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COUPLING OF WAVE AND ENERGY CONVERSION IN A SLOWLY VARYING NONUNIFORM DUSTY PLASMA

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Propagation of electromagnetic and acoustic waves through non-uniform plasma and their mutual coupling due to various factors has important application in space and astrophysical bodies, ionosphere, etc. In this paper the propagation of low frequency ion acoustic waves with dust acoustic waves and electromagnetic radiation through slowly varying unmagnetized dusty plasma have been studied. When low frequency ion acoustic wave is incident at the boundary of each layer of the varying plasma medium, a part will be transmitted consisting of three components of waves such as ion acoustic, dust acoustic and electromagnetic wave and other part will be reflected back consisting of these components stated above. The generation of the excited dust acoustic wave and electromagnetic wave were evaluated using W.K.B. approximation for slowly varying plasma parameters.

Introduction

In a non-uniform plasma, compressional wave and e. m. waves are coupled and energy is transferred from compressional waves to e. m. waves and vice versa. In a slowly varying plasma, Bremmer[1], Tidman[2] and Chakraborty[3] studied the wave propagation and their mutual coupling using the technique of W.K.B. approximation. Also Khan and Paul[4] have investigated the coupling of electron-acoustic and e. m. waves in a slowly varying two component plasma and obtained the power flux of transmitted and reflected e. m. waves using W.K.B. approximation generated by electron acoustic wave.

However, the presence of charged dust grains has an important impact on the total scenario of coupling waves in a non-uniform plasma. Dusty plasmas produce some new electrostatic modes such as 'dust acoustic wave'[5], 'dust ion acoustic wave'[6] in the very low frequency limit.

In this paper, we have investigated the propagation of low frequency ion acoustic wave through unmagnetized slowly varying dusty plasma medium using the technique of W.K.B. approximation. When low frequency ion acoustic wave is incident at the boundary of each layer of the varying plasma medium, a part will be transmitted consisting of three components of waves such as ion acoustic, dust acoustic and e. m. wave and other part will be reflected back consisting of those components.

Basic Equations

We consider a three component non-uniform, collisionless, warm dusty plasma having electrons, ions and dust particles in absence of any static magnetic field. The dust particles are negatively charged satisfying the charged neutrality condition $n_{e0} + z_d n_{d0} = n_{i0}$. The non-uniformities of the plasma medium is due to the density gradient of the plasma constituents which varying slowly along the Z-direction and the scale length of the variation (L) is much smaller than the wave length (λ) of the incident wave. The basic equations are the equations of motion, the continuity equations for three fluids and the Maxwell's equations. We assume that the wave phase velocity $\omega / k << V_{te}$ so that we can neglect the electron

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inertia term from the corresponding momentum conservation equation. We assume that the charged on the dust grains are constant, so that we can neglect the damping arising due to the charged fluctuations.

To find the linearized plasma equations, we consider the following first order field quantities as

Now following[3], we obtain the following linearized coupled equations between transverse fields variables and the pressure perturbation for all relevant quantities,

$$\left(\nabla^2 + k_t^2\right)\vec{H} = \frac{4\pi ei}{c\omega\mu}\left[\left(\vec{\nabla}\mu\right)\times\vec{\nabla}\left(\frac{z_d\delta p_d}{m_d} - \frac{\delta p_i}{m_i}\right)\right] - \left[\frac{\vec{\nabla}\mu\times\left(\vec{\nabla}\times\vec{H}\right)}{\mu}\right] \quad (1)$$

$$\left(\nabla^2 + k_{li}^2\right)\frac{\delta p_i}{m_i} - \frac{z_d\omega_{pi}^2}{m_d V_d^2}\delta p_d = \frac{\left(\vec{\nabla}\omega_{pi}^2\right)\cdot\vec{\nabla}\left(\frac{z_d\delta p_d}{m_d} - \frac{\delta p_i}{m_i}\right)}{\mu\omega^2} + \frac{ic\left[\vec{\nabla}\omega_{pi}^2\cdot\left(\vec{\nabla}\times\vec{H}\right)\right]}{4\pi e\mu\omega} \quad (2)$$

$$\left(\nabla^2 + k_{ld}^2\right)\frac{\delta p_d}{m_d} - \frac{\omega_{pd}^2}{m_i V_i^2}\delta p_i = \frac{\left(\vec{\nabla}\omega_{pd}^2\right)\cdot\vec{\nabla}\left(\frac{z_d\delta p_d}{m_d} - \frac{\delta p_i}{m_i}\right)}{\mu\omega^2} + \frac{ic\left[\vec{\nabla}\omega_{pi}^2\cdot\left(\vec{\nabla}\times\vec{H}\right)\right]}{4\pi e z_d\mu\omega} \quad (3)$$

$$k_t^2 = \frac{\omega^2 - \omega_{pd}^2 - \omega_{pi}^2}{c^2}, \quad \mu = 1 - \frac{\omega_{pd}^2 + \omega_{pi}^2}{\omega^2}, \quad k_{li}^2 = \frac{\omega^2 - \omega_{pi}^2}{V_i^2},$$

$$k_{ld}^2 = \frac{\omega^2 - \omega_{pd}^2}{V_d^2}$$

$$\delta p_j = m_j V_j^2 \delta n_j, \quad V_j^2 = \chi T_j / m_j, \quad \omega_{pj} = \left(\frac{4\pi z_j^2 e^2 n_{j0}}{m_j}\right)^{1/2}.$$

Where χ, T_j, m_j, n_{j0} are respectively the Boltzmann constant, kinetic temperature, mass and equilibrium density of jth species.

W.K.B. Solution

We assume that the ion pressure perturbation is the only source of the excitation of the longitudinal waves and coupling occurs only by density gradients. We also assume that the equilibrium densities change slowly but continuously along OZ in such a way that the wave length (λ) of radiated waves is much less than the density gradient scale length i.e. $\lambda << L^2$, for which we can make the W.K.B. approximation. Under this condition the equation (1)and (3) reduces *t* the following form,

$$\left(\nabla^2 + k_t^2\right)\vec{H} = \frac{4\pi ei}{c\omega\mu}\left[\left(\vec{\nabla}\mu\right)\times\vec{\nabla}\left(\frac{z_d\delta p_d}{m_d} - \frac{\delta p_i}{m_i}\right)\right] \quad (4)$$

$$\left(\nabla^2 + k_{ld}^2\right)\frac{\delta p_d}{m_d} - \frac{\omega_{pd}^2}{m_i V_i^2}\delta p_i = 0 \quad (5)$$

The disturbances propagate in the (z, x) plane and the space dependence of the field quantities are of the form ~ $\exp(ik_0x)$.

The differential equation for the dust acoustic wave excited by the incident ion acoustic wave is

$$\left(D^2 + k_1^2\right)p_d(z) = \frac{m_d\omega_{pd}^2}{m_i V_i^2}p_i(z), \quad k_1^2 = k_{ld}^2 - k_0^2 \quad (6)$$

For W. K. B. solutions of the dust acoustic wave, using the method of variation of parameters[7], considering only real part we get

$$\delta p_d^t(z) = \frac{A_+}{\sqrt{k_1}}\exp\left[i\left(k_0x + \int k_1 dz - \omega t\right)\right] \quad (7)$$

$$\delta p_d^r(z) = \frac{A_-}{\sqrt{k_1}}\exp\left[i\left(k_0x - \int k_1 dz - \omega t\right)\right] \quad (8)$$

Where A_+ and A_- are the pressure amplitude of the transmitted and reflected part of the dust acoustic wave.

The differential equation of the excited transverse e. m. wave is of the form

$$\left(D^2 + k_2^2\right)\vec{H}(z) = \frac{4\pi ei}{c\omega\mu}\left[\left(\vec{\nabla}\mu\right)\times\vec{\nabla}\left(\frac{z_d p_d}{m_d} - \frac{p_i}{m_i}\right)\right],$$

$$k_2^2 = k_t^2 - k_0^2 \quad (9)$$

In the low frequency limit, the e. m. wave is attenuated.

Discussion

There is an extreme anomalous situation in the Saturn's F-ring, where the number density of free electrons is much smaller than the number density of ions. Since for this type of wave mode $v_{td},\ v_{ti} << \omega / k << v_{te}$ a very little amount of electrons is required in the back ground of collision less dusty plasma[6], this phenomena is relevance to low frequency noise in the F-ring of Saturn.

The optical depth of F-ring ~ 0.1 and there are micron to submicron ranged dust grains. Thus the number density of the dust grain varies as the grain radius. Hence Saturn's F-ring may be considered as the slowly varying medium and W.K.B. approximation is valid in this medium.

Thus for the ion acoustic wave incident on the Saturn's F-ring may generate the transmitted and reflected dust acoustic waves and the corresponding energies can be calculated by using W. K. B. technique as mentioned above. Due to the presence of varying grain distribution in such a media, the conversation of energy may be more efficient than the ordinary electron-ion plasma. ❐

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