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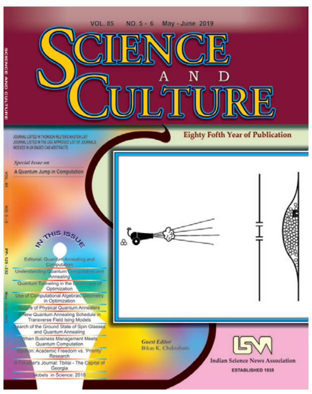
EDITORIAL

QUANTUM ANNEALING AND COMPUTATION:
THIRTY YEARS OF RESEARCH ON DELOCALIZATION IN QUANTUM SPIN GLASSES



The idea of computation by breaking it up into elementary operations like addition, subtraction, etc, allowing them to recast into number crunching arithmetic, has become very popular. This is because, it could finally be achieved using the basic logic gates AND, OR, NOT, etc, and binary digitization of the numbers then made way for the digital computers employing ever faster electronic gates.

The development and success of digital computers eclipsed the parallel development of an intriguing physical method of computation, often called the analog (way of) computation. Imagine a bowl on the table and you need to 'locate' its bottom point. Of course, one can calculate the local depths (from a reference height) everywhere along the inner surface of the bowl and search for the point where the local depth is maximum. However, as every one would easily guess, a much simpler and practical method would be to allow rolling of a marble ball along the inner surface of the bowl and wait for locating its resting position. Here, the physics of the forces of gravity and friction allows us to 'calculate' the location of the bottom point in an analog way! In principle, a similar trick would work for cases where the bowl becomes larger and its internal surface gets modulated, as long as the surface contour or 'landscape' has valleys all tilted towards the same bottom point location. Problem comes when these valleys are separated by 'barriers', high and low.

It is now known that the computationally hard problems, like the N-city Traveling Salesman Problem where the salesman has to visit all the N cities in a trip with minimum travel cost (distance), can be mapped to a (geometric) problem of locating a minimum in a rugged landscape of size $N!$ (factorial N). Generally, for such minimum 'cost' search from among $exp(N)$ or higher number of trips or congurations, the search time can not be bounded by any polynomial in N. Such problems are called Non-deterministic Polynomial (or NP)-hard problems.

In their seminal paper 'Optimization by Simulated Annealing' [*Science, Vol. 220, pp. 671-676 (1983)*], Kirkpatrick, Gelatt and Vecchi proposed a novel stochastic technique, inspired by the metallurgical annealing technique: To search for the optimized cost (travel distance or energy of the ground state 'crystal') at eventually vanishing noise (or temperature), one starts from a high noise (temperature) 'melt' phase, and tune slowly the noise level. In this 'simulated' process, the (classical) noise at any intermediate level of annealing allows for the acceptance (with Gibbs-like probability determined by the change in cost, distance or energy, scaled by the noise factor) of even higher cost (energy or travel distance) fluctuations. As the noise level (T) is slowly reduced during the annealing, the gradually decreasing probability of accepting higher cost values, allows the system to come out of the local minima valleys and settle eventually in the 'ground state' of the system with lowest cost (energy or travel distance) value. It has been a remarkably successful trick for 'practical' computational solutions of a large class of multi-variable optimization problems. Though some 'reasonable' optimization can be achieved very quickly using appropriate annealing schedules, the search time for reaching the lowest cost state or conguration for NP-hard problems however grows again as $exp(N)$.

The bottleneck could be identified soon. Extensive study of the dynamics of frustrated random systems like the (two state Ising) spin glasses, in particular of the Sherrington-Kirkpatrick model [*Physical Review Letters, vol. 35, pp. 17921796 (1975)*], showed that its (free) energy landscape (in the spin glass phase) is extremely rugged and the barriers, separating the local valleys, often become N order for an N-spin glass (search for the degenerate ground states from 2^N states is NP-hard). In the macroscopic size limit (N approaching infinity) therefore such systems become non-ergodic or localized and the classical (thermal) fluctuations like that in the simulated annealing fail to help the system come out of such high barriers (at random locations or congurations, not dictated by any symmetry) as the escape probability is of order $exp(-N/T)$ only. Naturally, the annealing time (inversely proportional to the escape probability), to get the ground state of the N-spin Sherrington-Kirkpatrick model, can not be bounded by any polynomial in N.

The idea proposed by Ray, Chakrabarti and Chakrabarti [*Physical Review B, vol. 39, pp. 11828-11832 (1989)*] was that quantum fluctuations in the SK model can perhaps lead to some escape routes to ergodicity or quantum fluctuation induced delocalization (at least in low temperature region of the spin glass phase) by allowing tunneling through such macroscopically tall but thin barriers which are difficult to scale using classical fluctuations. This is based on the observation that escape probability due to quantum tunneling, from a valley with single barrier of height N and width w, scales as $exp(-\sqrt{N}w/\Gamma)$, where Γ represents the quantum fluctuation strength (or tunneling probability). This extra handle through the barrier width w (absent in the classical escape probability) can help in a major way in its vanishing limit. Indeed, for a single narrow ($w \rightarrow 0$) barrier of height N, when Γ is slowly tuned to zero, the annealing time to reach the ground state or optimized cost, will become N independent! It has led to some important clues. Of course, complications (localization) may still arise for many such barriers at random 'locations'. In any case, with this observation and some more developments, the quantum annealing technique was finally launched through a landmark paper by Kadowaki and Nishimori [*Physical Review E, vol. 58, pp. 5355-5363 (1998)*]. Since then, as mentioned earlier, a revolution has taken place through a surge of outstanding papers both in theory [see e.g., Das & Chakrabarti, *Reviews of Modern Physics, vol. 80, pp. 1061-1081 (2008)*, Albash & Lidar, *Reviews of Modern Physics, vol. 90, art. no. 015002 (2018)* and Mukherjee & Chakrabarti, *Journal of the Physical Society of Japan, vol. 88, art. no. 061004 (2019)*, for reviews on theoretical developments] and in technological applications [see, for example, Brooke et al., Science, vol. 284, pp. 779-781 (1999) for its demonstration in random magnets, and the development of successive generations of D-Wave quantum annealers or computers; Johnson et al., D-Wave group, *Nature, vol. 473, pp. 194-198 (2011*) and Denchev et al., NASA-Google Group, *Physical Review X vol. 6, art. no. 031015 (2016)*]. These intense and massive researches in the last two decades led finally to the birth of this new age of quantum Information and technologies.

Feynman first proposed in 1982 [*International Journal of Theoretical Physics, vol. 21, pp. 467 488 (1982)*] the utilization of the superposition principle in quantum mechanics for the development of parallel gate-based efficient digital quantum computers and the best level achieved so far for such digital quantum computers (by IBM) today employs at most 20 qubits [see e.g., Moran, *Frontiers in Physics, vol. 6, art. 69 (2018)*]. In contrast, the latest D-Wave quantum annealing (analog) computers, commercially available today in the market, already employs successfully more than 2000 qubits, and many encouraging high performance capabilities have already been demonstrated. However, the full potential of these quantum annealing machines (particularly of the most successful and commercially available D-Wave 2X and 2000Q machines today), in the context of its claimed success in scaling down the NP-hard problems, is yet to be established.

In this special issue on Quantum Annealing and Computation, in order to commemorate thirty years since the publication of Ray, Chakrabarti & Chakrabarti (1989), we have been extremely fortunate to be able to gather a number of excellent reviews and papers from some of the most authentic scientists who are stalwarts in the field. Tamir from *Bar-Ilan University, Israel*, presents an outstanding introduction to the intriguing story of the development of quantum computation and annealing in a unique reader-friendly language with excellent cartoons and without any equation. We believe, this article will become a popular science classic in this particular domain. Das from the *Indian Association for Cultivation of Science*, who contributed significantly from India in this development, has given next a brief overview of the developments in quantum annealing, in a semi-popular language. In the next, a somewhat technical review by Dridi, Alghassi and Tayur from *Carnegie Mellon University, USA*, the readers are introduced to the intriguing use of computational algebraic geometry in the study of adiabatic quantum computing employing the D-Wave quantum processors, as originally conceived by them. The review ends with a brief yet delightful summary section, exposing the key contributions to the developments of 'practical mathematics' in the last 4000 years from across the major cultures over human history and civilizations, ending with the present development. This section of the paper may be a must for every reader, in particular of Science and Culture! In the next article, Albash and Hen, stalwarts from *University of Southern California, USA* and one of the most intensely active group of research on quantum annealing and adiabatic quantum computation, indicate the remaining challenges and future prospects. More importantly, they indicate to some alternative potentially promising uses for such quantum annealers to provide advantages outside of optimization. In the next technical paper by Yamaguchi of the *Tokyo Quantum Computing, Japan*, an intriguing proposal for a new quantum annealing schedule or algorithm has been made. Finally, Mukherjee from *Saha Institute of Nuclear Physics, India*, reviews briefly the quantum spin glass physics as precursor to these developments. A very enthusiastic and graphic collection presented in one appendix of this paper helps partially establishing the main point of this special issue. We have also added in the end a brief conversation among Tayur and his students, colleagues of Carnegie Mellon University on a topic connecting business management and quantum computation.

It may be noted, due to unavoidable reasons, the section-subsection, figure and contents style followed in this special issue has been a little different from that in the regular issues.

We do hope, this special issue on 'quantum annealing and computation' will offer a very enjoyable feast of ideas in the making, and will inspire the young readers to explore further. We are extremely grateful to the contributors and thankful for the support received from the Editor-in-Chief of *Science and Culture* helping publication of this special issue. ❐

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Prof. Bikas K Chakrabarti is Emeritus Professor of Saha Institute of Nuclear Physics, S. N. Bose National Centre for Basic Sciences, and Visiting Professor of Economics in Indian Statistical Institute, Kolkata. His research interests include classical and quantum statistical physics and modelings of economic and social dynamics. He has authored, co-authored more than 185 papers, 7 reviews [including 4 in Reviews of Modern Physics] and more than 10 books [including 3 from Cambridge University press, 2 from Oxford University Press and 3 from Springer, 2 from Wiley]. He is Editorial Board member of several journals including European Physical Journal B (EPS; Executive Editor), Journal of Economic Interaction and Coordination (Springer) and Scientific Reports (Nature). He received the S. S. Bhatnagar Award of CSIR (Govt. of India) in 1997 and Outstanding Referee Award of the American Physical Society in 2010. He is Fellow of the Indian National Science Academy & the Indian Academy of Sciences, and is presently J. C. Bose National Fellow of the DST (Govt. of India).

Editor's Note : *Richard Feynman and Yuri Manin in 1980s conceived the need of a quantum computer to simulate quantum systems when the number of probabilities becomes unimaginably large. Quantum computation, which was a theoretical artifact at the beginning, has now reached a stage of realization through the sustained efforts of various scientists world-wide. We are very happy to announce that the name of our Guest Editor of this Special issue features in the timeline of quantum computation. A part of the 'Timeline of Quantum Computing' has been presented at the end of this issue (p.232).*

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