DOI: https://doi.org/10.36094/sc.v89.2023.Quantum_Droplets_in_Ultra_Cold.Siddik_Sekh.383

ARTICLE

Sci. and Cult. 89 (11–12) : 383-386 (2023)

QUANTUM DROPLETS IN ULTRA COLD ATOMIC SYSTEMS

SK. SIDDIK AND GOLAM ALI SEKH*

We consider Bose-Einstein condensates (BEC) beyond mean-field theory by taking account quantum fluctuation. The quantum fluctuation due to Lee-Huang-Yang term along with mean field interaction gives an effective Gross-Pitaevskii equation that describes quantum droplets. Density distributions of droplets are localized with flat top and they are linearly stable. We also calculate Shannon entropy of the quantum droplet and find that it starts to increase abruptly at the threshold of droplet phase.

Introduction

Since the experimental observation of Bose-Einstein condensates (BECs) by Eric Cornell and Carl Wieman of the University of Colorado and Wolfgang Ketterle of MIT[1,2], the system ultra cold atoms have been exploring to study different physical phenomenon both experimentally and theoretically due to its flexibility to control different parameters. Several phenomena like vortex formation, Mott insulator, superfluid-Mott transition, Anderson localization and delocalization, supersolid, Josephson and Bloch oscillation have already been observed.

Atoms in the condensates are weakly interacting through mean-field interaction. This interaction can, however, be controlled with the help of Feshbach resonance technique that allows to generate matter-wave solitons in the BECs. A soliton is a solitary wave which is a localized travelling wave that does not spread or disperse, but retain its size, shape, and speed when it moves. Bose-Einstein condensate supports dark solitons for repulsive interactions and bright solitons for attractive interactions. In a periodic potential the BEC with repulsive inter-atomic interaction can permit matter-wave gap solitons.

Beside the mean-field interaction, one can expect another type of interaction due quantum fluctuations. An interplay of quantum fluctuation and effective mean field interaction allows the formation of self- bound liquid like states. Thus, the generation of quantum droplets is purely a manifestation of quantum nature[3]. Quantum droplet is one of the most remarkable discoveries in contemporary science. Its generation in very dilute ultra-cold gases does not follow the classical liquid. The main advantage of creating QDs in ultra-cold atomic systems is that it is highly controllable with external parameters[4,5].

Our objective in the present paper is to study the properties of quantum droplet in the BEC and analyse its linear stability. It is seen that the attractive mean-field interaction leads to a sharp-peaked density distribution. Top of the distribution becomes flat due to quantum fluctuation. Both the states are linearly stable. We see that the change in the properties of density distribution in quantum droplet phase is manifested in the change of Shannon entropy. Several studies based on Shannon entropy clearly reveal the fact that it is efficient to detect global changes of the properties of a system[6-8].

In section 2, we discuss the theoretical formulation and write an effective equation to describe the effects of quantum fluctuation. More specifically, we get a modified Gross-Pitaevski equation with Lee-Huang-Yang term. In section 3, we present linear stability analysis and measure Shannon entropy of the droplet. In section 4, we make concluding remarks.

* Department of Physics, Kazi Nazrul University, Asansol-713340, WB, India

Email: skgolamali@gmail.com

The Oretical Formulation for Quantum Droplets

Bose Einstein condensate (BEC) is a state of matter in which all particles coalesce into a single quantum mechanical lowest energy state when cooled below the absolute zero Kelvin temperature. Ground state dynamics of BEC with mean-field interactions and confined in an external potential $V_{ext}(\vec{r})$ can be described by Gross-Piteavaskii equation (GPE)

$$i\hbar\frac{\partial\psi(\vec{r},t)}{\partial t}$$

$$=\left(-\frac{\hbar^2}{2m}\nabla^2+V_{ext}(r)\right)\psi(\vec{r},t)+g\left|\psi(\vec{r},t)\right|^2\psi(\vec{r},t)\cdot(1)$$

Here $\psi(\vec{r},t)$ is the wave function which is normalized to N and $g=4\pi\hbar^2 a_s/m$. By substituting $\psi(\vec{r},t)=\varphi(r)e^{-i\mu t/\hbar}$ in Eq.(1), one can obtain the following time-independent GP equation to study properties of stationary solution.

$$\mu\varphi(r)=\left(-\frac{\hbar^2}{2m}\nabla^2+V_{ext}(r)\right)\varphi(r)+g\left|\varphi(r)\right|^2\varphi(r), \quad (2)$$

where μ represents the chemical potential of the condensate. It gives a self-bound soliton in attractive interactions (g < 0), called bright soliton[9]. For weakly interacting systems at temperature much below the critical temperature, the quantum fluctuations are negligible. However, there are some situations where quantum fluctuations are not negligible. The underlying theory relies on the Lee-Huang-Yang (LHY) correction to the mean field Gross-Pitaevskii equation (GPE). The effective GP equation with LHY term in describing Q1D Bose-Einstein condensates is given[10]

$$i\hbar\frac{\partial\psi}{\partial t}=-\frac{\hbar}{2m}\frac{\partial^2\psi}{\partial x^2}+V(x)\psi+\delta g\left|\psi\right|^2\psi-\frac{\sqrt{2m}}{\pi\hbar}g^{3/2}\left|\psi\right|\psi \quad (3)$$

For convenience, we reduce equation (3) into dimensionless form by adopting new variables $t=t_0\tilde{t}$, $x=x_0\tilde{x},\ \psi=\psi_0\tilde{\psi},\ V'(x)=\frac{V(\tilde{x})}{E_0}$. Equation (3) is turn out to be

$$i\frac{\partial\psi}{\partial t}=-\frac{1}{2}\frac{\partial^2\psi}{\partial x^2}+V(x)\psi+\left|\psi\right|^2\psi-\left|\psi\right|\psi \quad (4)$$

Here the tildes are omitted. In Eq. (4), length, time, energy and wave function are rescaled by $x_0=\frac{\pi\hbar^2\sqrt{\delta g}}{\sqrt{2mg^{3/2}}}$, $t_0=\frac{\pi^2\hbar^3\delta g}{2m\,g^2}$, $E_c=\frac{\hbar^2}{mx_0^2}=\frac{\hbar}{t_0}$, $\psi_0=\frac{\sqrt{2m}}{\pi\hbar\delta g}g^{3/2}$ respectively. For a stationary solution $\varphi(x)$ with chemical potential $\mu<0$, one can write

$$\mu\varphi=-\frac{1}{2}\frac{\partial^2\varphi}{d\,x^2}+V(x)+\varphi^3-\varphi^2. \quad (5)$$

Considering tight trapping in the transverse direction and flat trapping in the x-direction, the effect of trapping on the BECs can be ignored near the centre of the trap. Under this approximation, eq. (5) gives rise to the known family of exact QD solutions[11]

$$\varphi(x)=-\frac{3\mu}{1+\sqrt{1+\frac{9\mu}{2}}\cosh\left(\sqrt{-2\mu}x\right)}, \quad (6)$$

where μ lies between $0<-\mu<\frac{2}{9}$. The norm of Eq. (6) is given by

$$N=\frac{4}{3}\left[\ln\left(\frac{1+3\sqrt{-\mu/2}}{\sqrt{1+\left(\frac{9\mu}{2}\right)}}\right)-3\sqrt{-\frac{\mu}{2}}\right]. \quad (7)$$

In the limit of $\mu\to-0$, density profile $\rho(x)=|\psi|^2$ describes traditional bright soliton dominated by the mean-

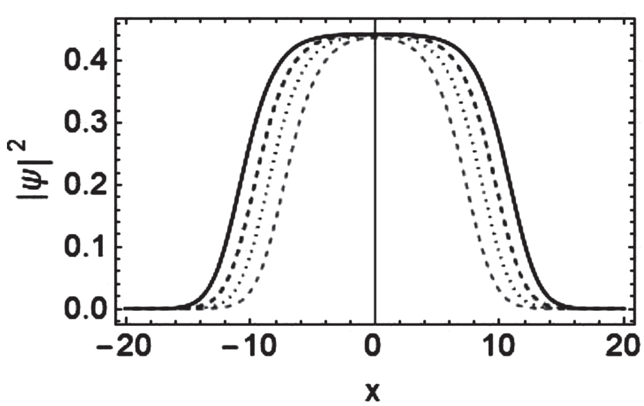

Fig. 1: Spatial density distribution, $|\psi(x)|^2$ for different values of N. Here, black solid, red dashed, blue dotted and black dotted-dashed give the densities for N=9.2633, 8.23159, 7.26615, 6.12967 respectively.

field cubic non-linearity. However, density profiles changes in $\mu \to -\frac{2}{9}$ limit (Fig.1). The top of the density profile becomes flat as the system tends towards the QD regime due to interplay between mean-field attractive interactions and repulsive interaction arising from quantum fluctuations.

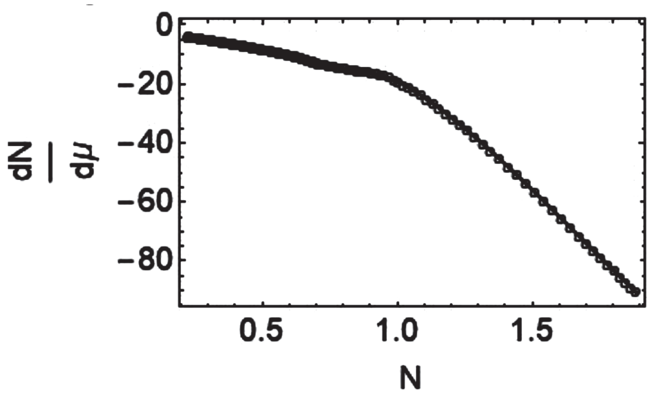

Fig. 2: Variation of $\frac{dN(\mu)}{d\mu}$ with the norm *N*.

With a view to check linear stability of quantum droplet we make use of Vakhitov-Kolokolov (VK) criterion which states that the solution in Eq. (5) will be linearly stable if $\frac{dN(\mu)}{d\mu} < 0$. In figure 2, we display $\frac{dN(\mu)}{d\mu}$ as a function of *N*. It clearly shows that $\frac{dN(\mu)}{d\mu}$ remains negative and thus droplets are linearly stable[11]. It may be noted that VK criterion is a necessary condition for the stability of localized states supported by any self-attractive nonlinearity[10].

Shannon Entropy of Quantum Droplet

In order to identify the changes of traditional soliton and flat top soliton as the chemical potential varies $\mu \to -0$ to $\mu \to -\frac{2}{9}$, we make use of Shannon entropy. This entropy possesses a global character and thus efficient to detect global change in the density distribution. The Shannon entropy $\left(S_\rho\right)$ in the position space is defined by[6-8,12]

$$S_\rho = -\int \rho(x) \ln \rho(x)\, dx . \tag{8}$$

Here $\rho(x)$ stands for density distribution corresponding to the wave function ϕ normalized to . It is interesting to note that the entropy suddenly increases as $\mu \to -\frac{2}{9}$ indicating the appearance of droplet phase. The entropy is minimum near the droplet phase. It again increases as the system tends toward soliton phase. However, the increment in this is much slower than that in the case droplet phase (Fig. 3). These two phases are clearly visible if one looks the change of S_ρ with $\frac{dN(\mu)}{d\mu}$. More specifically, it finds a distinct minimum in between the soliton and droplet phases (Fig. 4). Negative values of $\frac{dN(\mu)}{d\mu}$ implying that both the phases are linearly stable.

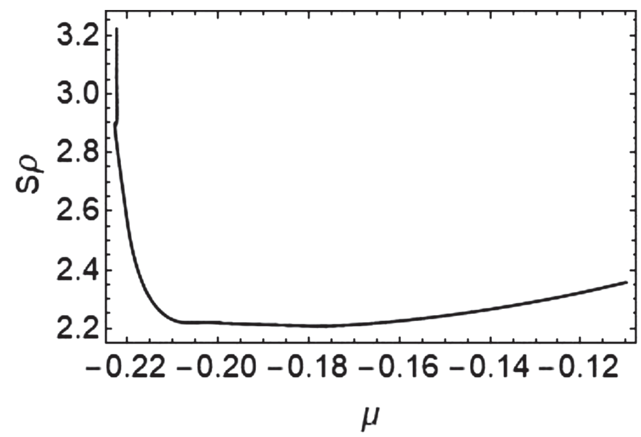

Fig. 3: Shannon entropy with chemical potential.

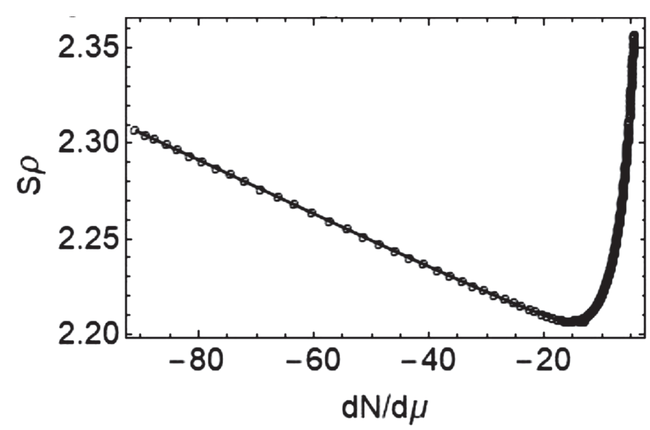

Fig. 4: Shannon entropy versus $\frac{dN(\mu)}{d\mu}$.

Conclusion

Quantum droplets is one of the manifestations of quantum fluctuation in Bose-Einstein condensates. It needs a theoretical description beyond mean-field theory. A mean-field approximation at zero temperature leads to the formation of bright matter-wave solitons due to interplay between dispersion and attractive nonlinear interaction. In presence of quantum fluctuations, we can expect to observe an interplay between the bright solitons and quantum droplets. At the point of transition from soliton to droplet state, the three energy scales; kinetic energy, mean-field energy and quantum fluctuation are comparable. We have measured the Shannon entropy and found that it increases abruptly in the quantum droplet phase while its increment is slower in the soliton phase. In between these two phases, the entropy finds its minimum value.

Acknowledgement:

S. Siddik would like to thank "West Bengal Higher Education Department" for providing Swami Vivekananda Merit Cum Means Scholarship. GAS would like to acknowledge the funding from the "Science and Engineering Research Board (SERB), Govt. of India" through Grant No. CRG/2019/000737. ❐

References

1. M.R. Anderson, J.R. Ensher, M.R. Matthews, C.E. Wieman, and E.A. Cornell, *Science*, **269,** 198-201(1995).
2. K.B. Davis, M.O. Mewes, M.R. Andrews, N.J. Van Druten, D.S. Durfee, D.M. Kurn, and W. Ketterle. *Phys.Rev. Lett.* **75,** 3969 (1995).
3. L. Chomaz, S. Baier, D. Petter, M. J. Mark, F. Wachtler, L. Santos, and F. Ferlaino, *Phys. Rev.* X **6**, 041039 (2016).
4. C.R. Cabrera, L. Tanzi, J.Sanz, B. Naylor, L.Thomas, P. Cheiney, and L. Tarruell, *Scince*, **359**, 301-304(2018).
5. Joseph C. Smith, D. Baillie, and P.B. Blakie, *Phys. Rev. Lett.* **126**, 025302 (2021).
6. K.K. Dey, S. Das and G.A. Sekh**,** *Eur. Phys. J.* D **73**, 18 (2019).
7. Golam Ali Sekh, A. Saha and B. Talukdar, *Physics Letters A* 382, 315-320(2018).
8. S. Chatterjee, G.A. Sekh and B. Talukdar, *Reports on Mathematical Physics* **85**, 281-291(2020).
9. S. Novikov, S.V. Manakov, L.P. Pitaevskii, V.E. Zakharov, Theory of solitons: the inverse scattering method (Springer Science & Business Media, 1984).
10. D. Petrov and G. Astrakharchik, *Phys. Rev. Lett.* **117**, 100401 (2006).
11. A. Debnathand A. Khan, *Ann. Phys. (Berlin)* **533**, 2000549 (2021).
12. B. Layek, S. Sultana, and G.A. Sekh, *Can. J. of Phys.* **101**, 303-308(2023).