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ARTICLE

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THE PHYSICS BEHIND THE PITCHED DRUMS OF KERALA

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Drums are an integral part of the cultural life in India. One special characteristic of many Indian drums is that they produce a well-defined pitch, compared to western drums. In this article, the harmonic nature of the strokes of four pitched drums Maddalam, Mridangam, Tabla and Edakka commonly used in Kerala are studied. Our studies show that Edakka, a unique traditional musical drum from Kerala is the best musical drum among the other drums studied.

Introduction:

Many drums are used in various art forms to produce music in different states and regions of India[1]. The most ancient and popular drum that produces sound with a definite pitch is the Mridangam. There are many other drums in various parts of India that resembles Mridangam in their structure, like Maddalam in Kerala, Khol in West Bengal, Pung in Manipur and Pakhawaj played in Dhrupad style of music are some among them[2–5]. All of them produce pitched sounds. Mridangam is the largely used instrument in South Indian classical music and Tabla is its counterpart in North Indian music having a key role in concerts. In Kerala, a southern state of India, four drums-Maddalam, Mridangam, Tabla and Edakka with clear pitch are commonly played. We made a study of the harmonics produced by these drums and also the fractal nature of the drums, which is a measuring tool of the musical content. These drums are shown in Figure 1. Before going to the detailed study, we will give a brief description of these drums in the next section.

Description of the Drums:

Mridangam: Mridangam is an ancient drum made with wood and covered with animal skin [6]. In Mridangam, the central region of the drum head is stretched by applying a black paste made by mixing a fine powder of puranakeedam stone and rice. This loading creates fine harmonics in the drum[7]. Mridangam tones are loud and they persist for a longer period[8]. To play a Mridangam, the artist needs high energy and coordination of all parts of the hand[9]. To make the body of the drum, Redwood, Jackwood and Margosa wood are used and among them, Margosa wood is referred as ideal in ancient Tamil literature, "Pacha Marabu"[10]. A Mridangam consists of valamthala that produces fine pitched tones and thoppi that produces bass sound and both are attached to a cylindrical barrel made with wood using leather straps made from buffalo skin[11]. The wooden

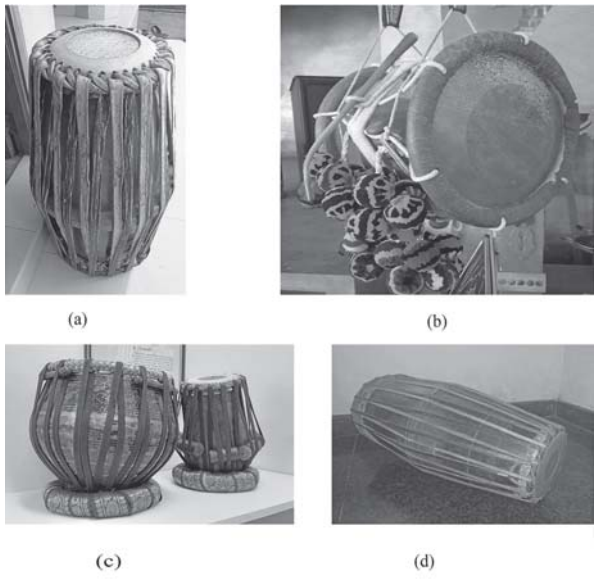

Figure 1: Images of (a) Maddalam (b) Edakka (c) Tabla and (d) Mridangam

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barrel has only a small influence on the harmonic tones of Mridangam as only the axial modes of the shell is combined with axis-symmetric modes of the membrane[12,13].

Maddalam: Maddalam is a percussion drum that resembles Mridangam in structure and is played in percussion ensembles and art forms like Kathakali. There are three types of Maddalam namely Thoppimaddalam, Veeramaddalam and Sudhamaddalam differing in their size and use[14]. The construction and size of Thoppimaddalam are similar to Mridangam and it is the smallest among the three variants of Maddalam played at temple rituals. The drum was played at some art forms like OttanThullal that was subsequently replaced by Mridangam[15]. Presently, Thoppimaddalam is played only at Krishnanattam in Guruvayoor temple, a famous temple in Kerala[16]. Veeramaddalam is a rare variant and the biggest drum used during army parades and royal ceremonies in the past[17]. Suddamaddalam is the commonly used variant with size midway between two other variants and is played in Panchavadhyam and other art forms. The similarities in construction, shape and lack of clear contrast in ancient literature between Mridangam and Maddalam indicates that Maddalam could be a variant of Mridangam and evolved later to the present form[18]. It's different variants are played in Odisha, Assam, Jharkhand, West Bengal and Nepal[19].

Tabla: Tabla is a popular drum set, that consists of two drums-dayan and bayan. The dayan has central loading on its drum head, whereas in bayan eccentric loading is present[20]. The air is enclosed in the body of Tabla drums. The weight added by the surrounding air and the air enclosed in the cavity of the drums increases the duration of vibration and produces prolonged tones in Tabla[21]. Both drums are covered with different layers of membranes made of goat and buffalo skin loaded with black paste as in Mridangam. The drum dayan is a small drum whose body is made with wood and played with the right hand whereas the body of bayan is made with copper and played with left hand[22]. The black paste on the dayan produces a pitched sound that sustains more and it also generates overtones that give the characteristic tone quality for the drum[23]. In Tabla, the region struck by the player on the drum head and his hand gesture are important in the generation of a particular tone[24]. In Tabla the tuned frequency and the diameter of the drumhead are related inversely and hence, a high pitched Tabla has a small diameter in comparison with low pitched Tabla[25].

Edakka: Edakka is a musical drum played mainly in temples of Kerala and also in various art forms and ensembles in Kerala and its reference is found in ancient literature[26]. The drum is capable to generate both rhythm and music[27]. When played with stick, two parallel strings fixed across the drum head below the membrane produces vibratory effect[17]. While playing the drum, the artist moves the barrel upward and downward that stretches the membrane and the change in tension helps to shift the tones to a wide range[28]. The body of the instrument is made with jackwood and the drum head is made of hide obtained from the intestine of cow[29]. Four wooden rods are placed on the ropes connecting the drum heads termed Jeevakol that represent four Vedas[30]. As decoration to the drum, 64 multi-coloured balls that represent 64 types of arts are also attached to wooden rods. After playing the drum, it is disassembled and the drum is not kept on the ground in the assembled form. The drum is played near sanctum sanctorum in temples and its music connects the deity and the devotee as the word "Edakku" means the centre in Malayalam, the language of Kerala[31].

In this paper, the frequency ratios of the four pitched drums described above are first studied and are given in the next section. From the frequency spectra of different strokes, the ratios of different modes are found.

Harmonics:

Mridangam: [1]In Mridangam four strokes are found resonant and clear on hearing. The four strokes include Dheem, Chapu, Dhin and Nam. The sound sample of Mridangam tuned to a pitch "d sharp" is used in the study. On a standard musical scale, the pitch is 155.6Hz. The first five peaks in the spectrum are noted in this study. The frequency spectrum of strokes are given in Figure 2. All the strokes in Mridangam produce good harmonics. The frequency ratios of different strokes are given in Table 1. In the spectra of strokes, more than one peak is present with nearby frequencies and these are degenerate modes of vibration. The histograms of strokes of Mridangam are shown in Figure 3.

From the frequency spectra of different strokes, the ratios of various modes are analysed and given in Table 2. The ratios obtained in the study are as predicted by C. V. Raman. The ratios of modes (0, 2), (1, 2) and (2, 2) are not identified from the peaks. These are degenerate modes and they are not excited in the collected samples of the strokes.

[1] A part of the studies on Mridangam are presented in XXXV Annual IAPT Convention & National seminar on innovation in physics teaching and research held at Indore on 28-30 November 2021.

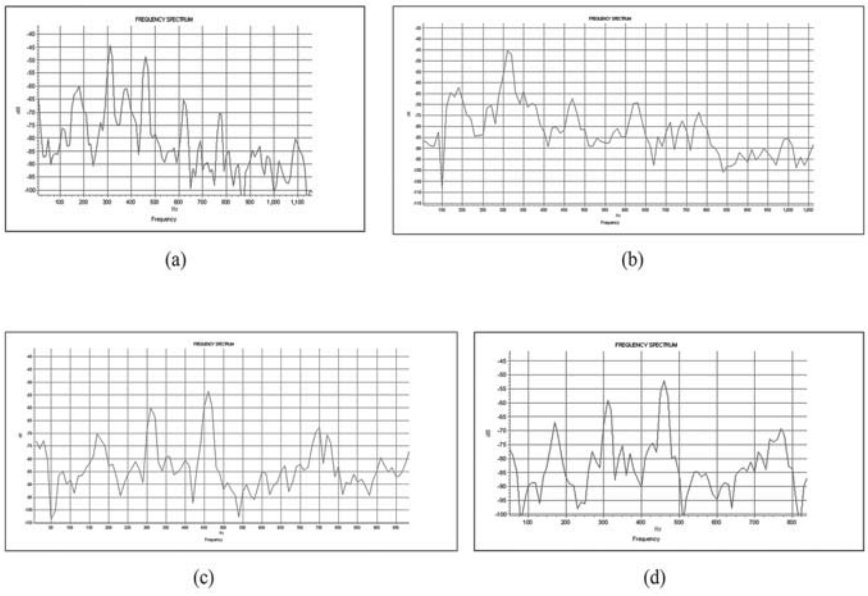

Figure 2: Frequency spectrum of (a) Dheem (b) Chapu (c) Dhin and (d) Nam strokes in Mridangam

Table 1 : Frequency ratios of strokes of Mridangam

Peak	Dheem		Chapu		Dhin		Nam	
	Frequency (Hz)	Ratio	Frequency (Hz)	Ratio	Frequency (Hz)	Ratio	Frequency (Hz)	Ratio
1	180.351	1.15	190.136	1.22	169.667	1.09	170.446	1.09
2	310.238	1.99	310.225	1.99	309.71	1.99	309.9	1.99
3	460.108	2.95	469.773	3.01	460.236	2.95	459.599	2.95
4	621.968	3.99	630.178	4.04	600.26	3.85	———	——-
5	771.838	4.96	780.29	5.18	770.039	4.94	769.806	4.97

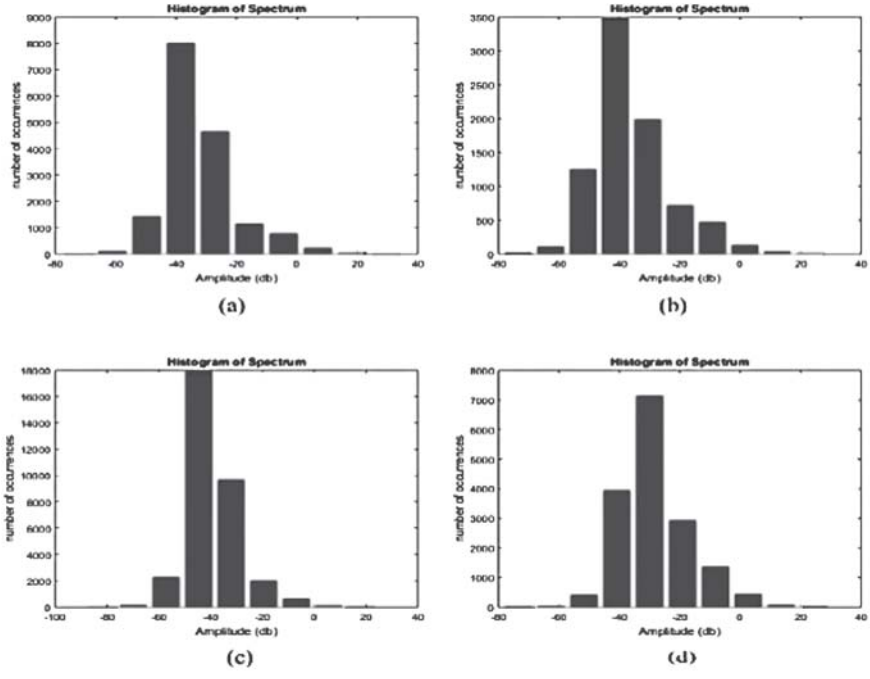

Figure 3: Histogram of (a) Dheem (b) Chapu (c) Dhin and (d) Nam strokes of Mridangam

Maddalam: In Maddalam three clear and resonant strokes are identified. They are Dheem, Dhi and Nam. Dheem is an open stroke played by striking at the centre in the pasted region. Dhi is played at the central region by resting the fingers at the centre to give a suppressing effect. Frequency spectra of strokes of Maddalam are shown in Figure 4. Nam is played near the boundary. The frequencies and their ratios of Dheem, Dhi and Nam strokes are given in Table 3. The peaks in Dheem stroke form integer relation with lowest peak. For Nam and Dhi strokes, the relation is not perfectly harmonic. The values indicate that the drum Maddalam is not a perfect pitched drum in comparison with Mridangam. The drum is played mostly as a rhythmic drum and hence its pitched character of strokes is not well established. Changes in the shape and construction could be the factors that make it a less harmonic drum. The distributions of amplitudes in each stroke of Maddalam from their frequency spectra are represented in the form of a histogram in Figure 5.

Tabla: In Tabla three strokes are studied as they are resonant on hearing. The strokes include Tun, Tin and Na. The observed peaks in Tun, Tin and Na strokes are given in Table 4. The frequency spectrum of Tun, Tin and Na strokes are given in Figure 6. The histogram of different strokes of Tabla are shown in Figure 7. The ratio of frequencies for different modes of vibration are analysed from the frequency ratios obtained for different peaks in the frequency spectrum and are given in Table 5. Like Mridangam, Tabla also has degenerate modes of vibration. Like Mridangam, we got an integer ratio for various modes of vibration in Tabla. The ratios for different strokes

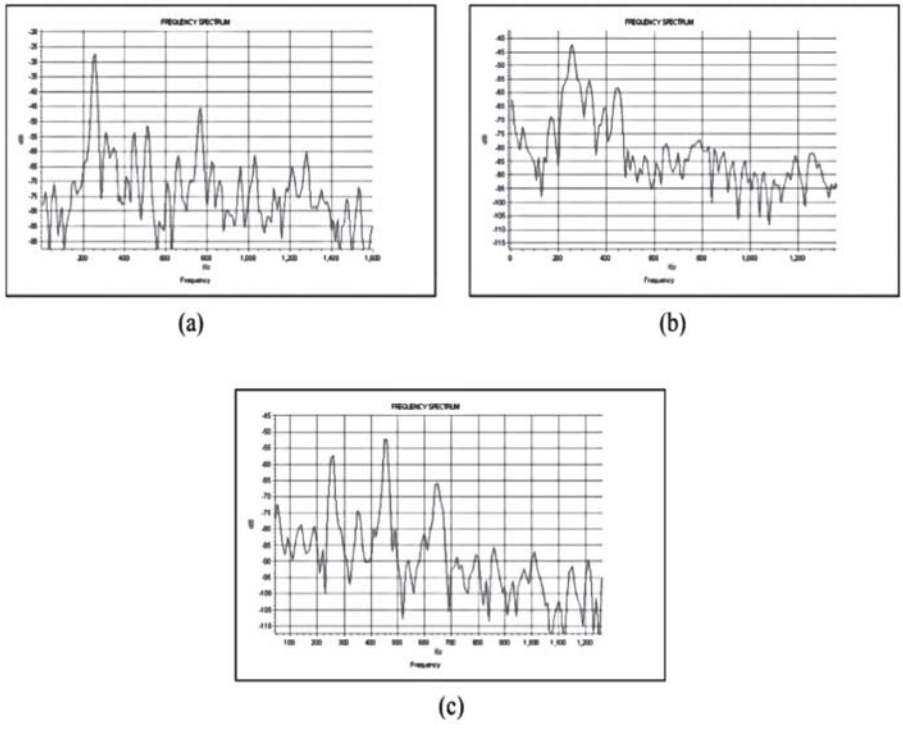

Figure 4: Frequency spectrum of (a) Dheem (b) Dhi and (c) Nam strokes of Maddalam

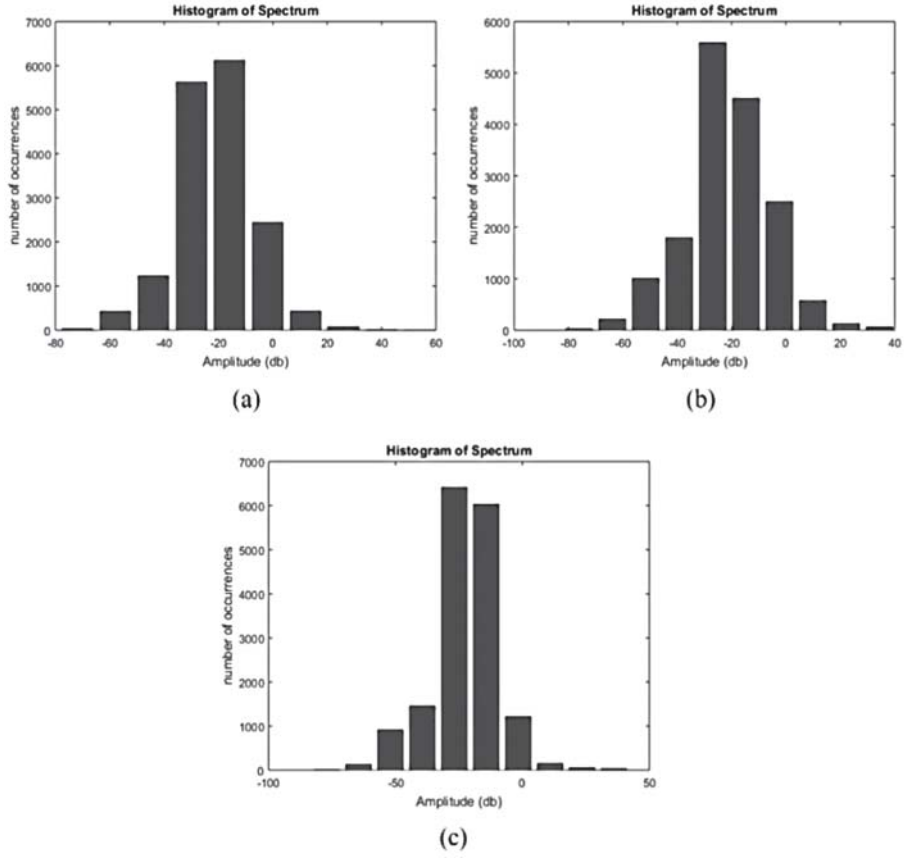

Figure 5 : Histogram of (a) Dheem (b) Dhi and (c) Nam strokes of Maddalam

of Tabla are better compared with Maddalam strokes. Hence Mridangam and Tabla are more harmonic than Maddalam.

Edakka: In Edakka, the sound producing mechanism is different from loaded drums. But, the drum creates good harmonic tones. This is possible by the alteration of tension on the drum head by pushing the barrel upward and downward while playing the instrument. The frequencies present in a stroke on Edakka are plotted and is given in Figure 8. The ratios of frequencies in Edakka sample are shown in Table 6. In Edakka, there are no names for the strokes. Different notes are played by changing the pitch. The distribution of amplitudes is plotted for three sample notes named sample 1, sample 2 and sample 3 of Edakka and are shown in Figure 9.

Discussion

From the histograms of all the strokes, it is seen that all of them have normal distribution leaning towards the left for their amplitudes in the spectrum. For all Maddalam strokes, the maximum number of amplitudes occur around -20 dB. In Mridangam strokes Dheem, Chapu and Dhin, the maximum amplitude is seen around -40 dB and for Nam stroke maximum is seen between –20 dB and -40 dB. For Tabla stroke Tun, maximum amplitude occurs around -20 dB and -40 dB. For Tin stroke, the maximum is below -50dB and for Na stroke maximum is seen around -20 dB. In Edakka samples maximum occurrence of amplitudes are seen for sample1, sample2 and sample 3 around -30 dB, -25 dB and -25 dB respectively. From the study of the

Table 2: Frequency Ratios of Modes of Mridangam

Mode	Frequency ratio
(0,1)	1.09
(1,1)	1.99
(2,1)	2.95/3.01
(0,2)	—
(3,1)	3.85/4.04
(1,2)	—
(4,1)	4.97
(2,2)	—
(0,3)	5.18

Table 3: The Frequency Ratios of Strokes of Maddalam

Peak	Dheem		Dhi		Nam	
	Frequency (Hz)	Ratio	Frequency (Hz)	Ratio	Frequency (Hz)	Ratio
1	258.629	1.00	259.329	1.00	260.728	1.00
2	511.171	1.97	449.214	1.73	459.887	1.76
3	769.203	2.97	650.966	2.51	650.572	2.49
4	1029.98	3.98	——	——-	860.325	3.29
5	1279.776	4.94	———	———	1010.754	3.87

Table 4: Frequency Ratio of Strokes of Tabla

Peak	Tun		Tin		Na	
	Frequency (Hz)	Ratio	Frequency (Hz)	Ratio	Frequency (Hz)	Ratio
1	318.766	1.10	318.906	1.07	319.778	1.08
2	579.211	2.00	590.909	2.00	590.238	2.00
3	880.624	3.04	869.977	2.94	879.745	2.98
4	1179.111	4.07	1180.838	3.99	1180.679	4.00
5	1489.303	5.14	1459.906	4.94	1489.232	5.04

Table 5: Frequency Ratios of First Nine Modes in Tabla

Mode	Frequency Ratio
0,1	1.10
1,1	2.00
2,1	2.94/2.98
0,2	3.04
3,1	3.99/4.00
1,2	4.07
4,1	5.04
2,2	5.14
0,3	4.94

frequency spectrum, the harmonic nature of drums are analysed. Among the drums with a pasted region on the drum head, Mridangam and Tabla contain the most harmonic tones. It is possible to identify the frequency ratio of the first nine modes from three resonant strokes in both drums. Earlier, the acoustics of Tabla and Mridangam were studied by many authors[32–35]. C. V. Raman through his experiments found various modes involved in the tones of Mridangam and Ramakrishna and Sonthi explained the observed modes of vibration with their theoretical model[36,37]. Unlike these drums, Maddalam has only Dheem stroke with an integer frequency ratio. Other strokes in the drum have a better ratio in comparison with strokes on the ordinary membrane. Hence Maddalam is partially harmonic in comparison with Mridangam and Tabla. For Edakka, when tension on the drumhead is changed the pitch or fundamental frequency also changes. Hence it is hard to identify the modes with changing tension with the obtained frequency ratios only. Edakka is a drum in which two parallel strings are present beneath the animal skin on the drum head. The combined vibration creates different harmonics. There is only single study regarding the theoretical formulation of such a membrane-string system in Edakka[28]. The drum creates fine tones with an integer ratio like Mridangam and Tabla. Unlike

Table 6: Frequency Ratios of Edakka Stroke

Peak	Frequency (Hz)	Ratio
1	171.041	1.00
2	350.655	2.05
3	520.15	3.04
4	689.645	4.03
5	849.021	4.96

Table 7: Fractal Dimension of Strokes of Mridangam

Stroke	Sample 1	Sample 2	Sample 3
Dheem	1.703	1.672	1.71
Chapu	1.705	1.661	1.667
Dhin	1.705	1.702	1.673
Nam	1.63	1.686	1.64

Table 8: Fractal Dimension of Strokes of Maddalam

Stroke	Sample 1	Sample 2	Sample 3
Dheem	1.662	1.676	1.664
Dhi	1.678	1.645	1.64
Nam	1.655	1.678	1.655

Mridangam and Tabla, it has a clear sharp fundamental with a large number of higher harmonics that follow integer relation. In Mridangam and Tabla clear peaks are seen up to the fifth or sixth position and higher peaks are getting damped due to the outer annular ring membrane on the drum head. Hence Edakka can serve as a better musical drum with a wide range of tones. Its peaks are damped only when the player adjusts the tension. C. V. Raman identified the loading on drum heads as an ancient Indian contribution towards making musical sound producing drums[38]. The technique adopted in Edakka adds one more method to produce harmonics in drums. Hence, in India we can see that there are two techniques available, to generate musical sound in drums. To have more insight into the differences of these drums we study the fractal nature of the music produced by the drums in the next section

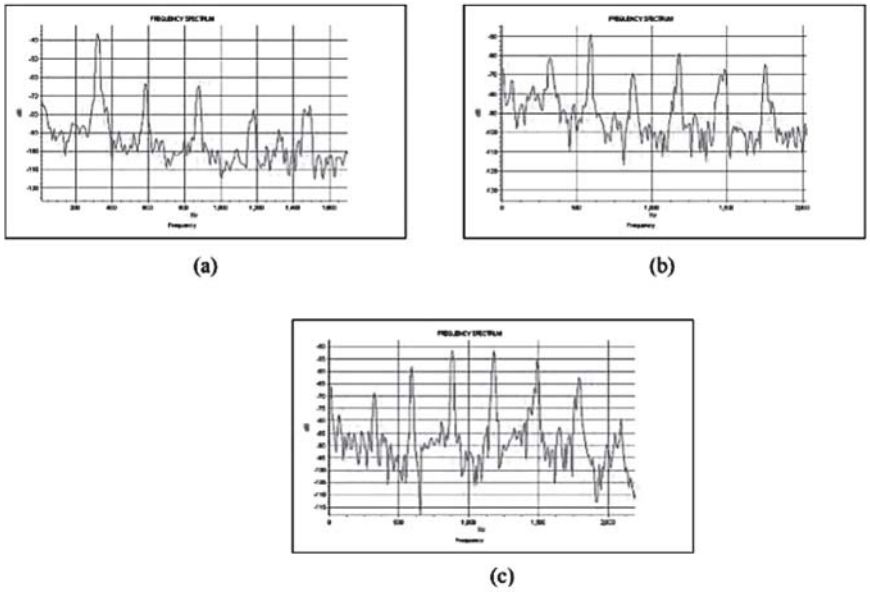

Figure 6: Frequency Spectrum of (a) Tun (b) Tin and (c) Na Strokes of Tabla

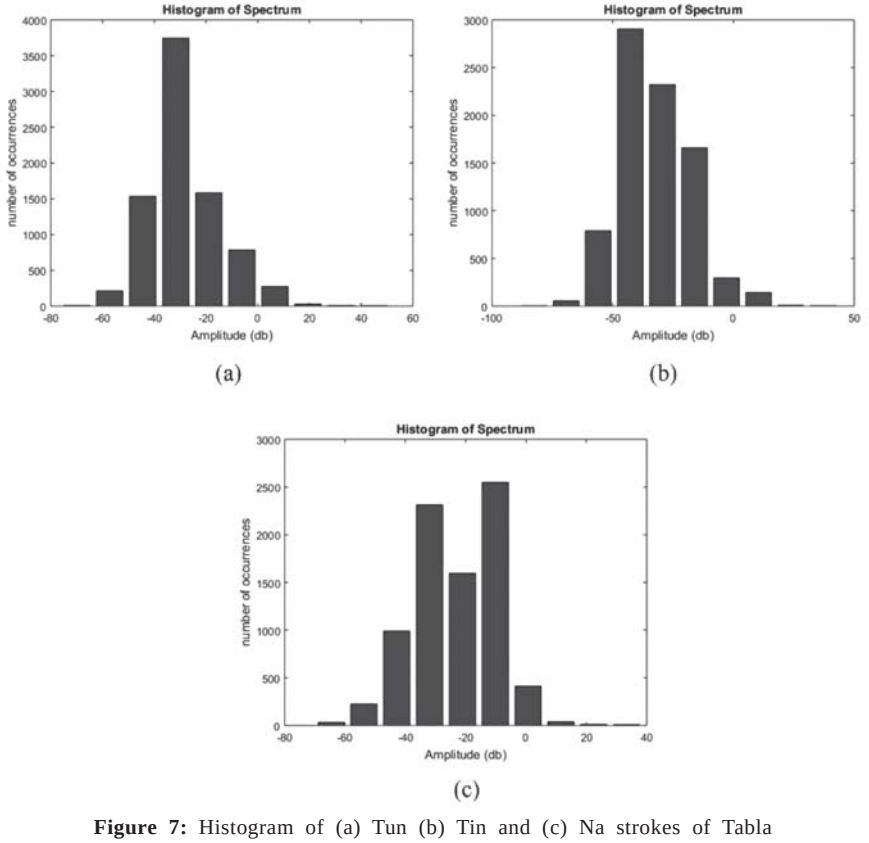

Figure 7: Histogram of (a) Tun (b) Tin and (c) Na strokes of Tabla

Fractal Study

It is known that musical sound has fractal nature[39]. Perrin S Mayer reported after studying different musical samples of various styles that the fractal dimension of the music is around 1.65[40]. Let us quantitatively find how much musical are the strokes of drums under study by finding the fractal dimension of different strokes.

The fractal dimensions of three samples of each stroke of drums are found. The obtained values of fractal dimension of strokes of Mridangam, Maddalam, Tabla and Edakka are given in Tables 7, 8, 9 and 10. The fractal dimension of different samples of Mridangam, Maddalam, Tabla and Edakka strokes are plotted in Figure 10. All samples have a fractal dimension between 1.506 and 1.71. It is seen that some stroke samples of Mridangam and Tabla have low value for fractal dimension than other samples. This shows that harmonic nature changes with different strokes in them as higher fractal dimension is an indication of better harmonic nature[41]. It is also seen that Edakka samples have an almost consistent value. The results suggest that Edakka can create musical sound with consistent harmonic character.

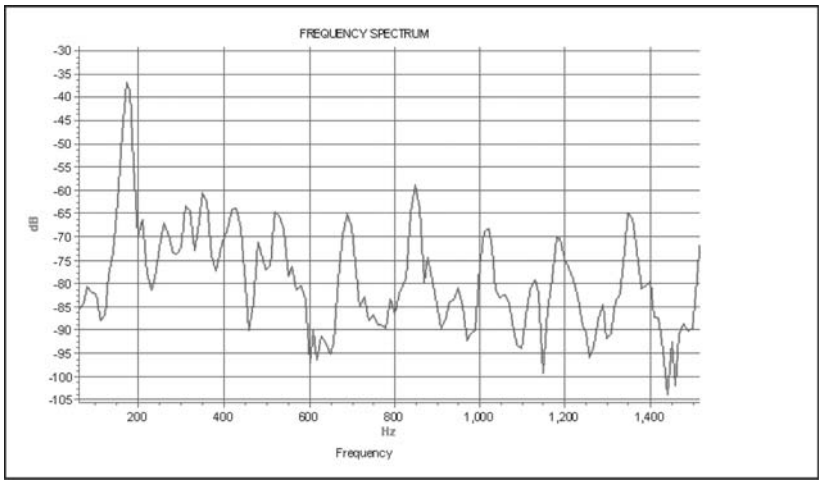

Figure 8: Frequency spectrum of stroke of Edakka

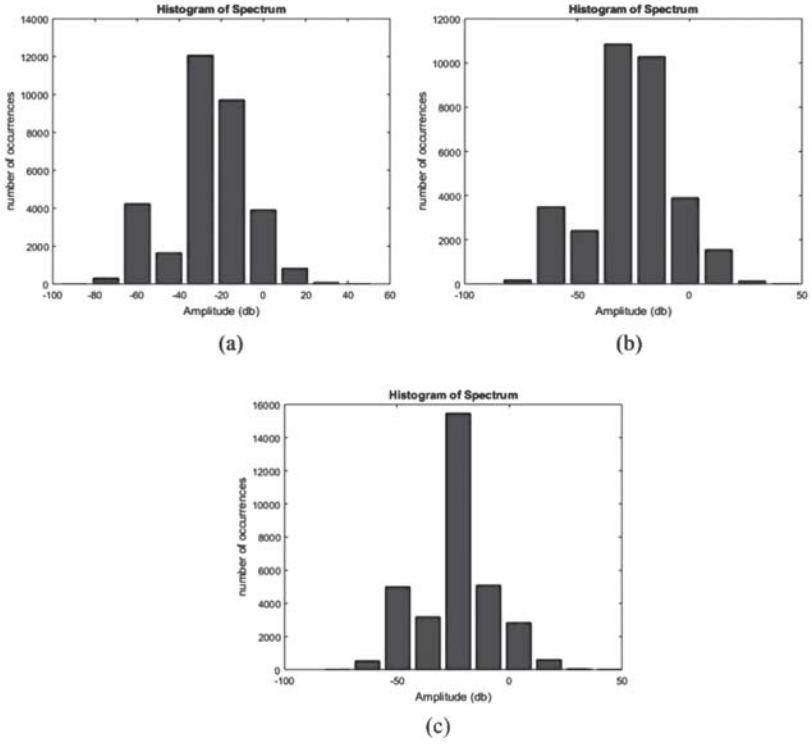

Figure 9 : Histogram of strokes of Edakka

Table 9: Fractal Dimension of Strokes of Tabla

Stroke	Sample 1	Sample 2	Sample 3
Tun	1.644	1.574	1.653
Tin	1.637	1.66	1.684
Na	1.625	1.506	1.575

Table 10: Fractal Dimension of Samples of Edakka

Sample Number	Fractal Dimension
1	1.678
2	1.68
3	1.672

Discussions

The fractal studies of strokes in Edakka showed that Edakka can generate harmonics consistently with different strokes. Hence it is a better music producing drum in comparison with Tabla and Mridangam. Edakka is only played in Kerala and traditionally kept inside a temple and it is considered as a divine instrument. Hence the drum is not popular among the people. The various possibilities of Edakka as a musical drum is not yet fully explored.

Conclusions

Drums are native to Indian culture and are the results of the cumulative efforts of the people who lived in India from ancient days onwards for the creation of better musical sound in drums. The loading at the centre of a drum head is the most popular and widely used technique in various drums in many parts of India. But, stretching the membrane by varying tension is a lesser-known technique and not commonly used. Incorporation of additional strings on the drum heads, making the barrels that move while playing a drum etc. could produce a new variety of drums that produce pitched tones. The favourable techniques and methods to make harmonic tones in such drums is a promising research topic.

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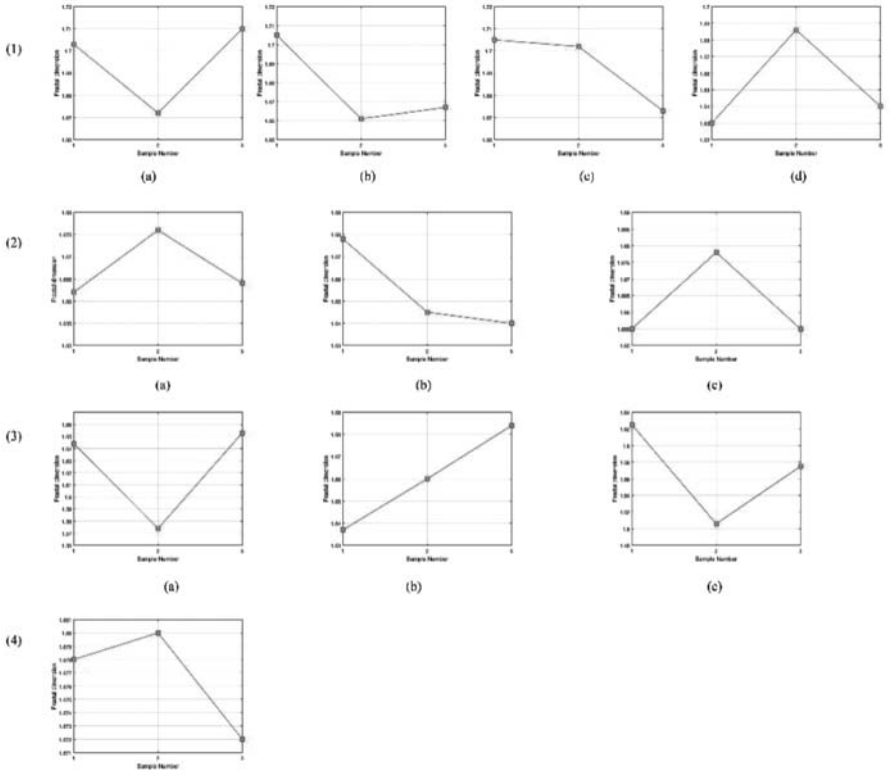

Figure 10: Fractal dimension of (1) Mridangam strokes (a) Dheem (b) Chapu (c) Dhin and (d) Nam ; (2) Maddalam strokes (a) Dheem (b) Dhi and (c) Nam; (3) Tabla strokes (a) Tun (b) Tin and (c) Na; (4) Edakka strokes

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