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OPINION

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BOSE'S LETTER TO EINSTEIN

SUSHANTA DATTAGUPTA*

In this article we analyze S. N. Bose's letter to Albert Einstein, its background and its consequences, from a historical perspective.

Introduction

This year we are celebrating the centenary of the famous letter that Satyendra Nath Bose wrote to Albert Einstein from Dacca University, East Bengal of then undivided India (now Dhaka, in Bangladesh), on 4 June 1924. This letter is believed to be the harbinger of Quantum Statistics. The purpose of this article is to take an incisive look at the letter, the paper that ensued with Einstein's recommendation and the profound messages they contain. The relevant portion of the letter reads:

Respected Sir,

I have ventured to send you the accompanying article for your perusal and opinion. I am anxious to know what you think of it. You will see that I have tried to deduce the coefficient $8\pi\nu^2/c^3$ in Planck's Law independent of classical electrodynamics, only assuming that the ultimate elementary region in the phase space has the content h^3 ....I do not know whether you still remember that somebody from Calcutta asked your permission to translate your papers on Relativity in English. You acceded to that request. The book has since been published. I was the one who translated your paper on Generalized Relativity.

How did Bose arrive at the expression $8\pi\nu^2/c^3$, ν being the frequency and c the speed of light, and the importance of the Planck constant h, the Boltzmann constant k_B as well as the temperature T, in the rest of the Planck formula for thermal radiation? This will be the principal focus of our writeup, discussed in Section IV. But, before that, we comment in Section II, on the special geopolitical situation that Bose grew up in. Much of the material in Section II is borrowed from Venkataraman's monograph, written in a pedagogical style[1]. Further in Section III, we provide a background in which Bose was inspired to write the celebrated letter, especially in the context of the radiation formula of 1900[2] and Einstein's analysis of it in 1905[3]. Finally, in Section V, we offer our concluding remarks. After completing the manuscript G. Baskaran made me aware of his paper on 'Boson Bloom' that elaborates on many of the issues and anecdotes discussed in the present article, albeit with a different focus and stylistic emphasis[4].

Bose's Personal Background

Satyendra Nath Bose, known as S. N. Bose to the scientific world, was born on 1 January 1894. His ancestral home was in Boro Jagulia, a village adjacent to the site of the Indian Institute of Science Education and Research, Kolkata. His father Surendra Nath worked in the Railways, the great institution under the British Raj that had employed so many middle-class Indians. Satyen was the eldest and the only son amongst six younger sisters.

Extraordinary scientists and philosopher-poets come in two categories – one apparently disenchanted with regimented school curricula and the other with brilliant academic records. Albert Einstein† and Rabindranath Tagore, both of whom had greatly influenced Bose, are of the former type while Bose belonged to the latter. Apart from the superlative marks in his examinations (see Fig. 1),

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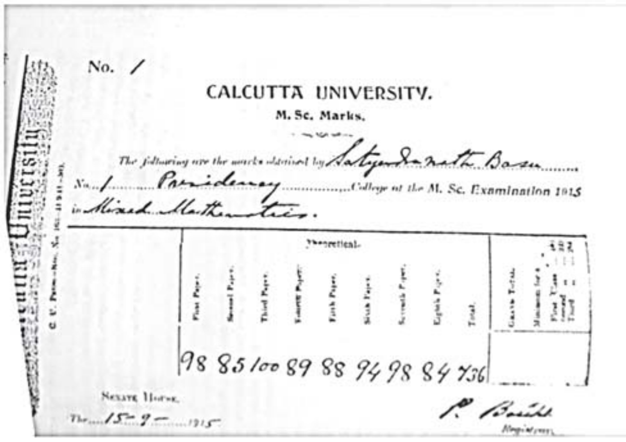
it is interesting to note that Bose's subject was 'Mixed Mathematics' – a truly interdisciplinary topic. Such was the approach of that generation of students who did not want to get compartmentalized within their narrow disciplines – an example to ponder on, for today's students.



No. 1

CALCUTTA UNIVERSITY.

M. Sc. Marks.

The following are the marks obtained by Satyendranath Basu

No. 1 Presidency College at the M. Sc. Examination 1915

in Mixed Mathematics.

Theoretical.									
First Paper.	Second Paper.	Third Paper.	Fourth Paper.	Fifth Paper.	Sixth Paper.	Seventh Paper.	Eighth Paper.	Total.	Grand Total.
98	85	100	89	88	94	98	84	736	

Senate House,

The 15-9-1915

Registrar.

Fig. 1. Bose's M. Sc. marksheet

Bose studied in the Hindu School of Calcutta. After the school final examination in 1909, he had entered the Presidency College. At that time, there were several excellent teachers in the college including J. C. Bose and P. C. Ray (see Fig. 2 and 3). The individuals depicted in these two figures are a representative set of scientists amongst a slew of creative poets, philosophers and writers who had adorned the landscape of the great Bengal renaissance. Two of the illustrious scientists: M. N. Saha and P. C. Mahalanobis, were Bose's classmates during his B. Sc. and M. Sc. days. Bose outclassed Saha and came first while Saha came second. After M. Sc., Bose and Saha were invited by the-then Vice Chancellor of Calcutta University Ashutosh Mukherjee, to join as Lecturers in the Physics Department, just as Mukherjee had anointed P. C. Ray and C. V. Raman into the Palit Chairs of Chemistry and Physics.

Both Bose and Saha started teaching Physics, but as Bose said later: When we began to teach, we had to do things seriously. We had to try to do something original. This is a wonderful illustration of the realization that there is a synergy between teaching and research – a fact which we seem to have forgotten in the present separation of universities and research institutions in our country. It is also pertinent to keep in mind the general socio-political atmosphere in which Bose and Saha worked. A great 'Swadeshi' movement was sweeping across India, Bengal in particular. The British had machinated to partition Bengal in 1905. Tagore penned several swadeshi songs at that time, one of which is now the national anthem of Bangladesh. In the face of severe opposition, the government had to retract their ploy to divide the people, in 1911. The year 1911, incidentally was also the year in which the Mohan Bagan club of Calcutta was the first truly Indian team to win the IFA shield of football. The point is, Bose, Saha and Raman, to name a few, were all members of that generation which was out to prove that the natives were no inferior to their British masters in the pursuit of excellence in science and other fields, including sports.

In 1921, the Dacca University was initiated. Dr. Philip Hartog, the first Vice Chancellor, wanted to build from scratch viable departments and chose Bose as Reader in Physics. This, he felt, was in order, as the existing Professor was considered not that capable! His name was Mr. Jenkins about whom Bose had written[1]:

He was a clever man but not of any eminence whatsoever, nor had he done any research work. He had been a Professor of Physics in the old Dacca College and when the University was created, they took him. He was from Cambridge. He belonged to the Indian Educational Service and was just passable in Physics. His special qualication was that he was a good football

Fig. 2. Seated (from left to right) M. N. Saha, J. C. Bose and J. C. Ghosh; Standing (left to right): Snehomoy Dutt, S. N. Bose, D. M. Bose, N. R. Sen, J. N. Mukherjee and N. C. Nag

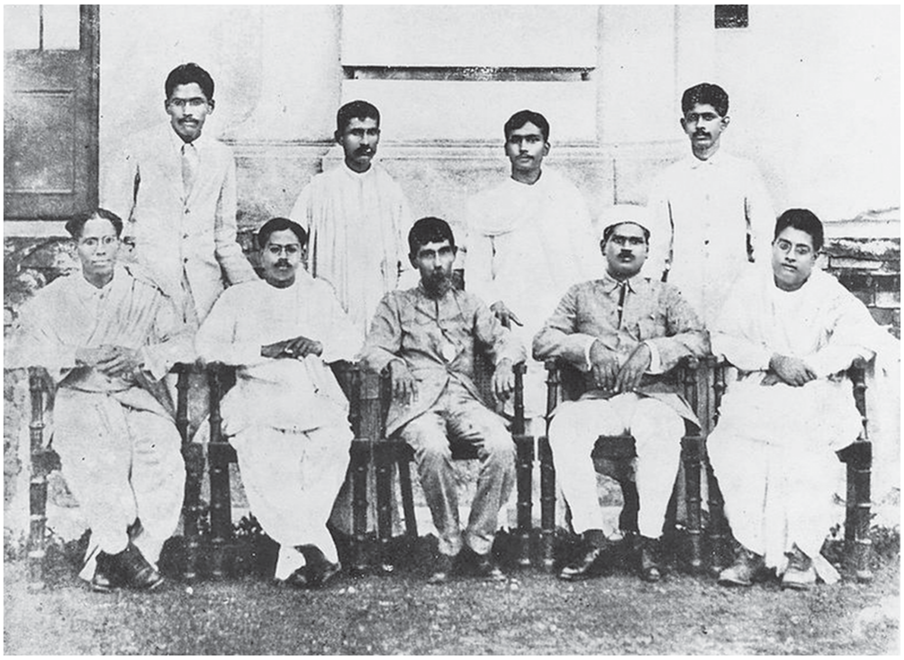


Fig. 3. Bose is seated with (left to right) P. B. Sarkar, Amaresh Chakravarty, P. C. Ray and T. S. Muttu; Standing (left to right) are M. N. Saha, U. N. Karmakar, J. C. Ghosh and J. N. Mukherjee. Two of the founders of the Indian Science News Association and its mouthpiece: `Science and Culture', Meghnad Saha and Prafulla Chandra Ray, figure in this photo.

player and with that merit, he was appointed to the Professorship of Physics.

Sometime in March 1924, Saha had visited Dacca, his hometown. Soon the two met (see Fig. 4) and started to talk physics. Bose told Saha that he was trying to teach Planck's law of radiation to his students but was not at all happy with the available derivations. Saha in turn drew Bose's attention to some other publications in German by Einstein and Pauli. (Incidentally, Saha was knowledgeable in German, Bose was not.) About the latter paper Bose subsequently recalled[1]:

What seems to be happening in Pauli's work was that in order to apply the quantum condition, you had to know exactly what was going to happen afterwards. So, there were certain difficulties, and Saha pointed them out to me.

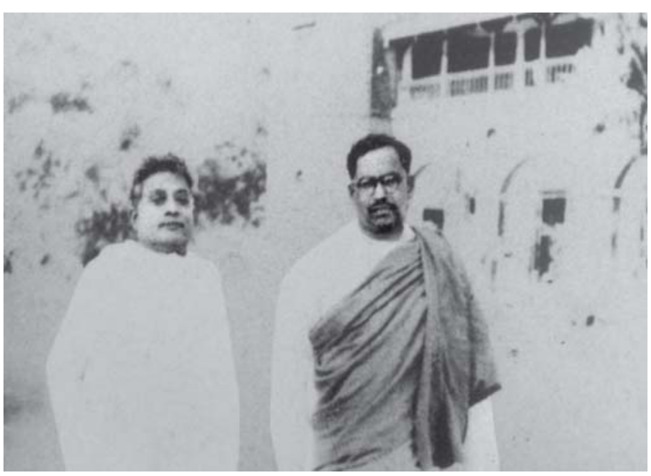


Fig. 4. Bose and Saha in Dacca

The anecdote above is a relevant pointer to how by asking a question in teaching a class, a pathbreaking advance in research can be made. Before discussing in Section IV, what prompted Bose to write that 1924 letter to Einstein, we take the reader to Section III, in which we lay down the prevailing knowledge that triggered Bose's mind.

Background To Bose's Letter

The time we are talking about, namely the early part of the twentieth century, witnessed a revolution in the world of physics. This has been aptly discussed in a popular book by Gamow: *Thirty years that shook physics*[5]. We may alternately dub the title as 'Thirty years of physics that shook the world', which began in 1900, almost after the discovery of the electron, and culminated in 1930, following the theory of the electron. In 1900, Planck[2] considered a cavity in thermal equilibrium with electromagnetic radiation, that may be called 'The black body of radiation'. Kirchhoff had earlier shown that the energy $E(\nu, T)$ of radiation emitted per second per unit volume within the frequency range ν and $\nu + d\nu$, is a function only of ν and the temperature T of the cavity (see Fig. 5). The challenge before experimentalists and theorists was to determine what this functional relation was. Two of the proposals deserve special mention – one by Rayleigh and Jeans – at low frequencies, and the other by Wien – at high frequencies:

$$E(\nu, T) \sim \nu^2 T, \text{ (Rayleigh-Jeans);}$$

$$E(\nu, T) \sim \nu^3 e^{-\alpha\nu/T}, \text{ (Wien),}$$

α being a constant. It was up to Planck to bridge the gap between the low and the high frequency results, captured by Fig. 6.

What did Planck do?: Planck's first step, in October 1900, was to interpolate the Rayleigh-Jeans and Wien limiting forms in order to obtain a satisfactory interpretation

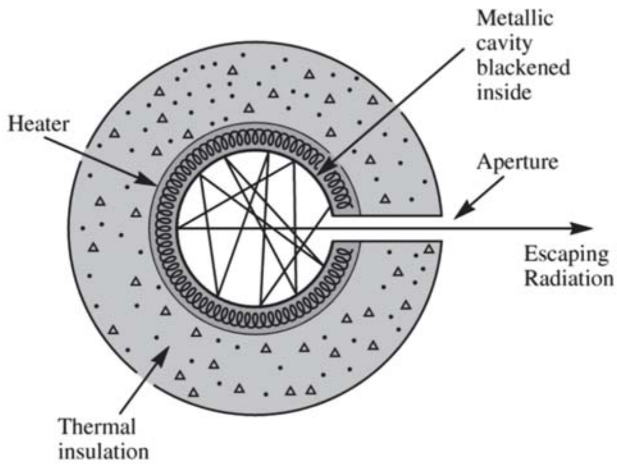


Fig. 5. A cavity blackbody

of the experimental data. Then, in December of the same year, he made a revolutionary surmise that light radiates in a burst of discrete packets. It is pertinent to remark here that the name 'photons' came much later — long after the contributions of Planck and Einstein and indeed of Bose — when it was coined by the chemist Lewis in 1926[6], otherwise famous for his theory of chemical bonding, Lewis acids, etc.[7] Thus, Planck proposed that the energy, viewed as a fluctuating dynamical variable **ε**, in the sense of classical statistical mechanics of Gibbs and Boltzmann, is given by[8]

$$\boldsymbol{\varepsilon} = \boldsymbol{n}\, h\, \nu, \tag{1}$$

wherein, it is the thermal average of $\boldsymbol{\varepsilon}$ and hence of $\boldsymbol{n}$ (the 'values' of which are discrete integers), that ought to yield the energy as a function of ν and T. The physical picture that Planck adopted was that the radiation, when it impinges on the cavity walls, must cause transitions

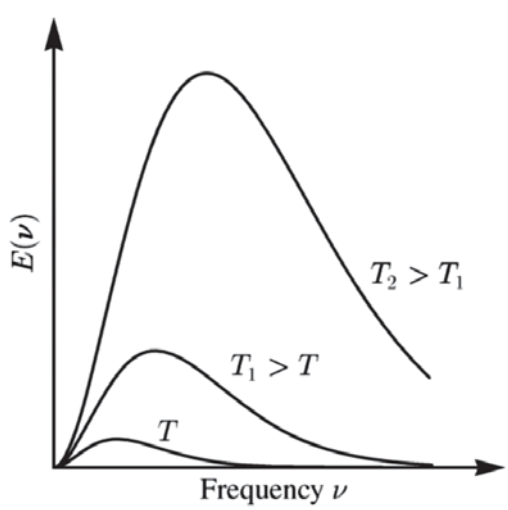


Fig. 6. Blackbody radiation captured by the energy density E(ν, T) as a function of ν at different temperatures.

between energy levels of atoms that constitute the walls[9]. And, because the radiation comes in discrete bursts, the consequent energy absorption and subsequent re-emission, must also involve discrete energy levels of the cavity. Planck now invokes the Boltzmann idea of 'statistical weight'; for instance, the probability that a single energy component is absorbed is given by

$$p_1 = \exp\,(-\,h\nu/k_BT)\,/Z, \tag{2}$$

and similarly, $p_2 = \exp\,(-\,2h\nu/k_BT)\,/Z$, and so on.

The constant Z, called the 'partition function', is introduced to ensure that the total probability (= $p_0 + p_1 + p_2 + p_3 + \ldots$) sums to unity:

$$Z = 1 + \exp(-\,h\nu/k_BT) + \exp(-\,2h\nu/k_BT) + \ldots$$
$$= [1 - \exp(-\,h\nu/k_BT)]^{-1}. \tag{3}$$

The average of $\boldsymbol{n}$, denoted by angular brackets, is then given by a straightforward probability argument as

$$\langle \boldsymbol{n} \rangle = n(\nu, T) = \frac{\sum_{n=0}^{\infty} n\, exp\left(-nh\nu/k_BT\right)}{Z}$$

$$= -\frac{\partial}{\partial\left(h\nu/k_BT\right)} \ln Z\,. \tag{4}$$

From Eq. (3) then

$$n(\nu, T) = \frac{1}{exp\left(h\nu/k_BT\right) - 1}. \tag{5}$$

It may be stressed that apart from the drastic assumption embedded in Eq. (1), the derivation of (5) only involves application of classical Boltzmann statistics.

We are now in a position to write down the Planck radiation formula, which is exhibited as the product of three different bracketed pieces, as follows

$$E(\nu, T)\, d\nu = (2)\left(\frac{4\pi\nu^2 d\nu}{c^3}\right)\left(h\nu n(\nu, T)\right), \tag{6}$$

where $n\,(\nu,\,T)$ is given by Eq. (5) which, along with the energy $h\nu$ for a single mode, constitutes the third round-bracketed term in Eq. (6), that reproduces the Rayleigh-Jeans and the Wien limits.

The next question is, how does one arrive at the second bracketed term ($4\pi\nu^2 d\nu\,/\,c^3$) that equals the number of modes of electromagnetic waves the cavity can sustain between ν and $\nu + d\nu$ per unit volume? In order to answer this, we imagine light as waves like in the strings of an Esraj that Bose was so fond of playing (Fig. 7; see Section

V for a photo of Bose playing Esraj). (The fact that light, unlike the sound waves in the Esraj, does not have a longitudinal component is not germane to the issue here.) It is clear that for a fixed length L at which the Esraj strings are anchored and bowed, the number of modes, dependent on the nodes and antinodes, keep changing; the smaller the wavelength the larger is the frequency. For a given speed of wave propagation, the number of modes that can be sustained is proportional to the volume of a spherical shell of radius between ν and $\nu + d\nu$ that equals $4\pi\nu^2 d\nu$, which appears above. Planck had to divide this by c^3 to obtain the right dimension of Eq. (6), which is the energy per unit volume per unit time.

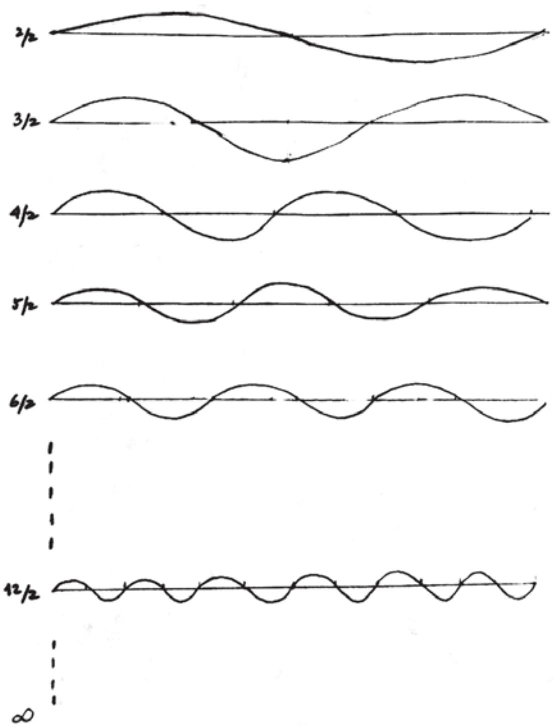


Fig. 7. Standing waves in an Esraj – the first entry has a wavelength one-half of the string-length

That leaves us with the factor of 2, highlighted by a script symbol and embraced by the first set of round brackets in Eq. (6). This factor has generated a lot of debates and speculations, in the context of Bose's letter that we will come later to, in Section IV. At this point in time, it is fair to say that Planck attributed this factor to the two states of polarization of light, wherein the polarization, though a three-dimensional vector, is constrained to lie in the two-dimensional plane normal to the direction of propagation of light. The underlying transverse nature of light waves was already an established fact. Eq. (6) was in perfect agreement with known experimental results.

We end this subsection with a remark which may arouse curiosity. In the first two bracketed terms in Eq. (6), Planck uses purely classical concepts. Only in the third term he introduces an idea, encoded in Eq. (1), that appeared bizarre at that time, the proper justication of which came only much later after the establishment of Quantum Mechanics. This interplay of classical and 'quantum-like' notions is also a feature of Einstein's work (Section III B below) and interestingly, of Bose's as well, as we will argue in Section IV below. In my mind there is nothing strange about it; this is the way science progresses when half-baked ideas find fruition through imaginative experimental and theoretical developments, carried out subsequently.

What did Einstein do?: Einstein had abandoned the Planck idea of using excitations in the cavity walls and instead concentrated on the radiation inside the cavity. But, like Planck, he was greatly influenced by Boltzmann's work on statistical fluctuations and in particular, the latter's formula for the entropy. The Boltzmann concept is somewhat like this: A state of the system under consideration is defined by a coarse-grained object called the macrostate. The macrostate can be realized by counting all possible microscopic ways of generating it. A deep assumption of Boltzmann is that these microstates, obtained by looking at the state of each constituent individually, are equally probable and their number W leads to the entropy S by the formula

$$S = k_B \log W. \tag{7}$$

It is in the counting of W that Bose made a path-breaking ansatz which we will amplify in the section below. Planck had also employed a counting method to estimate W but that is a bit more cumbersome than the straightforward scheme we discussed above.

For applying the entropy formula, Einstein, in his last of the five papers in the 'Miraculous Year' of 1905, looks at the regime in which the discrete nature of the radiation described by what he termed light quanta, would be most relevant, namely for the high-frequency, low-temperature Wien limit, in which

$$\varepsilon(\nu, T) = \varsigma \nu^3 e^{-\alpha\nu/T}, \tag{8}$$

where ζ is another constant to be fixed. He then uses the thermodynamic relation connecting the inverse of the temperature T to the derivative of the entropy with respect to energy at constant volume *via* the second law of thermodynamics as

$$\frac{1}{T} = \frac{\partial s}{\partial \varepsilon}. \tag{9}$$

Calculating $1/T$ from the logarithm of Eq. (8) then gives

$$\frac{\partial s}{\partial \varepsilon} = -\left(\frac{1}{\alpha \nu}\right) \ln\left(\varepsilon / \varsigma \nu^3\right). \quad (10)$$

Here s is the entropy per unit volume which, upon multiplication by the volume V, leads to the extensive entropy S. Integrating Eq. (10), we have

$$S = \left(\frac{E}{\alpha \nu}\right)\left[1 - \ln\left(\frac{E}{V \varsigma \nu^3}\right)\right] + \text{constant}, \quad (11)$$

where the total energy of the radiation $E(= \varepsilon V)$ is a constant (because the blackbody cavity is an isolated system). Introducing another reference entropy S_0 (associated with the volume V_0) the integration constant can be eliminated. Thus,

$$S - S_0 = \left(\frac{E}{\alpha \nu}\right) \ln\left(\frac{V}{V_0}\right). \quad (12)$$

Finally, Einstein leans on Planck to view the radiation as an ideal gas of n independent light quanta for which

$$W = V^n, \quad (13)$$

and a concomitant expression for W_0 associated with V_0. Combining then with the entropy formula in (7), Einstein arrives at

$$E = n\,(\alpha k_B)\nu, \quad (14)$$

which fixes α as h/k_B, in relation to the Planck formula in Eq. (1). Evidently E is the averaged energy as we are using thermodynamics.

The high-temperature, low-frequency Rayleigh limit is much more straightforward to analyze, in which

$$\varepsilon(\nu, T) = \frac{\varsigma \nu^2 T}{\alpha}. \quad (15)$$

As before, assuming *a la* Planck that the blackbody radiation is an ideal gas of light quanta which are governed at high temperatures by the classical equipartition theorem, we must have

$$\varepsilon(\nu, T) = \frac{4\pi\nu^2}{c^3} k_B T. \quad (16)$$

The pre-factor in Eq. (6), within round brackets, was adapted by Einstein from Planck's argument, given in the paragraph below Eq. (6).

This then determines the constant ζ in Eq. (15). The remaining 'factor of 2' in Eq. (6), is again fixed by Einstein, like Planck, from the two states of polarization. Once more, we notice a juxtaposition of wave and particle-like concepts that was alluded to, in the last paragraph of Sec. III A.

The remarkable feature of the contributions by Planck and Einstein was that they broke away from the shackles of Maxwell's concept of light – that light is just a wave that exhibits interference, diffraction and polarization. In some sense they resurrected the corpuscle idea of Newton in treating light!

At the end of his paper on light quantum, Einstein was guided by the experiments of Lenard which revealed that the maximum electron energy ejected from a metallic surface due to impinging light showed "not the slightest dependence on the light intensity". This led to the celebrated equation $\varepsilon_{\max} = h\nu - \phi,\ \phi,$ being the work function of the metal, which forms the cornerstone of the photoelectric effect. Speaking of this equation Millikan says: "I spent 10 years testing that 1905 equation of Einstein and contrary to all expectations, I was compelled to assert its unambiguous verification in spite of its unreasonableness since it seems to violate everything we know about the interference of light." Millikan further deduced from his experiments the value of the Planck constant h as 6.4×10^{-34} Joule-sec.

On Boltzmann's impact on Einstein, we quote the latter from 1917 [10]:

.... Because of the understanding of the essence of Brownian motion and the photoelectric effect, suddenly all doubts vanished about the correctness of Boltzmann's interpretation of the thermodynamic laws.

In the words of Pais [11]:

Had I to compose a one-sentence scientific biography of Einstein, I would write, better than anyone before or after him, he knew how to invent invariance principles and make use of statistical fluctuations.

In the context of "invariance principles", Pais was of course referring to the work of Einstein on the special theory of relativity, in the same 'miraculous year', namely 1905 [3].

Analysis of the Letter

We now come to discuss the core issue, namely the letter that Bose sent to Einstein in June 1924. The letter was accompanied by a manuscript and a request to Einstein to comment and help its publication. It is amazing that in just about a week after receiving the letter Einstein forwarded the paper to Zeitschrift für Physik[12] with an unusual footnote of strong endorsement:

Note by translator: Bose's derivation of Planck's law is an important step forward. The method here employed also yields the quantum theory of the ideal gas as I will show in another place.

The English translation of the aforesaid paper is available from the special birth centenary publication[13]. It is evident that Bose was intent on giving a 'proper' derivation of the Planck radiation formula as he found the earlier derivations by Planck and Einstein unsatisfactory. In particular, Einstein had relied on macroscopic concepts of thermodynamics. Bose wanted to go into the microscopics just as statistical mechanics provides microscopic justication for the macroscopic subject of thermodynamics. Besides, Bose was unhappy with the entropy calculation by Einstein discussed above in Section III B.

It is well in order to point out that nearly four-fifths of the 'thirty years that shook physics', had already elapsed[5] by 1924 and Bose was aware of monumental contributions already made before him. As Gamow pointed out, the subject of quantum mechanics got developed, slowly – step by step – through the exemplary contributions by several physicists, especially notable amongst them being Planck (1900), Einstein (1905) and Bohr (1913) before Bose (1924) – and then through an accelerated growth, after Bose, by Dirac (1925), Schrödinger (1926) and Heisenberg (1925, 1927), spurred by the pilot-wave model of de Broglie (Ph. D. thesis in 1924, paper in 1925). Mukunda has made a delineation of this period as[14]:

Old Quantum Theory 1900 – 1924

Quantum Mechanics 1925 – 1927.

The year 1927 is the culmination of non- relativistic quantum mechanics. We may suffix that by 1928, a la Gamow, the year of the Dirac equation of relativistic quantum mechanics. Bose's letter happens to fall right at the junction of these two periods.

The enigma of quantum mechanics persists even after the 1930 milepost of Gamow and continues to baffle us, even today[14,15]. Because our aim is to decipher Bose's letter in this centenary year of its dispatch from Dacca, we will endeavor to be factual, reliant only on stuff that is documented and not get distracted by hearsay. Also, because the paper sent by Bose was amended by Einstein before sending it for publication, both of them ought to be accountable for the contents. We believe that a transparent assessment is very important for an honest appraisal of the History of Science.

In this context it is appropriate to refer to the special July 2024 issue by 'Bangiyo Sahityo Parishad', which incidentally was founded by Bose himself with the objective of popularizing science through Bengalee. Several authors[16] commented on the Bose letter and in particular on the 'enigmatic factor of 2' but the clearest, dispassionate exposition on what Bose actually did and he did not – vis a vis Einstein – came through the writings of Pal[17].

Given this preamble I turn to Bose's letter and the publication based on it[13,14]. At the outset, we examine the second round-bracketed expression in (6). Bose proceeded as follows. Any one light quantum has a position in space $x; y; z$ and a momentum $p_x; p_y; p_z$ such that

$$p_x^2 + p_y^2 + p_z^2 = \left(\frac{h\nu}{c}\right)^2 . \tag{17}$$

Here use is made of special relativity that the energy of a light quantum (being of zero rest- mass) is pc which also equals $h\nu$. In the phase space, with the coordinates $x; y; z$ and $p_x; p_y; p_z$, a light quantum is constrained within a volume V and with regard to $p_x; p_y; p_z$, to a spherical shell of radius $h\nu/c$ and thickness $(h/c)d\nu$. This spherical shell has the volume in momentum space $4\pi\left(h\nu/c\right)^2 hd\nu/c$. It is this step, in which the Planck constant h appears, that had been claimed by Bose in his letter to be "independent of classical electrodynamics". However, the aforesaid factor was divided by h^3 by Bose to get the number of quantum cells in this phase volume, yielding an identical phase space factor as Planck and Einstein had! It is noteworthy also that only the momentum p appears in enumerating the phase space volume while the position q simply contributes an overall volume factor of V. This is understandable for a free gas of noninteracting light quanta and hence to define a cell by both coordinate and momentum seems to be an overkill! (This point gets amplified by a later paper by Wigner in which the underlying wave function incorporates the spin of the photon but has no reference to its position[18].)

We now come to discuss the enigmatic 'factor of 2'. We prefer to go by what is written in the published paper of 1924; we quote: "it seems, however, appropriate to multiply this number once again by 2 in order to take into account the fact of polarization..." In a recent paper though Ghatak and Mani[19] have suggested that Bose was the first to postulate the intrinsic angular momentum of the photon and referred to certain statements which are based on hearsay. Because no copy of the original Bose manuscript is available, we do not wish to venture into realms of speculation. The authors themselves have however pointed out that much earlier, Poynting[20] had made "a suggestion as to the angular momentum in a beam of circularly polarized light". As it turns out, the covariant form of the angular momentum density of the electromagnetic field has also been discussed in the classic book by Jackson[21]. But, as Arvind et al pointed out, the rigorous quantization of the angular momentum and its elucidation as the intrinsic spin of the photon, occurred only during the development of the quantum field theory of electromagnetism[22].

Ghatak and Mani, in the first part of their paper, have correctly asserted that the electron's intrinsic spin got accepted quite rapidly through the experiments of Gerlach and Stern[23], the theoretical proposal by Uhlenbeck and Goudsmit[24], and finally the theoretical base provided by Dirac who had combined quantum mechanics with the special theory of relativity[25]. By contrast, the photon spin took a long time to sink in, as indicated above.

Returning to the issue of the wave function, which was developed by Schrödinger in 1926 following the doctoral thesis of de Broglie (2025), we note that for an assembly of n quantum particles like an electron, the wave function is antisymmetric under the exchange of any pair *i* and *k*, i.e.,

$$\psi(x_1, x_2 \ldots x_i \ldots x_k \ldots x_n) = -\,\psi(x_1, x_2, \ldots, x_k, \ldots, x_i, \ldots x_n), \quad (18)$$

where x_i represents position and one spin component, or momentum and one spin component, depending on whether we employ coordinate or momentum representation, respectively. For photons, the sign on the right-hand-side is positive implying symmetry of the wave function though, for reasons cited earlier, the coordinate-dependence of the wave function is a dubious idea for photons.

It is indeed this aforesaid symmetry (or antisymmetry) that is directly linked with the 'indistinguishability' of quantum particles that was tacitly employed by Bose in his phase space counting of the third bracketed term in the Planck formula, though the notion of wave function was unknown in 1924. This is why Delbrück[26] had entitled his paper as: "Was Bose-Einstein Statistics Arrived at by Serendipity". The dictionary meaning of the epithet 'serendipity' is: 'the faculty of making happy discoveries by accident'. The point that Delbrück is making: how could Bose take his particles to be indistinguishable when the quantum theory of wave function was still to be established? The upshot is: although it is the 'factor of 2' that seems to have captured a lot of attention, an element of 'second-guessing' is also inherent in Bose's way of entropy calculation, as elaborated below[27].

What then is the "accidentally correct" method of Bose that yields the right derivation of the third factor in the Planck formula, according to Delbrück? Instead of giving algebraic details of the counting procedure in Bose's 1924 paper, which are a bit clumsy, we provide a schematic diagram a la Delbrück that has the essence of Bose's derivation. Consider three individuals to be placed in two rooms: the living room and the kitchen. Classically, these guys are distinguishable and there are several possibilities. Keeping one of them in the kitchen, we show three possible arrangements in Fig. 8. However, Bose took these blokes as indistinguishable and came up with only one arrangement as shown in Fig. 9. This is how the total

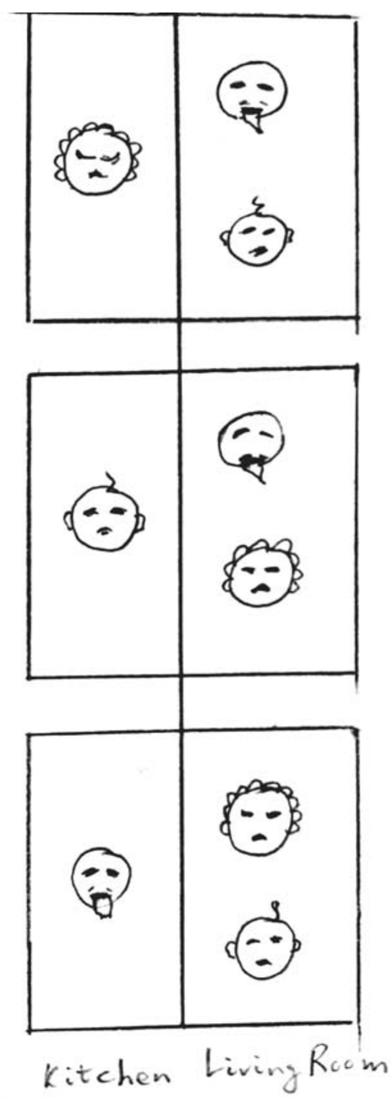


Fig. 8. 'Classical' counting with one bloke in the kitchen to the left

number of microstates was counted by Bose and once again, by the use of the Boltzmann expression for entropy, he arrives at the Planck radiation formula. Before concluding this section, we may mention one other interesting aspect in Bose's derivation. Even though the arguments advanced were purportedly more 'microscopic', Bose employed the same classical entropy idea of Boltzmann when he had maximized the total number of ways in which the microstates were counted. Years later Bose said, quoted by Venkataraman[1]:

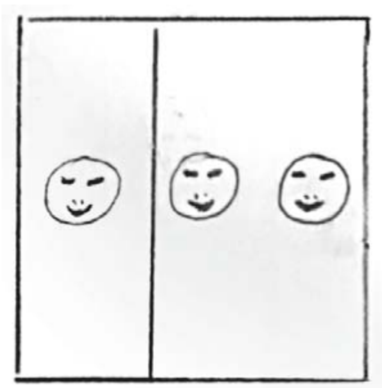


Fig. 9. Bose's counting stratagem – all the chaps are taken to be indistinguishable

I had no idea that what I had done was really novel.... I was not a statistician to the extent of really knowing that I was doing something really different from what Boltzmann would have done.

Only today we know how to rigorously characterize the entropy in Quantum Mechanics via the density operator, thanks to von Neumann[28].

Conclusions

Summarizing, the story of what we now know as Bose-Einstein statistics is an amazing one. A young man in colonized India writes a post card to an already famous Nobel Laureate physicist to, far-away-Berlin from Dacca, with all his adulations embedded in the obsequious way he addresses: "Respected Sir". In absence of airmail, it must have taken at least three weeks before the letter reached Einstein. To his credit, Einstein did not throw the card from an unknown man to the trash can but immediately found a profound novelty in Bose's treatment, which impressed him. It appears that Einstein was in a hurry and did not notice that the indistinguishability that Bose had presumed would lead to the pressure of an ideal monatomic Bose gas to be lower than what one would get by treating the gas as classical. This 'pseudo' attractive pressure is purely of quantum origin related to the symmetry of the wave function associated with the Bose particles. Thus, even though the gas is ideal, i.e., there is no mechanical force between the atoms, they are not independent, since the atoms would want to come closer. The opposite antisymmetry in the case of electrons would result in Pauli pressure keeping two electrons apart, by what is called the exclusion principle. This realization prompted Einstein to publish two back-to-back papers[29], after he had endorsed Bose's manuscript for publication, which led to the idea of superuidity and what is called the Bose-Einstein condensation[30].

As I see it, life is a mixed bag – of losses and gains. Some ardent followers of Bose may feel that he was sidestepped by Einstein in not having him as a collaborator in the 1924-25 papers (though Einstein had been transparent in his footnote to Bose's paper in asserting: ... as I will show in another place). On the other hand, Dirac compensated by calling the Bose particles 'Bosons' just as he was magnanimous in not appropriating the label 'Fermions'. Similarly, as Pal pointed out[17] – it is not just in the discovery of the bosons – Bose had very little to do with superuidity albeit the underlying phenomenon is often referred to as Bose Condensation.

There is no evidence that Bose himself felt ignored by Einstein. By all accounts, he was a happy-go-lucky, warmhearted person, given to play his favorite Esraj in his leisure (see Fig. 10). Bose's all-round Bengali personality

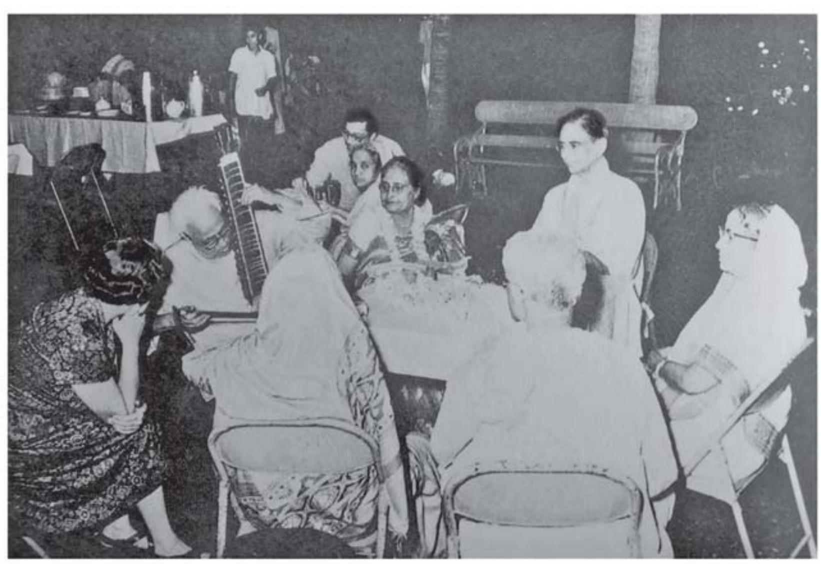


Fig. 10. Bose playing Esraj at a wedding anniversary party of Rani and Prasanta Mahalanobis

Fig. 11. Bose with Jamini Roy and Bishnu De in an 'Adda'

is reflected in Fig. 11. The scientic community too held him in high esteem as illustrated by Fig. 12. Bose returned from Dacca to Calcutta in 1945 to become the Khaira Professor of Physics at Calcutta University. He retired from Calcutta University in 1956 and went to Santiniketan as Vice Chancellor of Visva-Bharati, founded by Rabindranath Tagore in 1921 and enacted to be the first Central University in 1951[31]. In 1937, Tagore had published a book on Science in Bengalee, called Viswa-Parichay, which he had dedicated to Bose. That was also an inspiration for Bose to start writing about science in Bengalee through Bangiyo Sahitya Parishad.

In January 1974 Calcutta University organized an international conference in honor of S. N. Bose. Bose concluded his lecture by saying:

Well, after all, if one has lived through so many years of struggle, and at the end he finds that his work has been appreciated, he feels that he does not need to live long.

Prophetic words they sound like, for Bose passed away soon after, on 4 February 1974. Bose is no more but Bosons are there forever. I end by citing the Krishnan Memorial lecture delivered by Bose at the National Physical Laboratory (Fig. 13 and 14). At the end of his talk, he said:

Fig. 13. Bose delivering the Krishnan Memorial lecture at NPL on 12 March 1971

It is a perpetual challenge to the Indian genius as to why, even though the country is endowed with such natural resources, even though the country has had such a brilliant history, it continues to remain third rate in spite of so many resources and so much manpower. Well, gentlemen, I stop here with these questions asked to our young men.

Fig. 12. Neils Bohr vising Bose at his Calcutta home

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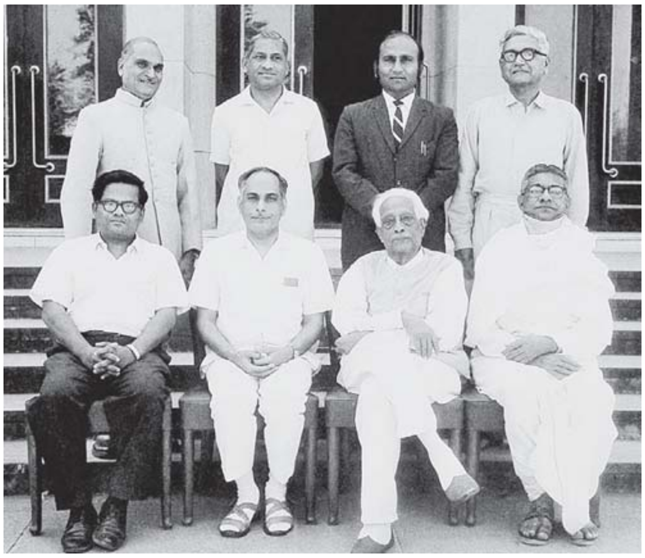

Fig. 14. Bose seated with A. P. Mitra and A. R. Verma; standing behind is V. G. Bhide, amongst others

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31. Like several Vice Chancellors, before and after him, Bose's administrative experience in Visva-Bharati was not a pleasant one and he could not complete his term. He returned to Kolkata after two years of his tenure, in 1958.

† While mentioning Einstein's school performance it is fair to say that there may be a difference in the German and Swiss ways of assigning grades. I thank Stefan Thomae for pointing this out.