

## ECONOPHYSICS AS CONCEIVED BY MEGHNAD SAHA

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*We trace the initiative by Professor Meghnad Saha to develop a (statistical) physics model of market economy and his search for the mechanism to constrain the entropy maximized width of the income distribution in a society such that the spread of inequality can be minimized.*

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### Introduction

Professor Meghnad Saha had been a great physicist and perhaps an even greater social scientist and reformer. It is this second aspect, which is not analysed often or highlighted much in the literature about him. In this brief article, I would like to discuss his science-based attempts to model social dynamics and about his inspiring attitude towards social science in general.

### Saha's Science-based Approaches for Social Problems

There are several writings about him which talks about Professor Saha's deep interest and involvement in matters of our day-to-day social concern. Let me quote from a book and a recent article on Saha, as examples: Dilip M. Salwi, in his book titled 'Meghnad Saha: Scientist with a Social Mission'<sup>1</sup> writes "...Meanwhile in 1943, Damodar river went into spate, its flood waters even surrounded Kolkata and cut it off from rest of India. ... Saha took upon himself the task of studying the flood problem of Damodar river in totality because it also caused soil erosion and siltation. In addition to the study of the topography of the region, the annual rainfall at various spots, etc., he set up a small hydraulic laboratory in the college to simulate the actual conditions and understand the problem. He also conducted a first-hand survey of the region and even studied the

various flood control measures of river systems of the western world. On the basis of his painstaking research, he wrote a series of articles in his own newly launched monthly *Science and Culture*. ... To sensitize his colleagues, friends, and students to the menace of floods, he also set up a model of Damodar river system in the corridor of Science College!". Pramod V. Naik in his article<sup>2</sup> writes "... Meghnad Saha (1893-1956) ... was of the firm opinion that in a country like India, the problems of food, clothing, eradication of poverty, education and technological progress can be solved only with proper planning, using science and technology. ... While thinking about various issues of national importance, Saha realized the need for a scientific periodical, ... and founded a monthly titled *Science and Culture*. ... The range of topics in editorials and general articles was amazing; for example, the need for a hydraulic research laboratory, irrigation research in India, planning for the Damodar Valley, ..., public supply of electricity in India, national fuel policy, ... mineral sources and mineral policy, problem of industrial development in India, automobile industry in India, industrial research and Indian industry, industrial policy of the Planning Commission, ... and so on. Every article shows his inner urge for the goal of national reconstruction. ... show his direct or indirect role as a social thinker and reformer.". See also the paper by Vasant Natarajan in this special issue.

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### Saha and Srivastava's Kinetic Theory for Ideal Gases with 'Atoms' and 'Social Atoms'

No wonder, Professor Saha had thought deeply about the scientific foundation of many social issues. In particular

the omnipresence of social (income or wealth) inequalities in any society throughout the history of mankind must have caught his serious attention. Like all the philosophers through ages, and also as a born socialist, he must have been pained to see social inequalities cutting across the history and the globe at any point of time. Yet, his confidence in science, statistical physics in particular, perhaps convinced him that 'entropy maximization' principle must be at work in the 'many-body' system like the society or a market, and some amount of inequality might be inevitable! One Robinson Crusoe in an island can not develop a market or a society. A typical thermodynamic system, like a gas, contains Avogadro number ( $N$  about  $10^{23}$ ) of atoms (or molecules). Compared to this, the number ( $N$ ) of 'social atoms' or agents in any market or society is of course very small (say, about  $10^2$  for a village market to about  $10^9$  in a global market). Still such many-body dynamical systems are statistical in nature and statistical physical principles should be applicable. In the famous text book 'Treatise on Heat'<sup>3</sup>, written together with Professor Biswambhar Nath Srivastava, the students were encouraged, in the section on Maxwell-Boltzmann distribution in kinetic theory of ideal gas, to think about applying kinetic theory to the market and find the income distribution in a society, which maximizes the entropy (for stochastic market transactions). It may be noted, this point was not put at the end of the chapter. It was put right

within the text of the section on energy distribution in ideal gas. It provokes the student to search for the equivalent money or income (wealth) distribution in a stochastic market when money is conserved in each of the transactions.

One can present the derivation of the energy distribution in an ideal gas in equilibrium at a temperature  $T$  as follows: If  $n(\epsilon)$  represents the number density of particles (atoms or molecules of a gas) having energy  $\epsilon$ , then  $n(\epsilon)d\epsilon = g(\epsilon)f(\epsilon,T)d\epsilon$ . Here  $g(\epsilon)d\epsilon$  denotes the 'density of states' giving the number of dynamical states possible for free particles of the gas, having kinetic energy between  $\epsilon$  and  $\epsilon+d\epsilon$  (as dictated by the different momentum vectors  $\vec{p}$  corresponding to the same kinetic energy:  $\epsilon = |\vec{p}|^2/2w$ , where  $w$  denotes the mass of the particles). Since the momentum  $\vec{p}$  is a three dimensional vector,  $g(\epsilon)d\epsilon \sim |\vec{p}|^2 d|\vec{p}| \sim |\vec{p}||\vec{p}|d|\vec{p}| \sim \sqrt{\epsilon}d\epsilon$ . This is obtained purely from mechanics. For completely stochastic (ergodic) many-body dynamics or energy exchanges, maintaining the energy conservation, the energy distribution function  $f(\epsilon)(\equiv f(\epsilon, T))$  should satisfy  $f(\epsilon_1)f(\epsilon_2) = f(\epsilon_1 + \epsilon_2)$  for any arbitrary  $\epsilon_1$  and  $\epsilon_2$ . This suggests  $f(\epsilon) \sim \exp(-\epsilon/\Delta)$ , where the factor  $\Delta$  can be identified from the equation of state for gas, as discussed later ( $= kT$ ,  $k$  denoting the Boltzmann constant). This gives  $n(\epsilon) = g(\epsilon)f(\epsilon)$

$$\sim \sqrt{\epsilon} \exp(-\epsilon/\Delta).$$

Knowing this  $n(\epsilon)$ , one can estimate the average pressure  $P$  the gas exerts on the walls of the container having volume  $V$  at equilibrium temperature  $T$  and compare with the ideal gas equation of state  $PV = NkT$ . The pressure can be estimated from the average rate of momentum transfer by the atoms on the container wall and one can compare with that obtained from the above-mentioned equation of state and identify different quantities; in particular, one identifies  $\Delta = kT$ .

Saha and Srivastava's chosen example in this section (see Fig.1)

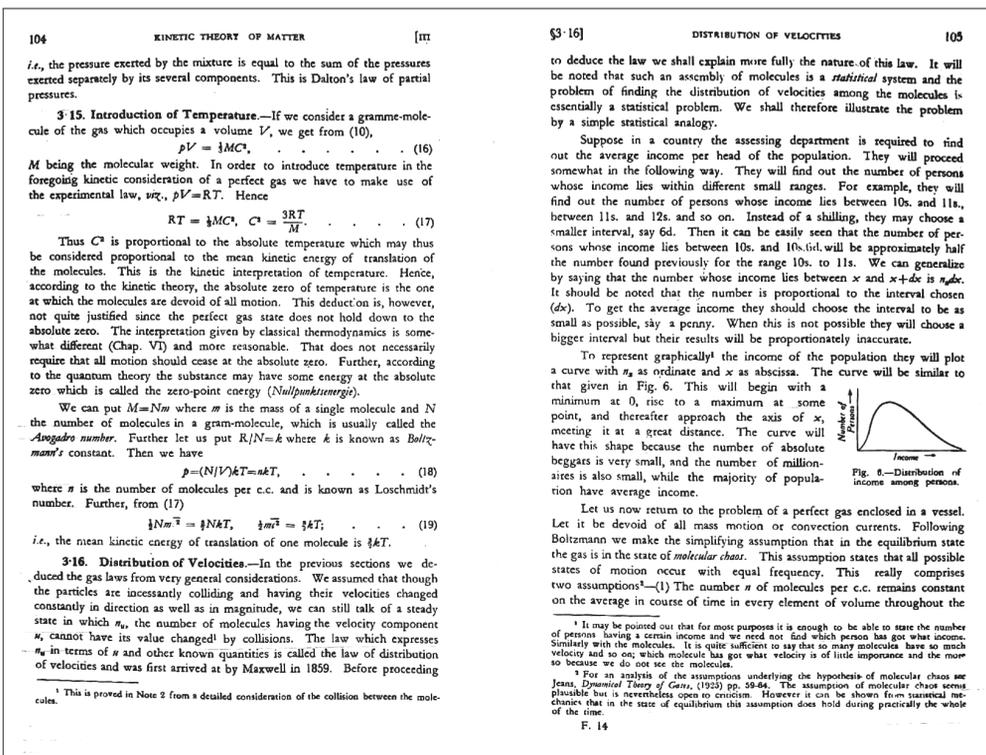


Figure 1: Photocopy of the pages 104 and 105 of Ref.<sup>3</sup>, where the authors urge the students to use kinetic theory to get the Gamma-like income distribution indicated in Fig. 6 of page 105.

indicated to the students that since in the market money is conserved as no one can print money or destroy money (will end-up in jail in both cases) and the exchange of money in the market is completely random, one would again expect, for any buyer-seller transaction in the market,  $f(m_1) f(m_2) = f(m_1 + m_2)$ , where  $f(m)$  denotes the equilibrium or steady state distribution of money  $m$  among the agents in the market. This, in a similar way, suggests  $f(m) \sim \exp(-m/\sigma)$ , where  $\sigma$  is a constant. Since there can not be any equivalent of the particle momentum vector for the agents in the market, the density of states  $g(m)$  here is a constant (each value of  $m$  corresponds to one market state). Hence, the number density  $n(m)$  of agents having money  $m$  will be given by  $n(m) = C \exp(-m/\sigma)$ , where  $C$  is another constant. One must also have  $\int_0^\infty n(m) dm = N$ , the total number of agents in the market and  $\int_0^\infty mn(m) dm = M$ , the total money in circulation in the country. This gives,  $C = 1/\sigma$  and  $\sigma = M/N$ , the average available money per agent in this closed-economy traders' market (as no growth, migration of labourers, etc. are considered). This gives exponentially decaying (or Gibbs-like) distribution of money in the market (unlike the Maxwell-Boltzman or Gamma distribution of energy in the ideal gas), where most of the people become pauper ( $n(m)$  is maximum at  $m = 0$ ). They asked the students to investigate, what could make the distribution more like Gamma distribution, as seemed to them to be observed phenomenon in most of the societies. Any understanding towards that would help to identify and develop public policies to reduce the extent of inequalities in the society. It may be mentioned here that although the statistical aspects of societies, markets in particular, had been addressed in some speculative papers earlier, the 1931 book 'Treatise of Heat'<sup>3</sup>] is the first ever text book addressing the question in the context of kinetic theory of gas and pleads for the search for a solution from statistical physics!

### **Saha's Econophysics Contribution as Noted in Some on-line Encyclopedias**

In the entry on 'History of Indian School of Econophysics', in the online encyclopedia (Hmolpedia)<sup>4</sup> Libb Thims (2016) writes: "... In 1931, Indian astrophysicist Meghnad Saha (1893-1956), an atheist, in his *Treatise on Heat*, co-authored with B.N. Srivastava, explained the Maxwell- Boltzmann distribution of molecular velocities according to kinetic theory in terms of the wealth distributions in society [Saha, Meghnad and Srivastava, B. N. (1931), *Treatise on Heat* (pg. 105), *The Indian Press Ltd. (1931)*]. Indian physicist Prasanta

*Mahalanobis (1893-1972), interested in Karl Pearson stylized biometrics, founded in 1931 the Indian Statistical Institute for developing physical and statistical models for social dynamics. In 1938, Saha began to form the Saha Institute of Nuclear Physics in Kolkata, India.*". The Wikipedia entry (June, 2018) on 'Kinetic exchange models of markets'<sup>5</sup> reads: "... *Kinetic exchange models are multi-agent dynamic models inspired by the statistical physics of energy distribution, which try to explain the robust and universal features of income/wealth distributions. Understanding the distributions of income and wealth in an economy has been a classic problem in economics for more than a hundred years. Today it is one of the main branches of econophysics. ... In 1897, Vilfredo Pareto first found a universal feature in the distribution of wealth. After that, with some notable exceptions, this field had been dormant for many decades, although accurate data had been accumulated over this period. Considerable investigations with the real data during the last fifteen years revealed that the tail (typically 5 to 10 percent of agents in any country) of the income/wealth distribution indeed follows a power law. However, the majority of the population (i.e., the low-income population) follows a different distribution which is debated to be either Gibbs or log-normal. ... Since the distributions of income/wealth are the results of the interaction among many heterogeneous agents, there is an analogy with statistical mechanics, where many particles interact. This similarity was noted by Meghnad Saha and B. N. Srivastava in 1931 [Saha, M. & Srivastava, B. N., *A Treatise on Heat. Indian Press (Allahabad; 1931). p. 105*], and thirty years later by Benoit Mandelbrot<sup>1</sup>. In 1986, an elementary version of the stochastic exchange model was first proposed by Jhon Angle [Angle, J. (1986). *The surplus theory of social stratification and the size distribution of personal wealth, Social Forces, 65 (1986), pp. 293-326*]."*

### **Brief Discussions on Recent Developments**

Saha and Srivastava had been in search of the mechanism that will ensure the initial fall (as in the Maxwell-Boltzmann distribution  $n(\epsilon)$  of energy for the Newtonian particles in the gas, due to the particle

<sup>1</sup> Mandelbrot, B. B., The Pareto-Levy law and the distribution of income. *International Economic Review*. 1 (1960) pp. 79-106; Indeed he wrote: There is a great temptation to consider the exchanges of money which occur in economic interaction as analogous to the exchanges of energy which occur in physical shocks between molecules. In the loosest possible terms, both kinds of interactions should lead to similar states of equilibrium. That is, one should be able to explain the law of income distribution by a model similar to that used in statistical thermodynamics: many authors have done so explicitly, and all the others of whom we know have done so implicitly.

momentum count of the density of states) in the Gibbs-like money distribution  $n(m)$  in the market<sup>2</sup>. They inspired the students to investigate the plausible reasons for this ‘Gamma’ distribution like initial fall (from the scenario where most people are pauper or  $n(m)$  is maximum at  $m = 0$ ), which they rightly conjectured to be the realistic form of the money distribution in any society. There was, however, another aspect of the distribution  $n(m)$  in any society. This was first observed and formulated in 1896 by Vilfredo Pareto (Engineer cum economist/sociologist of the Polytechnic University of Turin), known as Pareto law<sup>6</sup>. This law states that for the number of truly rich people in any society  $n(m)$  does not fall off exponentially (as in the kinetic theory model indicated above), rather fall off with a ‘fat tail’ having an ‘universal’ power law decay:  $n(m) \sim m^{-\alpha}$ , with  $\alpha$ -value in the range 2-3.

The market model considered above by Saha and Srivastava could at best be described as a trading market (with fixed value of  $N$  and  $M$ ) as it does not accommodate in this limit any growth, industrial development or migration of labours etc. If one consults a standard economics text book and looks for the discussions on trading market, one can not miss the discussions on ‘saving propensity’! This quantity characterizes the agents or traders by the fraction of their money holding at that instant they save before going in for a trade. Though, there can be some fluctuations in the value of this fraction from trader to trader, and can even change with time for any individual trader, their respective time-averaged values can characterize different traders’ attitudes. Even the traders from different countries seem to have clear differences in their average saving propensities! In kinetic theory, the atoms do not have any identity, while the social atoms seem to maintain their identity through their respective propensity to save in any transaction or trade (and also maintain the memory of the past transactions). This means, the kinetic exchanges in such trading markets are money conserving but non-Markovian exchanges.

In view of the above-mentioned observation by the economists for a trading market, and the realisation that kinetic theory of gas does not allow for any ‘saving’ of energy of any atom before its exchange with another, it was proposed (see e.g.,<sup>7</sup> and the references therein) that the kinetic theory of market may be extended a bit by allowing saving propensity of the ‘social atoms’, and the

money conserving exchange kinetic equations can be rewritten. It was obvious that, say with the same saving propensity of all the agents in the market, the Gibbs distribution will collapse to a Gamma-like distribution: Once any one having a finite saving propensity acquires some money, to become a pauper he/she will have to lose successively in the market in every one of the later trades or transactions. The probability of such an event, for any finite saving propensity value of the agent, is infinitesimal. This is a nonperturbative result:  $n(0)$  will drop down to zero from its maximum for any non-zero saving propensity of the agents, in the steady state. The most probable income per agent in the market will shift from zero to a value dependent on the saving propensity of the agents. It also suggests immediately that in a market with people having different or random values of saving propensity, there would be a net flow of money towards people with higher saving propensities and in the steady state, a robust power law tail for the distribution,  $n(m) \sim m^{-2}$  ( $m \rightarrow \infty$ ), will ensue for most of the non-singular probabilities of saving propensity among the population<sup>7</sup>. Many important developments and modifications on this kinetic exchange model, pioneered by Saha and Srivastava in<sup>3</sup> (see also<sup>7</sup>), have taken place since then, both in the mathematical physics (see e.g.,<sup>8</sup>) and in economics (see e.g.,<sup>9</sup>).

### Concluding Remarks

Apart from being one of the pioneering astrophysicists of our time, Professor Meghnad Saha had been a passionate and enthusiastic thinker in social science. His deep conviction about scientific approaches to social phenomena convinced him about the applicability of the laws physics, of statistical physics in particular, to social sciences. In view of the inherent many-body stochastic dynamical features of the markets, he realised that the entropy maximizing principle will not support a steady state narrow income or money distribution among the traders. As a diehard socialist, this conclusion must have been quite painful to digest. He was thus in search of the tuning parameters for controlling the width of the money distribution  $n(\epsilon)$ , around the most probable value of income (ideally the average money per agent  $M = N$  in circulation in the market). Saha-Srivastava’s kinetic exchange model for a traders’ market suggested that most traders will become pauper in the steady state ( $n(m)$  will be maximum at  $m = 0$ ). This comes due to the limitation of their model (where the social atoms can lose its entire accumulated money in the next trade with another, as in the case of real atoms in the gas) and also such money distributions do not quite compare either with the observational results

2 For a recent account on the extent and consequences of this Gibbs-like distribution in monetary econophysics, see: V. M. Yakovenko and J. Barkley Rosser, Statistical mechanics of money, Reviews of Modern Physics, **81**, 1703-1725 (2009)

in societies. They had, however, every confidence on the students, whom they expected to think about it and find eventually a proper solution. As mentioned above in section V, one such solution had been achieved later by adding finite saving propensities of the social atoms or traders in the Saha-Srivastava model, following the observations of economists in the trading markets. This additional feature in the model naturally gives Gamma-like income distribution with tunable dispersion of inequality near the most probable income and also induces the eventual Pareto-like slowly decaying numbers (fat tail distribution) of the ultra-rich people or traders in the society.

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