920-2000) stated that in the decay of isotropic turbulence, big eddies play an important role in the motion. However, "there is a range of eddy sizes which, during the initial period of decay, contains a negligible proportion of the total energy and is excluded from the similarity possessed by the smaller eddies."[38] He examined the motion associated with the small range of large wave-lengths in the more general case of homogeneous turbulence and found:

> "The biggest eddies of the turbulence are therefore permanent, being determined wholly by the initial conditions, and are dominant in the final period when the smaller eddies have decayed. The action of smaller eddies on the invariant big eddies is equivalent to that of a turbulent viscosity, the value of which may vary with direction."

T. van Karman's student, Chia-Chiao Lin (1916-2013), a Chinese-born American, applied Kolmogoroff's concept of turbulence at high Reynolds number and observed that for existence of the local similarity considered by Kolmogoroff and others is fulfilled only for extremely small eddies at ordinary Reynolds numbers. His results were reasonably in agreement with experimental data.[39]

In 1951, R.W. Stewart and A.A. Townsend, U.K., measured the double and triple velocity correlation functions and the energy function for the uniform mean flow behind turbulence-producing grids shapes at mesh.With wind-tunnel experiments, they proved the validity of the theories, which postulatedthe similarity or self-preservation decaying fields of isotropic turbulence.[40]

Nikhil Ranjan Sen stated that G.I. Taylor had shown that the important properties of isotropic turbulence could be studied by a spectrum function, which represent the distribution of turbulent energy among the wave numbers corresponding to the fluctuations of velocity in turbulence. From the equations of mechanics, unsuccessful efforts had been made to determine the empirical forms of the spectrum function. W. Heisenberg suggested a form which satisfies a general equation of decay. NRS gave a more general type of homologous solution than that given by Heisenberg (details below).[41]

According to Heisenberg, the asymptotic behaviour of a function[42]:

$$f(x) \sim x^{-5/3} \qquad x \to 0$$

G.K. Bachelor had shown that the asymptotic behaviour of the function at the end of the spectrum should be:

$$f(x) \sim x^{4} \qquad x \to 0$$

C.C. Lin was of the opinion that the above two results were not in agreement due to the failure of the similarity property at low frequency part of the spectrum at the initial stage of decay of turbulence. NRS resolved this limitation. He found that if the Reynolds number is large, then in Heisenberg's equation, the kinematic viscosity (v) of the turbulence fluid can be neglected. In that case Heisenberg's solution of the decay equation for v=0 is not the only possible solution. Sen gave an equation to represent the early state of decay of turbulence which had multiple solutions. For his equation, he tried the following solution:

$$f(x) \sim A\, x^{n}, \text{ where A is a constant} \qquad x \to 0.$$

This leads to asymptotic behaviour when n is replaced by (2-3c)/c.

$$f(x) \sim \text{const. } x^{(2-3c)/c} \qquad x \to 0,$$

where c is a constant having value <2/3, $\quad x \to 0$

$$f(x) \sim x^{-5/3} \qquad x \to \infty$$

Sen proved that for c=2/7, the fourth power law of Batechlor-Lin and Kolmogoroff spectrumcan be obtained:

$$f(x) \sim x^{4} \qquad x \to 0$$

$$f(x) \sim x^{-5/3} \qquad x \to \infty$$

In NRS' equation, if c=1/2, it gives Heisenberg-spectrum. In this equation, he showed that for c >1/2 and for c = 2/7, solutions are in general unstable. As the value approaches ½, the motion becomes steadier. "So that the number c = 1/2 (Heisenberg-Chandrasekhar solution) represents the most stable member of the family to which possibly all previous labile motions converge."[3]

Based on this knowledge, NRS explained the "velocity correlations in homogeneous turbulence"in his monograph. He stated that in the case of homogeneous turbulence, the average statistical properties are invariant under arbitrary translations of the coordinate. He calculated velocity functions and compared them with experimental results (Figure 4, Figure 5).

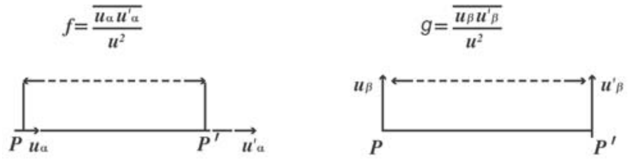

Figure 4: The scalars f and g.$u_\alpha$and $u_\beta$are the components of the velocities *u* and *u'* at point P and P' respectively. The two functions f and g are connected so that the second order correlations for homogeneous and isotropic turbulence depend on only one scalar function. Credit: IACS.

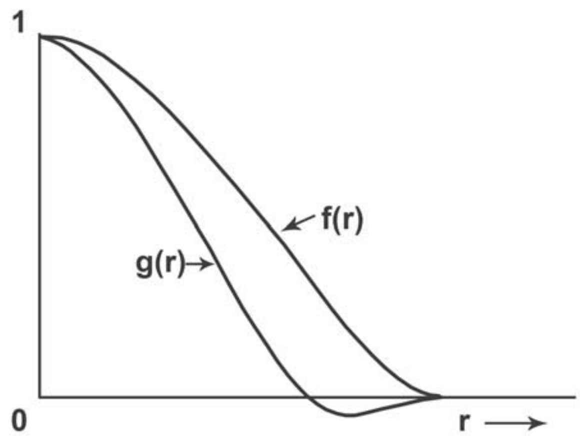

Figure 5: Measurements of f and g in the wind-tunnel. Credit: IACS.

NRS stated that the same principle could be applied to calculate the correlation between pressure at a point and velocity at any other point in an isotropic turbulent field. Consequently, in isotropic turbulence the static pressure at a point has no correlation with the velocity component at any other point.

In his lecture, NRS described the state of the art of the decay of isotropic turbulence in experimental and theoretical contexts and "Self-preserving solution of the Karman-Howarth equation" (Figure 6).

To sum up, in Chapter III, NRS discussed "one and three dimensional spectra of turbulence", "Influence of pressure and inertia forces on energy spectrum", "The spectrum function", "Decay of Isotropic Turbulence", "Solution of Heisenberg's Spectrum function equation" and "Character of the Energy Spectrum function for isotropic turbulence."[43]

After writing the above booklet, NRS continued research in this field. In 1957, he stated that six years ago, it was shown by him that during the earlier part of decay motion, the inertia terms of the equations of motion dominate the decay process.[44] Now, he has examined the family of self-preserving solutions of Heisenberg's equation for decay of isotropic turbulence. He found that the stability of the spectrum was associated with the fourth power law for small wavenumber; the solution of Heisenberg and Chandrasekhar represents "the most stable member of the family to which possibly all previous labile motions converge."

In 1955, Sen's student Kshetra Mohan Ghosh wrote on "Numerical solutions to find out spectrum function of isotropic turbulence with a fourth power law fitting at small eddy number" (Figure 7).[45]

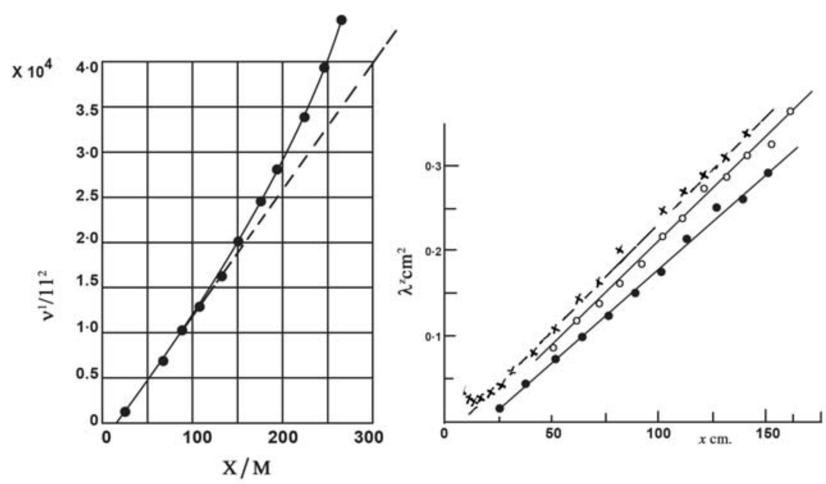

Figure 6: Left: Decay of isotropic turbulence. X-axis distance from the grid, and y-axis - Measured linearity. Right: Self-preserving solution of the Karman-Howarth equation, i.e. for which curves do not undergo a change of shape with time; the only change is a change of scale. The motion remains similar to itself. »- smallest eddies in the turbulent motion vs. distance x. Credit: IACS.

In another article K.M. Ghosh proved that the form of the spectrum "given by Sen is also associated with a more general decay equation"as suggested by Theodore von Karman.[46] In 1960, K.M. Ghosh wrote his D.Phil. thesis on "Some problems of statistical theory of turbulence".[47]

From the foregoing we see that Sen's papers from the year 1951 can be considered as a landmark achievement in the field of turbulence theory.

U.R. Burman, one of his associates called NRS "the leader of a team of workers investigating problems on the boundary layer, wave resistance, isotropic turbulence and shock waves."[3]

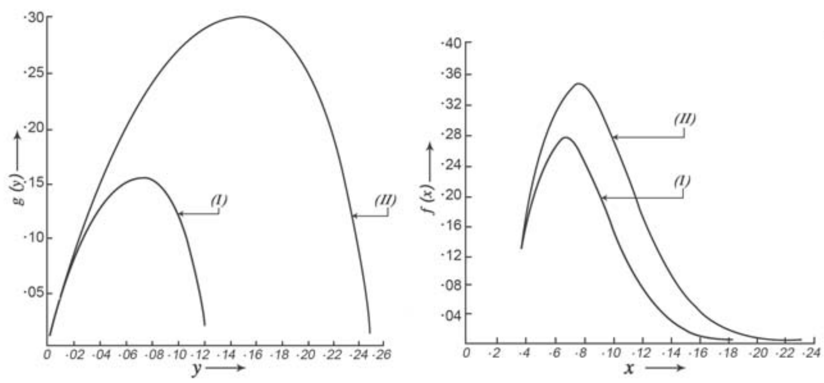

Figure 8: Left: Curves (I) and Curve (II) correspond to values given in Table 1 and Table 2, as given by K.M. Ghosh in his paper. The solution curves in a g-y plane. g_{max}(15711) at y =0.07.

Right: The decay spectra corresponding to curves in g-y plane.g_{max} (30646) at y= 0.14. Credit: University of Calcutta.

NRS passed away in 1963. One of his D.Phil. students, Asim Ray, in his thesis "on some ballistic problem", in 1966 acknowledged:

> "I desire to remember here my teacher (late) Prof. N.R. Sen, who by his active guidance, kind interest, constant good wishes and variety of ways supported and encouraged me throughout the course of this work."[48]

N.R. Sen's work in the field of ballistic will be discussed elsewhere.

Conclusion

Nikhil Ranjan Sen is known for his research in the theory of relativity. The present article shows that in the field of theory of turbulence, he was at par with the globally acknowledged renowned scientists of his time. In 1951, with his landmark paper, he proved the correctness and importance of the work on turbulence theories of W. Heisenberg, S. Chandrasekhar, Theodore von Karman and others. In spite of his tremendous contributions to mathematical science he did not receive the recognition that he deserved. This is evident from the fact that no biography has been written on NRS till this date. This suggests that Indian men of science give more importance to 'discoveries' and 'inventions' but pay little attention to scientists like Sen who helped to develop the theory of relativity and other fields of research at their very nascent stage.

Acknowledgement

Thanks are due to Prof. A.K. Singhvi – Vice-President INSA and Prof. Sudhanshu Aggarwal INSA, for sending some articles from the Proceedings of the INSA. One of the authors (SCR) is also thankful to Prof. Narayan Chandra Ghosh, formerly at Rabindra Bharati University for many helpful discussions. Last, but not the least, one of us (RS) is grateful to Prof. Michael Komorek, Head of Research Group – Physics Didactics and Science Communication, University of Oldenburg, Germany, for providing research facilities.

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