THE ECONOMICS OF THE KOLKATA PAISE RESTAURANT PROBLEM

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We provide a brief discussion on how the Kolkata Paise Restaurant (KPR) problem is modelled as a repeated game where players are assumed to coordinate their actions. Specically, we discuss some theoretical results obtained in this context. We also discuss why there is a need for more experimental research on the KPR problem.

Introduction

Kolkata was the first capital of British India, and has been one of the oldest trading ports in India. And so, it has a history of two hundred years of being an economic nerve center where laborers used to migrate from all parts of India. In recent decades, in spite of losing its pre-eminent position as an industrial hub, Kolkata has attracted a large amount of labor inow from nearby regions of Bihar and Jharkhand, with almost thirty thousand migrants reaching Kolkata annually on average in the decade 2009-2010.[1] The majority of these migrant laborers, however, are unskilled workers that provide manual labor to transportation or low-end construction businesses. These businesses invariably prosper in the unorganized sector without fixed working hours, secure wages or reasonable breaks. The workers, on the other hand, voluntarily submit to these unhealthy work conditions, as this is their only escape from abject penury that awaits them back home. Thus, everyday Kolkata accommodates a burgeoning class of poor migrant laborers in the underground economy, mostly in her slums spread over 107 wards of the municipal corporation (which has 141 wards in total).[2]

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A major component of this accommodation is the regular supply of lunch to these poor customers who are extremely price sensitive. This has led to cropping up of a large number of extremely cheap restaurants that serve very basic north Indian/ Bengali meals. On account of the sheer affordability of these roadside eateries as well as their modest infrastructure, they are called *Paise* restaurants. Unsurprisingly, this moniker derives from the fact that *Paise* is the smallest possible unit of India's national currency. It is observed that competition among these eateries drives the prices down to the basic costs of cooking, with their customers differentiating among them on the basis of the taste of food offered. That is, the laborers have a well defined ranking on these restaurants in terms of quality and flavor of service. Also, with most laborers coming from a reasonably homogeneous cultural and social background, these rankings do not vary from worker to worker. However, when looking for lunch, these laborers do not just care about the taste, they also care strongly about the time taken in service. The reason for this is the exploitative nature of these unorganized enterprises which allow very small lunch breaks, and impose steep fines upon missing time-lines. On the other hand, as mentioned earlier, these restaurants have very limited infrastructure with small servicing capacity. And hence, if the laborers do not manage to choose their lunch destinations carefully, they may end up having a wait time that makes them impossible to complete lunch in the allotted lunch break. On such occasions, the laborers would choose to forgo their lunch.

Everyday, these laborers face a complicated decision problem. They would love to have lunch at the commonly acknowledged restaurant with the tastiest dishes, but only if they can be sure of getting service at that restaurant. And the chances of them getting service at that restaurant depends on how many other laborers have flocked to the very same restaurant. Thus, the laborers end up interacting in a manner that is described in Economics as a *game*. Under certain modeling conditions that are discussed in detail below, this game has come to be known as the *Kolkata Paise Restaurant* problem or, in short, the *KPR* problem[3,4,5].

The Framework

KPR problem consists of a finite set of restaurants $R := \{1, \ldots, k\}$ and a finite set of customers $N := \{1, \ldots, n\}$. The customers have a commonly agreed preference over the set of restaurants denoted by a utility index $\{u_i\}_{i=1}^{i=k}$ such that $u_1 > \ldots > u_k > 0$. Everyday, all customers choose a restaurant for lunch in a simultaneous manner such that no one knows the choice of the other when deciding the destination for lunch. Each restaurant can serve only one customer in a day. In case, more than one customers arrive at a restaurant in a day, any one is chosen with equal probability for service. Each customer, therefore, faces a probabilistic outcome on getting service. We assume that every customer has particular preference over such lotteries that can be expressed by the expected utilities arising out of them. So if three customers end up at the same restaurant $l \in \{1, \ldots, k\}$, then each customer gets a utility of $\frac{u_1}{3}$.

Thus, KPR problem is a dynamic infinite horizon game where each customer i's deterministic strategy is a sequence $r^i := \{r_t^i\}_{t \geq 1}$ where $r_t^i \in R$ denotes the restaurant chosen by customer i for lunch on the t-th day.[3] That is, a pure strategy r^i is function defined on the domain of natural numbers with range in the set of restaurants R. Now, each possible collection of such strategies chosen by all customers, leads to a particular infinite stream of expected payofls for all. Therefore, in deciding the best course of action, a customer must be able to rank these infinite streams of payoff. A simple sum would not suffice, as it would render all possible streams of payoff to be equivalent. Upon substantial debate on this matter, modern economists have accepted the norm of weighting future payoffs down by some *discount* factor $\delta \in (0, 1)$, so that at any time $t \geq 1$, an infinite stream of payoffs in future p_t; $p_t + 1$; ... is indexed by the weighted infinite sum $p_t + \delta p_{t+1} + \delta^2 p_{t+2} + \ldots$. This sum is known as the *present value* of the payoff stream $p_t, p_{t+1}, \ldots$[1,2] (for details see).

Some Existing Results

Now, as economists, we are interested in the social norm that would prevail in this game. Note that such a social norm is a kind of steady state from which no social agent is observed to deviate. Further, if the agents are selfish payoff maximizing rational agents, then such a social norm would exist only if no agents finds it profitable to deviate from this norm. This implies that a necessary condition for existence of a social norm is that no customer should deviate from it. Indeed, this condition is formalized in game theory as the notion of *Nash Equilibrium*. That is, Nash equilibrium is the norm of a social interaction, or equivalently, the collection of strategies in a game such that : no customer finds it profitable to deviate unilaterally.

Therefore, in terms of the notation developed in the earlier part of this paper, Nash equilibrium of KPR game, is the collection of strategies $\bar{r} \equiv \left(\bar{r}^1, \ldots, \bar{r}^n\right)$ such that for each agent $i \in N, \bar{r}^i$ solves the following maximization problem

$$\max_{\{\tilde{r}_t\} \in R^{\infty}} \sum_{t=1}^{\infty} \frac{\delta^{t-1} u_{\bar{r}_t}}{\left|\left\{ j \in N : \bar{r}_t^j = \bar{r}_t \right\}\right|}$$

Therefore, $\bar{r}$ is a Nash equilibrium if and only if, for each agent i, $\bar{r}^i$ is the best strategy that i can play to maximize her present value, when all other agents $j \neq i$ play the strategies $\{\bar{r}^i\}_{j \neq i}$.

Note that there can be many Nash equilibria in KPR game, and hence, may social norms. To the extent that customers are rational, and so, avoid playing weakly dominated strategies, iterated or otherwise, it is desirable to ignore those Nash equilibria that do not satisfy *subgame perfection*.[4] Sub-game perfection of a strategy collection $\tilde{r} \equiv \left(\bar{r}^i, \ldots, \bar{r}^n\right)$ requires that on each day $t \geq 1$, irrespective of the past history of restaurant choices made by all customers, for each customer i, playing $\tilde{r}^i$ is the best response to all other agents $j \neq i$ playing $\{\bar{r}^i\}_{j \neq i}$. Note that subgame perfection of a strategy collection requires it to be a Nash equilibrium, and hence, the social norm implied by such strategy choices is known as *Subgame perfect Nash equilibrium (SPNE).* Note that it would appear impossible to characterize the SPNE of KPR game as it is an infinite horizon game, where the number of histories that can be conceptualized is infinite. Thankfully,[6] showed that a strategy collection or profile $\hat{r}$ constitutes SPNE if and only if $\hat{r}$ satisfies *one deviation property*. This property requires that no agent i can make a *particular* kind of profitable unilateral deviation, where she plays according to $\hat{r}$ on all days except for a *single* day t^* where she chooses restaurant $r^* \neq \hat{r}_t^i$.

[1] investigates the SPNE of KPR game under the restriction that $k = n$. They find that if $u_1 \leq 2u_k$, an interesting social norm called *Cyclically Fair Norm* constitutes an SPNE, irrespective of the value of discount factor . This norm signifies an ethical cooperative behavior where:

- Without loss of generality, on day 1, each agent i goes to restaurant i.
- On any day $t > 1$, i goes to restaurant k if she went to restaurant 1 in the last period $t - 1$.
- On any day $t > 1$, i goes to restaurant l if she went to restaurant $l + 1$ yesterday.

The attractive feature of this norm is that each customer gets a sure shot chance to enjoy lunch at each restaurant exactly once, every n days. And so, every customer gets to eat the best food as well as the worst food available in the market, within each block of n consecutive days. This is both fair and efficient as customers can coordinate to never end up at the same restaurant on any given day, while ensuring an equitable distribution of quality available in the market.[5]

Unfortunately, however, when $u_1 > 2u_k$ (and $n = k$), it is very difficult to show that the Cyclically fair norm constitutes an SPNE. A mildly positive result appears as we can show that when $n = 2$, there exists an SPNE where customers end up playing Cyclically fair norm if and only if customers discount future returns to a small enough extent. In particular, if customers have a discount factor greater than $\frac{u_1 - 2u_2}{u_1}$, then Cyclically fair norm is an SPNE outcome, and vice-versa. However, this result requires customers to choose strategies that incorporate a willingness to punish others for deviating from this agreed norm. This punishment mechanism works because any customer's deviation would lead to both customers ending up at the same restaurant on some particular day. Hence, customers can make their restaurant choices conditional on this event, and decide to punish the other by going to the best restaurant on all subsequent days. With customers patient enough to give sufficient weightage to any future loss in utility, this threat of punishment makes them stick to the agreed cooperative Cyclically fair norm.[6]

Experiments

Limited empirical work, that is, empirical analysis based on behavioral data (generated by human decision makers), whether observational or experimental, exists on the KPR game. An exception is the current experimental study of Ref.[7], which we briefly describe below[7] focus on the utilization rate as a proxy for efficiency (or full-capacity utilization). It is known that if players randomly choose restaurants each day with no memory or coordination, the average utilization rate converges to $1 - 1/e \simeq 0.63$ [3]), whereas if backward looking strategies are used using information from one or more past periods, the utilization rate may be more or less than this benchmark, depending on the strategy used[3,4]. The experiment in Ref.[7] has three conditions, with 15 players and 60 periods per condition. They assume the restaurants are all equal in terms of payoff, that is, not ranked. In the baseline condition, no information is provided about other players' past choices or their success or failure (in terms of getting served). The other conditions progressively provide more information, with success or failure information provided in one, and success or failure as well as choice information provided in the other. They find no difference in utilization rates across the conditions (average rates were around 70% in all conditions), though more information appears to induce players to change choices less often period to period, that is, induce more stability in choice.

Given the paucity of data on how players choose strategies or which factors promote efficiency, there is a definite need for more empirical research on the KPR game. Two possible avenues which can shed light on the latter set of issues are communication and information acquisition.

As long as going to an unoccupied restaurant is strictly preferred to not being served with positive probability (going to a restaurant with at least one other customer present), the set of pure strategy Nash equilibria in any KPR is coincident with the set of Pareto efficient allocations, that is, restaurant-customer assignments[3]. Selecting such an equilibrium may require coordination amongst players. Even selecting the symmetric equilibrium, which is in mixed strategies and inefficient, may require coordination, given the plethora of equilibria. This is unrealistic with many players. Theoretical analysis of the game has thus proceeded on the assumption that players do not have access to explicit coordination modes.

If there are a small number of players, coordination mechanisms such as communication may assume importance. Even with a large number of players, communication and other coordination mechanisms seem to evolve in practical situations where competition and coordination are simultaneously present, as in the KPR game. Thus, given that communication can possibly help coordinate players' actions, it may of interest to study how communication affects efficiency in the KPR problem. Some variables that can guide the choice of what experimental conditions to study may be: (i) the number of players, with the effect of communication on effciency is expected to be declining with the number of players, (ii) the communication protocol, for example, numerical cheap talk, free-form electronic, or face to face communication, or one-way or two-way communication, with the eect of communication on effciency expected to be lower as the communication protocol gets more structured, (iii) the communication frequency, for example, every period or every p periods etc., with communication expected to be less effective with reduced frequency, etc.

If communication does positively affect efficiency, this would normally suggest better coordination amongst players. A subsequent question of importance then is of fairness, if we assume the restaurants are ranked in terms of preference, with a common ranking across customers (as is indeed the case in our framework, that is, Section 2). Does coordination lead to some players being permanently excluded from better ranked restaurants, or do players settle into more equitable long-run distributions[1] (provide results for less than 4 players that fair equilibria can be sustained in any KPR if players are sufficiently patient, that is, have a sufficiently high discount factor, provided all information on past play of all players is available at all times to all players)?

The theoretical literature on the KPR game has focused on the types of learning behavior of and decision rules used by players which can promote eciency[3,4]. The general idea is that more sophisticated rules which take into account information on past outcomes and actions would be favored by players, and lead to greater efficiency. The data of Ref. [7], however, provide a counter to such arguments, as they find that providing more information to players on past outcomes and choices does not improve efficiency, which suggests some players may be using naive decision rules which do not condition on information.

Questions which emerge from this finding include a) whether players are heterogeneous in terms of their learning behavior, or decision rules, and b) whether information usage patterns would change if information had to be explicitly acquired, as opposed to being made available by the experimenter. In reality information is seldom freely available. Deliberate information acquisition may thus be an issue of importance in the KPR game. Experimental conditions could be implemented which vary the cost, type and extent of information that could be obtained, with information acquisition becoming a choice variable. Data from such conditions could be compared to data emerging from conditions where no information is obtainable or where information is available (on a mandatory basis) to determine if a) information acquisition patterns are homogeneous across players, b) deliberate information acquisition provides any advantage over the no information benchmark or over the mandatory information benchmark, c) greater information acquisition is associated with higher payo at the level of individual players, d) excessive information availability causes cognitive load, etc.

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2. See *https : //www:unescogym:org/wp – content=uploads=* 2015=11=*Migration – in – the – Slums – of – Kolkata – A – Gendered – Perspective:pdf*
3. Of course, i may also use stochastic strategy $\sigma^i := \{\sigma^i_t\}_{t\geq 1}$ such that at each time t, σ^i_t is probability distribution on R. However, considering such strategies only adds to the notational complexity without furthering the qualitative objective of this paper. Hence, we focus on pure strategies in the majority of this paper.
4. A strategy of a game is said to be weakly dominated for a player i, if there exists another strategy which gives i: no less payoff on each day, and a greater payo on some day.
5 Also see [4] for a discussion on fair strategies in section 4, page 7.
6 With three customers and $n = k$, dierent combinations of $\{u_l\}^k_{l=1}$ require different strategy proles to constitute an SPNE that leads to customers playing Cyclically fair norm. Each of these strategies dier in their exact punishment mechanisms. For more details see [1].

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