FRACTAL GROWTH PATTERNS IN SCIENCE AND TECHNOLOGY

ISHWAR DAS*[1], V. N. PANDEY[2], NAMITA R. AGRAWAL[3],
NEHA TIWARI[2] AND SHOEB A. ANSARI[3]

Fractals and growth patterns have relevance in different areas of science and technology. Fractals are observed in nature, environment, ecology, chemistry, physics, botany, geology, mathematics etc. It is also observed in different parts of human body and several non-living systems. In the present communication, we report basic concepts of Euclidean geometry, fractals and fractal dimension, diffusion limited aggregation (DLA) model, recent advances in fractal growth during electrodeposition of metals and growth of different bacterial colonies.

Introduction

A fractal as first conceived by Mandelbrot[1], is a geometrical structure which at a first look appears to irregular and complex. Fractal is a subject associated with the discipline of non- linear dynamics. It is not limited to science only but even in popular culture, nature and other aggregation processes. Fractals are characterized by dimension which is different from usual Euclidean dimensions having integral values D= 0, D= 1, D= 2 and D= 3 for different geometries as recorded in Table 1.

TABLE 1: Euclidean Dimension for Different Geometries

Geometry	Euclidean dimension
Point (.)	Zero
Line (-)	1
Circle or Square (o, m)	2
Cube ❒	3

1 Chemistry Department, D.D.U Gorakhpur University, Gorakhpur-273 009, India
2 Botany Department, D.D.U Gorakhpur University, Gorakhpur-273 009, India
3 Chemistry Department, St. Andrew's College, Gorakhpur-273 001, India
* Corresponding author, email: ishwardas.che@gmail.com

Fractals have non- integer (fractional) dimension D, lying between 1 and 2 or 2 and 3. Various stages in the growth of an exact fractal may be represented by Koch curves. The initial stages of the construction of Koch curves are shown in Fig. 1.

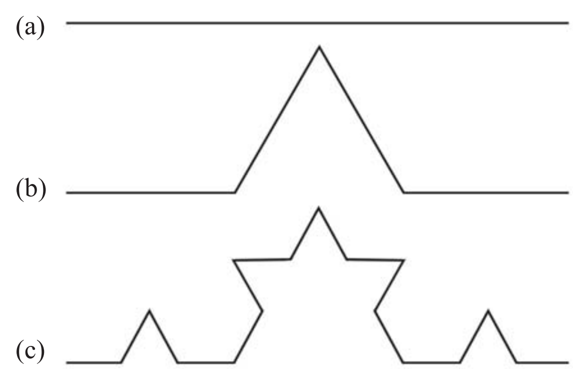


Fig. 1. The first three stages (a)- (c) of the generation of self- similar Koch curve.

The three stages shown in Fig. 1 can be extended an infinite number of times resulting in a curve consisting of infinite number of small segments as shown in Fig. 2.

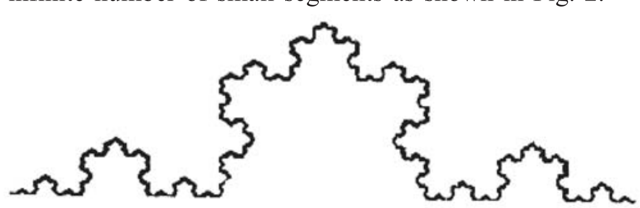


Fig. 2. Koch curve consisting of infinite number of small segments.

Determination of Fractal Dimension of Koch and Similar Mathematical Objects:

Consider a one dimensional curve of unit length that has been divided into N equal segments of length l so that

$N.l = 1$

l

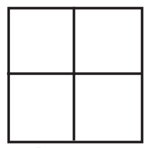
$D = 1$

or $\quad N = \frac{1}{l}$

As the l is decreased, N increased linearly. similarly if we divide a two dimensional square of unit area into N equal sub- squares of length l, we have for a two dimensional object,

$N.l^2 = 1$

or $\quad N = \frac{1}{(l)^2}$



l

l

In general, for a dimension D, we can write,

$N.l^D = 1$

or $\quad N = \frac{1}{l^D}$

where D is the fractal dimension of object.

On taking logarithm of both sides of the equation (1), we can write,

$$D = \frac{\log N}{\log\left(\frac{1}{l}\right)} \qquad ....(2)$$

Using equation (2), the fractal dimension of a Koch curve can be determined and will be non- integer.

In case of branched structures as shown in Fig. 3 (a), box counting method described by Das *et al*[2], is most commonly used method for fractal dimension calculation. It is a simple and reproducible method.

The relationship between the number of pixels N inside a circle of radius r should be a power law with non-integer exponent D as $N(r) \sim r^D$. A careful counting of the number of pixels for different values of r as shown in Fig. 3a, provides an information about the exponent D, the fractal dimension. Its value may be obtained from the slope of linear plot between $\log N$ and $\log r$ as shown in Fig. 3(b).

Diffusion Limited Aggregation (DLA) Model

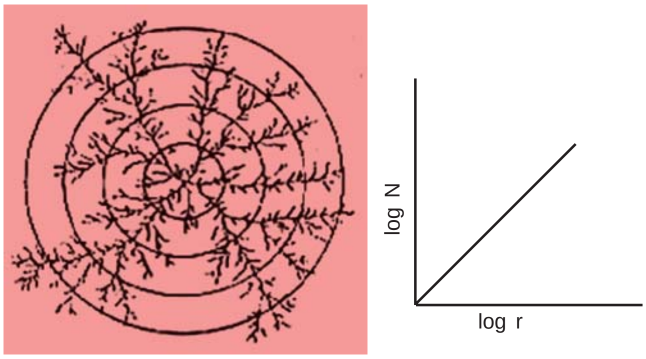
Although a large number of growth models such as (i) random walk model,[3] (ii) self avoiding random walk model (SAW), (iii) self attracting self avoiding walk model (SASAWS), (iv) true self avoiding walks model (TSAWS),[4] (v) diffusion limited aggregation (DLA) model, the single particle DLA model proposed by Witten and Sander[5] has been mostly studied. It is a model of random irreversible growth. The model can be summarized with the following rules,

(i) A seed particle starts at the origin of the lattice,

(ii) Another particle is allowed to walk at random and diffuse from far away and arrives at one of the lattice sites adjacent to the occupied site and then it is aggregated and finally, another particle is launched far enough from the seed and makes a random walk. The process is repeated and an aggregate known as DLA aggregate is obtained. DLA-like growth patterns are observed during crystallization, electrodeposition, bacterial growth etc where the particles undergo a random walk due to Brownian motion and cluster together as shown in Fig.4.

Fig. 4. Schematic diagram of Diffusion Limited Aggregation process

It is a successful model as large number of experimental systems displaying a structure similar to the DLA aggregates. The fractal theory is applicable to explain aggregation in any system where diffusion is the primary transport mechanism. The fractal analysis is a very useful tool for analyzing facts and outcome related to environmental engineering. In fact, fractals are also seen in nature. The fractal geometry gives a new idea to describe model and analyze the complex forms found in nature such as clouds, tree, mountains etc. Daccord *etal* [6] have observed fractal patterns by injecting water through pure plaster Most surfaces are fractal. In surface physics, the fractals describe the roughness of the surface. Biosensor interactions can be studied by using fractals. Fractal aggregates can also be obtained through polymerization of monomers under suitable conditions. The control parameters is generally observed that the morphology of the crystal strongly depends on the distance of the formation conditions from the thermodynamic equilibrium[7]. They demonstrated schematically a correlation between the distance of the formation conditions from the equilibrium and the resulting morphology ranging from polyhedral crystal to dendrites, fractals, diffusion limited aggregates (DLA) and dense branched morphology (DBM).



(a) (b)

Fig. 3. Branched structure of a cluster and log N versus log r plot for fractal dimension calculation by box counting method[2]

Pattern formation occurs in a variety of contexts with implications in chemistry, botany, physics, geology, pharmaceuticals, material science, biology, natural sciences and many more. The irreversible aggregation of separate small particles to form large aggregates[8] is one of the most common phenomena which can be seen in different areas of science and technology[9].

Recent studies have shown that fractal patterns are observed in nature, environment and many living and non living systems such as, growth during crystallization[10], periodic precipitation[11] ,electrodeposition[12], bacterial growth[13], electropolymerization[14], dielectric breakdown[15], viscous fingering[16],dissolution patterns[6], retinal vessels[17], solid-gas reactions[18], fracture of bones, growth of tumors /cancer, snowflakes and ecology etc. Besides fractal geometry, dendrites[19], dendrimers[20], tip splitting[21], dense branch morphology[22], spherulites[23], banded spherulites, burst[24], chiral patterns[25], spirals and chaos, stripes and bands etc are also observed in different disciplines. Some of these are shown in Fig. 5

Living cells can be viewed as complex systems that exhibit non-linear dynamics and fractals A human body is the best example exhibiting the fractal geometry in lungs, retinal vessel, brain , vascular systems and kidneys etc. The bronchial tree and its branching is fractal. Fractal geometry in different parts of body are shown in Fig. 6

Fractal Aggregates in Electrodeposition in Batch and Flow Reactors

Electrochemical deposition of materials is one of the most familiar aggregation phenomena in chemistry[26]. There is considerable interest in the interfacial pattern formation during electrodeposition. It has relevance to biological systems. A renewed interest in electrochemical pattern formation has been seen in recent years[27-34]. Electrochemical deposition is a suitable system for the study of morphogenesis under far from equilibrium. Non-equilibrium growth phenomenon has been widely recognized from both, basic and applied point of views. A

Fig. 5. Branched morphologies

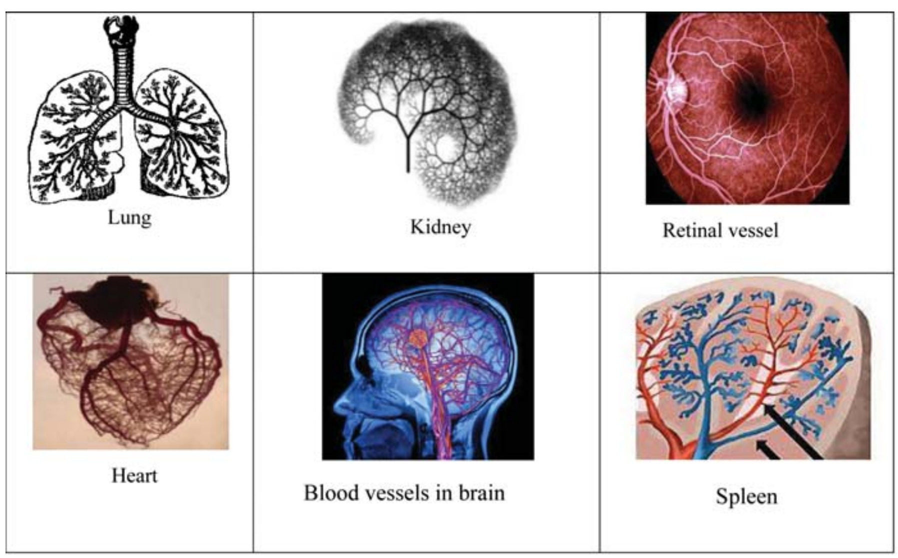
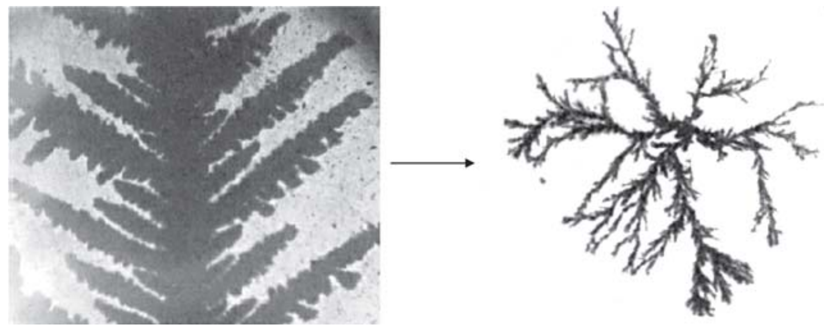


Fig. 6. Fractal geometry in different parts of body

metallic electrodeposit is grown from thin layer of aqueous electrolyte solution containing the metal cation to be reduced. Electrodeposition of various metals such as zinc, copper, lead etc gives rise to aggregates with complex shapes such as open, homogeneous complex, mixed stringy, dendritic, finger like including fractals[31]. It has been clearly shown that the electrodeposit morphology depend on experimental parameters such as cation concentration, applied potential difference, electrode set ups and separation between electrodes, cell geometry etc. The branched fractal structures formed by non-equilibrium electrodeposition of metals have been considered as a model system for the study of branching and fractal growth. Fractals and dendritic growth of metal electrodeposits have extensively been studied by several workers. Growth of gold fractals, morphologies and transitions has been investigated. It has been reported that the morphology of the deposit strongly depend on the kind of electrolyte used. Research on electrodeposition without any supporting electrolytes has received much attention after the discovery of fractal structures generated by aggregation and especially Diffusion Limited Aggregation. Liu etal[34] have studied the growth of gold fractal nanostructures by electrochemical deposition method. The fractal and dendrites during electrodeposition of metals were observed by Das et al[27-30] in mono and binary systems in batch and Continuously Stirred Tank Reactor (CSTR) using two vertical electrodes as shown in Fig. 7

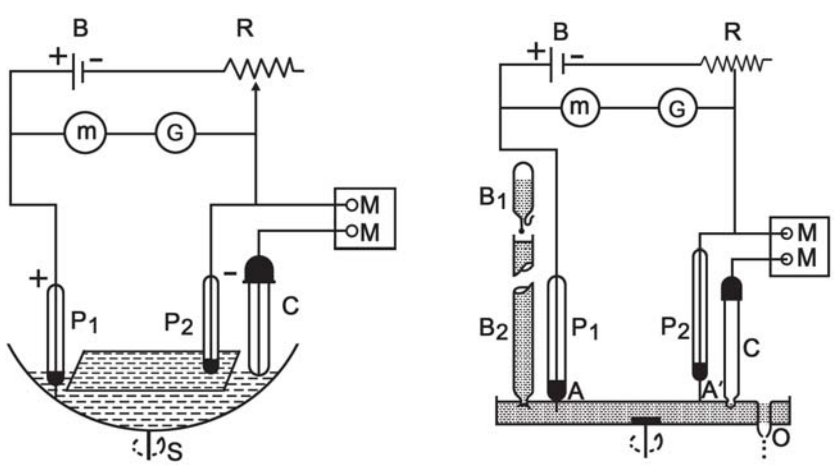


(a) (b)

Fig.8. Growth behavior of electrochemically deposited metal cluster at the tip of the cathode from (a) aqueous lead acetate solution(1M) and (b)solution containing 0.22% PVA [28]

Experiments in a Batch Reactor:

The experimental setup of a batch reactor (Fig 7a) is very simple and inexpensive. It consists of a porcelain dish containing 45 ml of an electrolyte solution. A microslide was put in the dish containing an aqueous solution of lead acetate in such a manner that a small volume of the solution was above the slide. Two identical clean platinum electrodes were inserted in the solution. The anode was extended below the surface of solution. A calomel electrode was inserted in the solution and attached to a digital multimeter. The entire assembly was kept in a thermostat which was maintained at constant temperature. Microphotographs of electrodeposited material were taken with sensitive camera and potential changes during the electrodeposition of lead metal at various experimental conditions were noted as a function of time. Dendritic growth was observed when aqueous lead acetate was used where as a DLA/Fractal type structure was observed when 0.022% polyvinyl alcohol (PVA) was added in this solution as shown in Fig. 8.



(a) (b)

Fig.7. Experimental setup of (a) Batch reactor and (b) Continuously stirred tank reactor to monitor potential changes and morphology during electrochemical deposition of Pb[28]

Experiments in a Continuously Stirred Tank Reactor (CSTR):

In case of a batch reactor, the concentration of solution decreases continuously as the structure grows. To maintain the system far from equilibrium for longer duration, electrolyte concentration was required to maintain to constant throughout the experiment. The experiment was thus performed in CSTR as shown in Fig.8 (b) designed by Rastogi etal [28]. The life time and amplitude of oscillation were increased in the case of CSTR as compared to Batch Reactor.

Das et al[29] have reported new results on the non-equilibrium growth patterns of silver in a Batch Reactor on microslides using cells of different electrodic arrangement as shown in Fig.9.

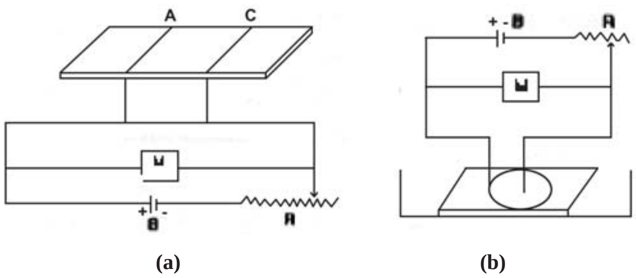


Fig.9. Experimental setup for electrodeposition of metal with (a) Two parallal electrodes (A, C) and (b) One circular anode and a point cathode[29]

Scanned picture of silver aggregate thus obtained were analyze and fractal dimension was calculated. Electrodeposition of lead, zinc and binary system containing lead and zinc were also carried out using the same experimental setup by Das etal [30(a)]. Electrodeposited fractal aggregates were obtained for the binary system containing $Pb(NO_3)_2$ -$Zn(CH_3COO)_2$ were mixed in 1:1 proportion. The fractal dimension was calculated and found to be 1.71± 0.04. It was in good agreement with the value for DLA cluster.

Das etal [30(b)] have investigated the non-equilibrium growth patterns of Zn-Cu alloys in then η and β phases. They reported the electrodeposition and growth behavior of the deposits obtained from the aqueous solutions of $ZnSO_4$, $CuSO_4$ and $ZnSO_4$ containing $CuSO_4$ in different proportions at various experimental conditions in batch and flow reactors using an experimental setup described by Rastogi etal[28] but different in electrode setup. In this case, two platinum electrodes (one vertical cathode and another circular anode) were used. Microphotographs of electrodeposited aggregates obtained from aqueous solutions of $ZnSO_4$ and $ZnSO_4$ containing $CuSO_4$ in different proportions and at different field intensities as shown in Fig.10.

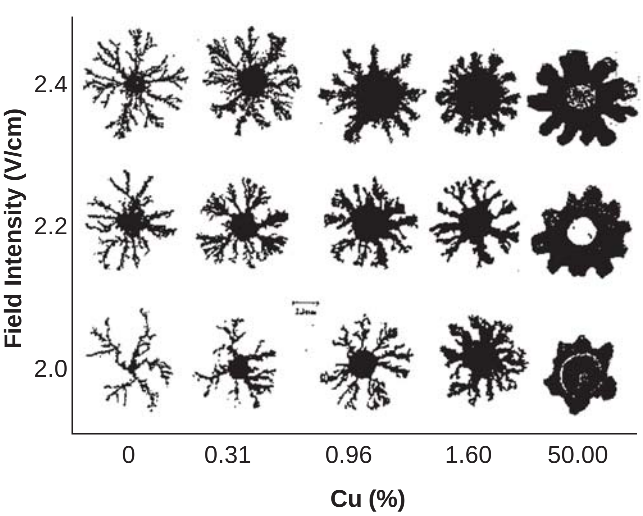


Fig.10. Microphotographs of electrodeposited aggregates obtained from aqueous solutions of zinc sulphate and zinc sulphate containing copper sulphate at different conditions [27]

The patterns adopted various irregular and fractal shapes with different fractal dimensions. Results showed that the patterns became more compact and dense on increasing (i) Copper content in the aggregate and (ii) Field intensity. The compactness was more in the β phase (high copper content) than those in the η phase (low copper content). The dynamics of growth of different intermettallic phases of Cu and Zn was also studied. Far from equilibrium phenomena including temporal oscillations are of considerable interest in chemical dynamics. Thus cathode potential changes with time during the process of electrodeposition of Zn, Cu and intermetallic η and β phases have been studied by Das etal[27] under potentiostatic condition in batch and flow reactors. In case of CSTR, cathode potential decreased with time in each case. Bi periodicity and multiple periodicities were observed in case of Zn and Cu electrodeposition respectively while the oscillation was periodic in ç and â phases. subsequently.

Fractal Patterns and Morphological Transitions During Bacterial Growth

Growth of bacterial colonies is another fascinating example of fractal type pattern formation. The bacterial colonies exhibit a variety of patterns depending upon the type of bacteria and the environmental conditions. The topic of fractal becomes more popular due to its connection with chaos. A fractal system is a complex non- linear disordered mathematical set that displays a statistical self similar repeating pattern on certain length. Fractal geometry can be useful for decreasing the pathological architecture of tumors and yields insights into the mechanism of tumor growth. Fractals are showing up in everything from computer graphics to strange attractor..

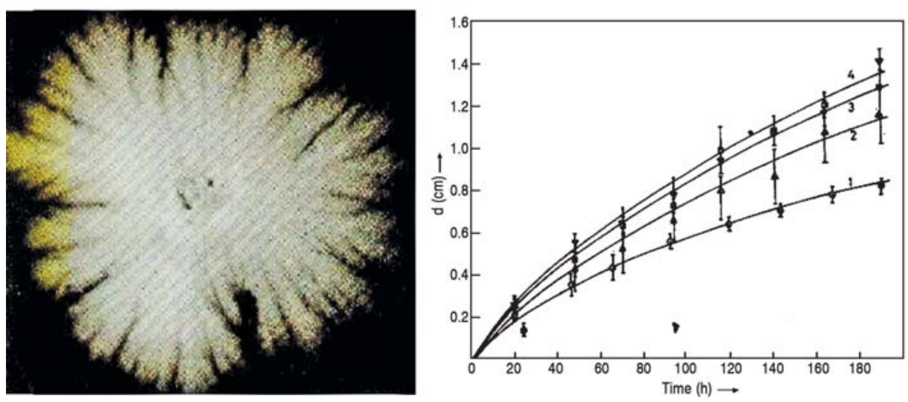
Microbes are present in versatile aspects of our life. We cannot ignore their immense potential in ecological cycles, food, industrial products agriculture and pharmaceutical etc. Today it is an urgent need to explore new avenues in microbial research to solve some challenges of this century like solution to health, agriculture and environment. The emergent spatial patterns generated by growing bacterial colonies have been the focus of intense study in physics[35].

Bacteria when grow on a semi-solid agar medium, forms a characteristic type of colony which differ in size, shape, surface, elevation, internal structure, colour, opacity etc. Most common colony shapes that are likely to encounter are shown in Fig.11

Fig.11. Different bacterial morphologies

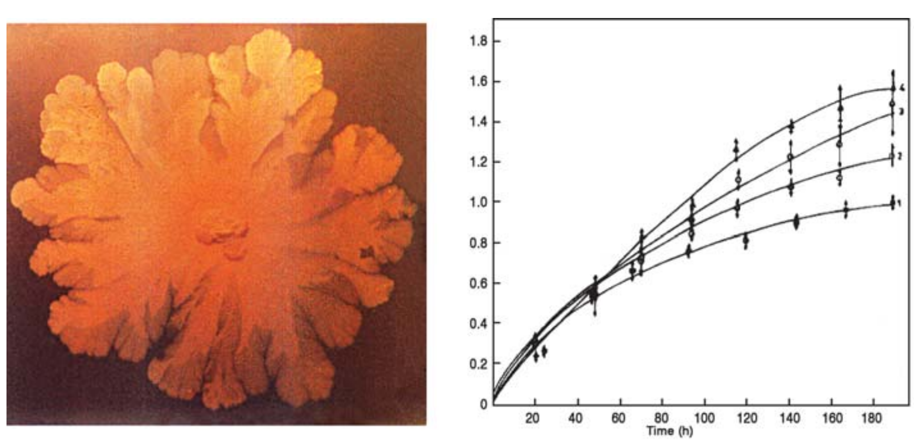
The growth of bacteria presents an inherent additional level of complexity compared to non-living systems. Bacterial colonies grow two dimensionally on agar surface because average pore size of the network of the prepared agar gel is smaller than the size of the bacteria. Agar is a polymer used as the gelling agent in the medium. The hardness of gel medium increases with increase in the agar concentration. Bacteria move more slowly through hard gel. Growth patterns depend on nutrient concentration. At lower nutrient density, fractal growth is favored. It is suggested that bacterial colonies grow in accordance with Diffusion Limited Aggregation(DLA) model on agar plate. Sawada etal[36] investigated that growth velocity of electrochemical deposition would depend on concentration. Ben-Jacob etal[37] have presented a study of interfacial pattern formation during the growth of bacterial colonies presented a non- local communicating walkers mode to study the effect of local bacterium bacterium interaction and communication via chemotaxis signalling and demonstrated how communication enables the colony to develop complex patterns in response to adverse growth conditions. Family[38] have studied the dynamics of growth of different strains of bacteria . *Bacillus subtilis* and *E. coli* under different conditions of low as well as rich nutrient concentrations and found that experimental data , the mean radius of the bacterial colonies grew with a power of time. The *Bacillus subtilis* is rod shaped[39] (about 0.7 μm in diameter and 2 μm In length) and can move in water by collectively retating fragella. However, when environmental conditions are adverse, the cells become spores. They observed the periodic colony formation during the growth of the bacterium Bacillus subtilis. Ben-Jacob et al[40] have grown bacterial colonies under different growth conditions, ranging from a very low level of nutrient concentration (0.1 g peptone per litre) to a very rich level (10g peptone per litre) and from a soft substrate (~1% agar concentration). On the basis of optical microscopy, Ben-Jacob etal[40] observed that colonies adopt various shapes as growth conditions were varied, compact → ramified on decreasing the peptone concentration. They also observed by optical microscopy that the bacteria performed a random walk- like movement.

To model the growth, Ben-Jacob etal[13] has suggested the following features, (i) diffusion of nutrients (ii) movement of the bacteria (iii) reproduction and sporulation (local communications. Meakin[41] on the basis of computer simulation demonstrated that when biological growth was governed by DLA process, the growth patterns showed characteristic features such as screening, repulsion etc. The



(a) (b)

Fig.12. (a) Bacterial colony of E. coli and (b) Growth of *E. coli* as a function of time for different peptone level[42]



(a) (b)

Fig.13. (a) Bacterial colony of K. ozaenae and (b) Growth of K.ozaenae as a Function of time for different peptone level[45]

behavior was examined and observed that many inner branches were found to stop growing afterwards inspite of their open neighborhood. They further observed the growth behavior of two neighboring colonies inoculated simultaneously at two points and incubated. They clearly observed that two neighboring colonies repel each other and never fused together. It has great similarity between the repulsion behavior of bacterial colonies and DLA clusters. Das et al[42] have reported the development of fractal growth of *E.coli* (Fig. 12) on nutrient agar medium plates containing peptone, agar-agar and beef extract.

Fractal dimension was calculated by box counting and found to be 1.55 $\pm$ 0.03. A transition from Eden –like to smooth spreading colony patterns was also observed. Kinetics of *E.coli* colony growth under different experimental conditions was studied and results are shown in Fig.12. Most of the curves obeyed an empirical equation $d^2 = mt + c$ for the growth of bacteria E. coli where d is the growth at any time t, m and c are slope and intercept respectively. Influence of vitamins, H_2O_2, antibiotics and the bacteria *Bacillus subtilis* on growth behavior were also studied. Vitamins C and B complex showed stimulatory effect to some extent while oxy tetracycline and H_2O_2 inhibited the growth of microorganisms. *E.coli* is gram negative, motile and capable of synthesizing all amino acids essential for its growth. Bacterial colonies grow two dimensionally on agar surface. It is an example of diffusion controlled growth process. The microbial growth could be thought of primarily as a consequence of nutrient diffusion and consumption, movement of bacteria and cellular division. At the cellular level, cell replication is an obvious auto catalytic process represented as:

$$X \rightarrow 2X$$

Under certain conditions replication leads to instability similar to the process of cancerization. When automatic strains of the bacterium *E. coli* were inoculated on semi-solid agar containing mixtures of amino acids or sugars, Budrene etal[43] observed that the cells sworm outwards in a series of concentric rings. Tsyganov etal[44] reported three types of population growth and development of chemotaxis motile bacteria, *E. coli* and observed the following behavior (a) stable develops-circular symmetrical waves (b) burst and (c) fractal like self organization. They supposed that fractal like behavior was based on the principle of successively forming multiple micro- burst and mathematical model was suggested to reproduce the experimental results.. Das etal[45] have also investigated the morphology and growth kinetics of the bacteria *K. ozaenae* on agar surface. Bacterial colonies grew two dimensionally with random branches as shown in Fig. 13 which is identical to other bacteria *E. coli, Bacillus subtilis*, retinal vessels and other non-living systems such as electrodeposition, crystallization, electropolymerization, dielectric break down, viscous fingering, dissolution patterns etc. Fractal diamension of the colony of *K. ozaenae*

TABLE 2. Values of fractal dimension (*D*) for different systems

S.No	System	Fractal dimension (*D*)	Reference
1	Oxalic acid	1.65	46
2	o-Toluic acid	1.53-1.61	46
3	Mandelic acid	1.84	46
4	Glucose	1.89	47
5	Cellobiose	1.74	47
6	Lactose	1.58	47
7	Saccharine	1.26-1.28	47
8	Ammonium chloride	1.671$\pm$ 0.002	48
9	KCl admixed with 0.2% agar agar	1.42-1.76	10
10	Electrodeposited copper metal	1.67 $\pm$ 0.04	49
11	Silver electrodeposits from silver nitrate solution	1.95	50
12	Electrodeposited binary system containing Pb and Zn	1.71-1.87	30(a)
13	Electrodeposited Polypyrrole	1.741.75-1.96	51,14
14	Electrodeposited Polyaniline	1.67-1.871.4-1.9	52,53
15	Glucose-glutamic acid reaction product	1.725	54
16	Fractal dimension of gold colloids by TEM	1.75	55
17	Fractal dimension of silver particles by TEM	1.51	56
18	Human retinal vessel	1.7	17
19	*Bacillus subtilis* Bacterial colony	1.72	39
20	*Escherichia* Bacterial colony	1.55	42
21	*Klebsiella* Bacterial colony	1.61	45
22	Radial viscous fingers	1.70 $\pm$ 0.05	16
23	Fractal dimension of dielectric breakdown	1.75 $\pm$ 0.02	15
24	Fractal patterns from chemical dissolution	1.6 $\pm$ 0.01	6

was calculated by box counting method and found to be 1.61, in agreement with the DLA model. Both phenotypic and genotypic adaptations were observed under different experimental conditions. Growth kinetics followed the same trend as observed in the case of *E. coli*. Changes in the bacterial growth patterns are due to the change in the bacteria's environment conditions. It can easily be controlled in the lab and possible to identify the specific quality of the bacteria and their growth behavior. A morphological change in bacterial colony formation.

A new phenomenon known as burst of morphology at a localized point along the interface was observed by Ben Jacob etal[25] during the bacterial growth. The new morphology has higher growth velocity and outgrows the original morphology. Chiral morphology[40] during the growth of bacterial colonies on soft agar is another interesting morphology. The colony shows chiral branches, all having the same handedness ie left handed or right handed as observed in different flowers namely harsingar (*Nyctanthes arbostrystis*) and Chandani (*Taberna /jasminum* sp)

The values of fractal dimension for a variety of fractal systems are recorded in Table 2

Conclusions and Outlook

Fractal geometry is a new language used to analyze and model the complex forms. Fractal patterns are observed in different areas of science and technology. Very often remarkably similar patterns are found in quite diverse systems. Salient features of fractal growth patterns observed during the processes of electropolymerization in batch and flow reactors for a variety of mono-and binary metal systems have been reported. Pattern formation in biological systems is believed to be much more complex than in chemical and physical systems. Bacterial colonies grow two dimensionally with random branches. Fractal growth of bacterial colonies such as *E. coli, K. ozaenae* and *Bacillus subtilis* have also been reviewed. Morphological transitions from fractal to dendritic and compact morphologies were also observed in both the processes depending on experimental conditions Fractal dimension of several systems are reported which are in agreement with DLA values. The study of fractals and fractal geometries has tremendous scope in polymer science including conducting polymers, copolymers, fractal like kinetics in solid gas reactions, topology of fractals, dielectric breakdown, viscous fingering and several other frontier areas of research.

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