MICE PROBLEM AND FRACTALS

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The celebrated Mice Problem – that of n objects chasing each other from corners of an n-sided polygon – in mathematics is a special case of more general Pursuit Problem. A new approach to solution of this problem, not found in standard texts, is given in this article using the concept involved in fractals, an unconventional geometric shape that can be divided into parts, each of which is a smaller duplicate of the whole. Similar parts of such objects as cloud or branches of a tree can be obtained by rotation and scaling of another part. Precisely, this mathematics of successive rotation and scaling has been utilized in finding the solution to the Mice Problem.

Introduction

In mathematics, the mice problem is a celebrated one in which a number of mice (or beetles, dogs, missiles, etc.) are placed at the corners of a regular polygon. In this problem, also called the beetle problem, ***n*** mice start at the corners of a regular ***n*** sided polygon of side length ***a***, each always heading towards its closest neighboring mouse in a counter-clockwise direction at constant speed *v*. The questions posed are:

(1) What is the distance covered by each mouse?

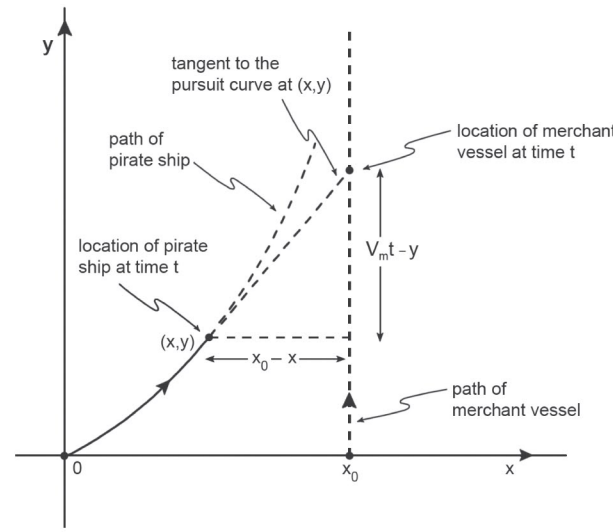
(2) When and where the mice meet?

(3) What is the trajectory traced out by each?

As a matter of fact, the mice problem belongs to the more general class of 'pursuit problems' – on a plane or on a curved surface such as a sphere as discussed below.

A Bit of History of General Pursuit Problems[1]

Modern mathematical pursuit analysis is generally assumed to begin with a problem posed and solved by the French mathematician Pierre Bouguer (1698– 1758) in 1732. Bouguer treated the case of a pirate ship pursuing a fleeing merchant vessel, as illustrated in the following figure:



The pirate ship and the merchant vessel are taken to be at (0, 0) and (x_0, 0) at time t = 0, respectively, the instant the pursuit begins, with the merchant vessel travelling at constant speed V_m along the vertical line x = x_0. The pirate ship travels at constant speed Vp along a curved path such that it is always moving directly toward the merchant, that is, the velocity vector of the pirate ship points directly at the merchant vessel at every instant of time. Bouguer's problem was to determine the trajectory which he called the curve or line of pursuit. It was also noticed from the earliest days of falconry that in its attack, the falcon always flies directly at the instantaneous location

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