

Pattern Formation of Heavy Metal Deposition and Related Hydrodynamics in a River Basin - A Case Study of the Subarnarekha River, Eastern India

Abstract: A number of heavy metals are observed in the Subarnarekha river sediments. The pattern of the said metal distribution curves shows periodicity, expressed by the sinusoidal nature of the curvatures. The periodic behavior results from the Hopf bifurcation phenomenon. Hopf bifurcation, in this case, describes a sub-critical bifurcation with sudden jumps in behavior indicating instabilities in fluid flow. Such bifurcations occur where the unstable cycle shrinks to zero amplitude and engulfs the origin (the stable point when the flow dynamics is laminar) rendering it unstable. The periodic bifurcation orbits appear to be thrown off from the origin. The distribution pattern of the heavy metals in the Subarnarekha river sediments is quite similar to those in different rivers in various parts of the world (published data have been critically analyzed). Interestingly, the distribution pattern of individual metals exposes a distinctive character, which may change as the specifics of the river system change.

Key words: Patterns, periodicity, Hopf bifurcation, instability, stability, flow dynamics, laminar, fractal dimension

The formation of patterns is apparent in natural systems; patterns in geography and geology are part of it. There are two types of patterns - natural and man-made. Patterns in natural systems range from clouds to animal markings and from ripples of blown sand to the intricate shells of microscopic marine organisms. These patterns of nature are self-organizing fractal structures and express a spontaneous formation (Self-organization is a kind of computation performed by the interaction of physical properties with the local environmental system. These natural fractal structures may arise in the fluvial landscape as a joint consequence of optimality and randomness). Their scientific study comprises one aspect of research into the so-called complex systems which typically show emergent behaviors that cannot be deduced or predicted by a focus on the properties of the individual component.

The features in the present case result from mathematical deductions/analogies and equivalences in the rules governing their formation. These rules can be expressed in terms of continuum equations describing transport of these minerals. Solutions are modified by specific circumstances like path-dependent hysteresis in transitions between patterns, thus exposing self-organization as an inherent property of the complex systems.

Geology is, in fact, a science of understanding patterns. The variability of patterns in river systems is, however, related to the non-linear hydrodynamic stability of the water flows and marked by the transition to turbulence. However, this transition to turbulence is more complex, the velocity of flow becoming an almost periodic function of time and that the auto-correlation of a turbulent velocity is observed to decay rapidly. According to Hopf¹, the turbulence arises through the repeated bifurcation of the solution representing the flow. But there are no general rules to determine the number and stability of steady solutions which exists as parameter varies, universal properties of bifurcation cannot be decided upon either; non-linear phenomena too as we know are much varied or dissimilar.

Over many years, considerable amounts of heavy metals have accumulated in the Subarnarekha river bed sediments and have been spatially and temporally modified by the flood events recurring each year. The deposition of metals has, however, been accentuated on account of the river traversing the Singbhum Multimetal Complex Shield, which includes base and precious metals and uranium. During investigation, it was felt that a better knowledge was needed of the processes that have caused the present day distribution of the metals in the river sediments giving rise to characteristic patterns. The metal concentrations in the river sediments have greatly varied along the flow length of the river, resulting in corresponding variations in the metal concentrations along the width.

Heavy metal distribution in the Subarnarekha river sediments (Fig. 1) shows a signature of periodicity.

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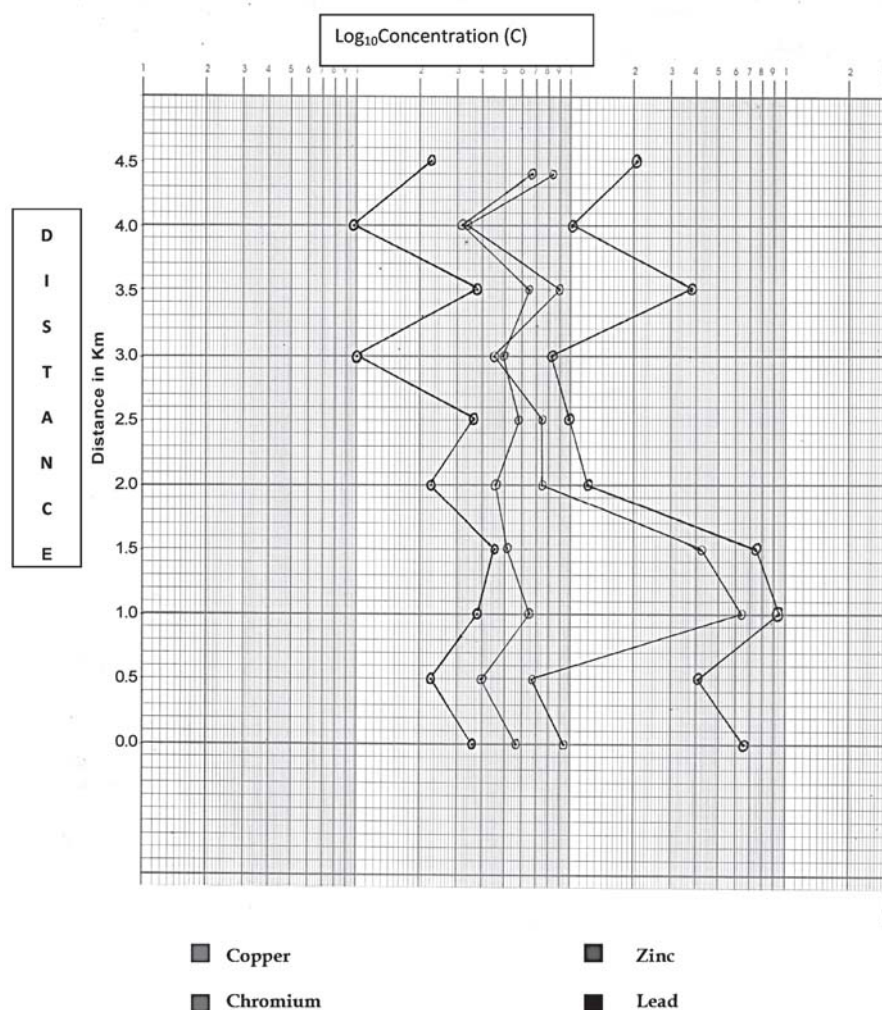


Fig. 1: Plots of Distances (x) vs Heavy Metallic Concentrations in Logarithmic Scale

Incidentally, a simple diffusion model for the heavy particles in water will give periodicity. But there are exceptions in the distribution patterns of Ni and Co in this case, which are totally dissimilar with the trend, probably because they are nanoparticulates². It is probable that the distribution patterns of heavy metals particularly, is greatly influenced by their grain size trends, varying with the specificity of the hydrodynamic environment.

Distribution patterns of the heavy metals in the Subarnarekha river (excluding Ni and Co) are compared with those of the major rivers all over the world. The comparative study shows that the heavy metals in the different rivers also expose a periodicity in their distribution/deposition pattern. Geometrically, the trends have an overall similarity. However, the distribution pattern of cobalt and nickel in the Subarnarekha river is different. Each one reflects a slightly wavy single straight line. We are not adequately equipped yet to explain their behavior.

Hopf bifurcations are negotiated in particle dynamics³. In the present case, the particles are defined as made up of heavy minerals. The variability of patterns are related to the hydrodynamic stability of the water flows in the river system and the depositional scenario, met with in the Subarnarekha river sediments points to sub-critical Hopf bifurcations. In the spectral evolution visualized by Landau^{4,5}, one of the successive bifurcations as the Reynolds Number⁶ increases could probably introduce a periodic oscillation into what were earlier steady flows. Such a periodic oscillation, a non-linear phenomenon, is too diverse and complex and could generate various patterns in the heavy metals distribution. Considerable research work has been carried out⁷⁻⁹, to prove how periodic flows bifurcate from a steady one when the real part of two complex conjugate eigen values of the linearized system passes through zero and occurs there if that eigen value has odd multiplicity.

Moreover, the bifurcated periodic solution is itself stable if it exists when the steady flow is supercritical and unstable when the steady flow is sub-critical. About a few kilometers down the slope of the Subarnarekha from the first sampling site, the pattern of heavy metals distribution shows a significant change. Such a change of geometric pattern with the change of scale suggests a disorder in the system's character indicating probability of the presence of chaos in the system. Bifurcation indicates sudden change where a system's previous character is disintegrated. Crisis events like floods are described in chaos theory as these points of system's bifurcation and their recurrence play an important role in the Subarnarekha in the development of different patterns in the heavy metal distribution.. In the Subarnarekha during the monsoon, floods recur. As the Reynolds Number increases, the flow intensifies and bifurcation takes place, so does the periodic oscillation giving rise to a specific pattern. Bifurcation represents changes in the qualitative

character of the transition in the system. The change of pattern in the periodicity curve as mentioned above, in our opinion, could be simply due to bifurcation. In the present sub-critical bifurcation a sudden jump in behavior might be due to a systems change in oscillations because of path dependent hysteresis in transitions between patterns, reflecting self-organization. $\square$

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