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ARTICLE

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GEOMETRICAL CONTRIBUTIONS OF BRAHMAGUPTA REGARDING CYCLIC QUADRILATERALS

SANATAN KOLEY*

The versatile Indian astronomer-mathematician, who enriched mathematics as well as astronomy is Brahmagupta. His major work Brāhmasphuṭasiddhānta, composed in 628 CE, consists of 24 chapters. Two of these chapters, namely 12th (XII) and 18th (XVIII), deal exclusively with pure mathematics (including some topics on geometry and mensuration). In this article an attempt will be made to discuss about his geometrical contributions regarding cyclic quadrilaterals only, which are included in the said chapter-XII. Thereafter, our aim will be to find the relevance (direct or indirect) of these early medieval contributions of Brahmagupta in the field of the present day high school geometry.

Introduction

The study of geonmetry as a science began first in India in the Vedic age. It was developed in the country from the rules for the construction of various types of altars for the Vedic sacrifice. The result of these operations was the compilation of a series of geometrical rules in the late Vedic period, which are contained in the extant *Śulbasūtra* texts and their commentaries. The period of the *Śulbasūtras* have been estimated to be between 800 BCE and 500 BCE. These texts give us a picture of development of early geometry in India before the advent and rise of the Jaina sect during c.500-300 BCE. Like the Hindus in the Vedic age, the Jainas had extended their geometrical practice for their religious needs. The formulae related to geometry and also mensuration given by the early Jainas are quoted in a number of their religious and semi-religious works like *Tattvārthādhigamasūtrabhāṣya* and *Jambudvīpasamāsa* of Umāsvāti (c.1st century CE), and others. They had unparallel influence in Indian mathematics, particularly geometry, after the end of Śulba period to the pre-Āryabhaṭīyan era, that is, up to c.500 CE.

* Former Headmaster, Jagacha High School (H.S.), Howrah-711 112; Present Address : Kadambari Housing Complex, BL-I, Flat-IA, 144 Mohiary Road, Jagacha, P.O.-GIP Colony, Howrah-711 112, W.B., India.
E-mail : sanatankoley2017@gmail.com

Mathematics in India attained one of its highest peaks during c. 6th-7th century CE with the arrival of distinguished mathematicians and astronomers like Āryabhaṭa I (c.476-550 CE), Varāhamihira (c. 505-587 CE), Bhāskara I (c. 600- c.680 CE) and Brahmagupta (c. 598-c.668 CE). Among them Varāhamihira and Bhāskara I are mainly astronomers; as a result most of their works are based on astronomy. Many of us probably know of the books *Āryabhaṭīya* composed in 499 CE by Āryabhaṭa I and *Brāhmasphuṭasiddhānta* written in 628 CE by Brahmagupta (at the age of 30). These two books are more or less based on astronomy and mathematics (including some topics on geometry and mensuration).

Brahmagupta was born perhaps in Bhillamāla, modern Bhinmal in Rajasthan and completed his studies there. He was the son of Jishnugupta and was a Hindu by religion, in particular, a Shaivite. He lived and worked in his own village for a good part of his life. Here he wrote his main work *Brāhmasphuṭasiddhānta* on astronomy and pure mathematics. Later, he moved to Ujjain in Madhya Pradesh, the then famous centre of learning and became the chief of the astronomical observatory situated there[1]. However, Brahmagupta composed here his next book *Khaṇḍa Khādyaka* based on astronomy in 665 CE at the age of 67.

It must be referred to here that, his very well known work *Brāhmasphuṭasiddhānta (Br.Sp.Si.* in brief), written in 1016 verses (different opinion : 1022 verses), is divided into 24 chapters. Major part of this treatise is based on astronomy, but the 12th (XII) and 18th (XVIII) chapters are based exclusively on pure mathematics. Brahmagupta named the 12th chapter comprising of 66 verses as '*Gaṇita*' and the 18th chapter comprising of 102 verses as '*Kuttaka*'. The chapter– '*Gaṇita*' includes some discussions on arithmetic, series and geometry (together with mensuration), whereas the chapter–'*Kuttaka*' includes discussion on algebra.[2]

Contextually, it is worth mentioning that Muhammad ibn Ibrāhīm al-Fazārī (746-806? CE), an Arabian mathematician-astronomer, translated the said famous treatise *Brāhmasphuṭasiddhānta* from Sanskrit to Arabic as Az-Zīj 'ala Sinīal-'Arab, or the Sindhind using an Indian scholar (of Ujjain) named Kanka who came to Baghdad by about 770 CE.[3] This translation was possibly the vehicle by means of which the Hindu numerals were transmitted from India to the Arabs. Furthermore, eminent British mathematician H.T. Colebrooke (1765-1837 CE) translated two aforesaid chapters, 12th and 18th, of this treatise from Sanskrit to English in 1817 CE.[4] By means of this translation, Brahmagupta's works reached Europe in the first half of 19th century CE and had a great influence thereafter.

We have already stated that the chapter-XII of the work *Br.Sp.Si.* deals with the topics on arithmetic, series, geometry and mensuration. Now, we will make an effort to discuss in detail on those verses of this chapter, which are based on geometry relating to cyclic quadrilaterals only. After that, we will try to explore the relevance of these geometrical works in the field of the present day high school geometry.

Area of a Cyclic Quadrilateral

It may be referred to here that, the formula for finding the area of a triangle, which was used in the *Śulbasūtra* texts during c.800 - 500 BCE is:

Area of a triangle = $\frac{1}{2}$ base × altitude, which is Datta's conjecture.[5]

The early Jainas had no application for the triangle. But this formula is found later in the work *Āryabhaṭīya* (verse II.6) of Āryabhaṭa I.

The above rule is applicable when the base and the altitude of a triangle are known. If three sides of a triangle or / and four sides of a quadrilateral are given, their 'approximate' and 'accurate' areas can be determined using Brahmagupta's rules. In the verse XII.21 of the *Br.Sp.Si.*, he states the concerned rules as follows:

स्थूल फलं त्रिचतुर्भुज बाहु प्रतिबाहुयोगदलघातः ।
भुजयोगार्धचतुष्टय भुजोनघातात्पदं सूक्ष्मम् ॥ 21 ॥

sthūla phalaṃ tricaturbhuja bāhu pratibāhuyogadalaghātaḥ|
bhujayogārdhacatuṣṭaya bhujonaghātātpadaṃ sūkṣmam‖21‖

This means: *The approximate area of a triangle or a quadrilateral is the product of half the sums of its opposite sides. The accurate (area) is the square root of the product of half the sum of the sides severally diminished by (each of) the four sides.*

That is, if *a, b, c, d* are four sides of a quadrilateral taken in order, then according to the rules we have its area (*A*) as :

$A = \frac{a+c}{2} \cdot \frac{b+d}{2}$ approximately, and

$A = \sqrt{(s-a)\,(s-b)\,(s-c)\,(s-d)}$ accuretely, where s = $\frac{1}{2}(a+b+c+d)$.

If we set one side, say *d,* equal to zero, then the rules can also be used for a triangle with sides *a. b, c.* Thus we get its area *(A)* as :

$A = \frac{a+c}{2} \cdot \frac{b}{2}$ approximately, and

$A = \sqrt{s(s-a)\,(s-b)\,(s-c)}$ accurately.

It is evident that, the approximate formula is of course exact for rectangles (including squares) but not for other quadrilaterals or triangles. However, the accurate formula, in the case of a triangle, was given before by the Greek mathematician Heron (c. 10 - c. 70 CE) of Alexandria.[6] Brahmagupta makes a remarkable extension of "Heron's formula" by giving its quadrilateral version. It is exact only for cyclic quadrilaterals, though Brahmagupta does not say so. Needful to say, all rectangles and isosceles trapezia as well as certain scalene quadrilaterals are cyclic figures.

Circumradius of an isosceles trapezium and a scalene quadrilateral

The rules for finding the circumradius of an isosceles trapezium and a scalene quadrilateral are given by Brahmagupta in the verse XII.26 of his treatise *Br.Sp.Si.*

The said verse is mentioned hereunder:

अविषम पार्श्वभुजगुण: कर्णो द्विगुणावलम्बकविभक्त: ।

हृदयं विषमस्य भुजप्रतिभुज कृतियोगमूलार्धम् ॥ 26 ॥

aviṣama pārśvabhujaguṇaḥ karṇo dviguṇāvalambakavibhaktaḥ|

hṛdayaṃ viṣamasya bhujapratibhuja kṛtiyogamūlārdham||26||

This means: *The diagonal of an isosceles trapezium (non-scalene) multiplied by its flank (lateral) side and divided by twice its altitude gives its circumradius. In case of a viṣma (scalene) quadrilateral, it is half the square root of the sum of the squares of the opposite sides.*

That is, if R be the radius of the circle circumscribing the isosceles trapezium (a, c, b, c) then the rule gives $R = \dfrac{dc}{2h}$ ... (i), where d and h respectively are the diagonal and altitude of the trapezium (vide Fig. 1).

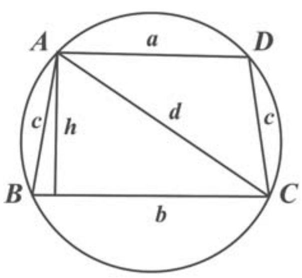

Fig. 1

In case of scalene quadrilateral (a, b, c, d), the circumradius (R) of it is $R = \dfrac{1}{2}\sqrt{a^2 + c^2} = \dfrac{1}{2}\sqrt{b^2 + d^2}$ .. (ii)

The formula (i) can be justified as follows :

From the triangle ABC (vide Fig. 1), using the properties of triangles (related to the sides and angles), we obtain $\dfrac{AC}{\sin \angle ABC} = 2R$ or, $d = \dfrac{h}{c}.2R$, hence $R = \dfrac{dc}{2h}$.

For the formula (ii), it must be noted here that this holds only in certain cyclic scalene quadrilaterals in which the diagonals intersect each other at right angles. For justification of the formula vide the section entitled 'Construction of rational cyclic scalene quadrilaterals' concerning explanation of the verse XII.38 (*Br.Sp.Si.*) of the running article.

Circumradius of a Triangle and a Quadrilateral

In the verse XII.27 of the *Br.Sp.Si.*, Brahmagupta states the rule for determining the circumradius of a triangle and a quadrilateral. He enunciates the verse as below:

त्रिभुजस्य वधो भुजयोर्द्विगुणितलम्बोद्धृतो हृदय-रज्जु: ।

सा द्विगुणा त्रिचतुर्भुजकोणस्पृग्वृत्तविष्कम्भ: ॥ 27 ॥

tribhujasya vadho bhujayordviguṇitalamboddhṛto hṛdaya-rajjuḥ|

sā dviguṇā tricaturbhujakoṇaspṛgvṛttaviṣkambhaḥ||27||

This means: *The product of the two sides of a triangle divided by twice the altitude is the circumradius (hṛdaya-rajju), and twice that is the diameter of the circle which passes through the vertices of the triangle and the quadrilateral.*

That is, if R be the circumradius of the triangle ABC and h be its altitude (vide Fig.2) then according to this rule $R = \dfrac{bc}{2h}$.

This result may be proved as follows:

In the triangle ABC, if AD (= h) be the perpendicular on BC and AE (=2R), the diameter of the circumcircle then the triangles ABE and ADC will be similar (vide Fig.2).

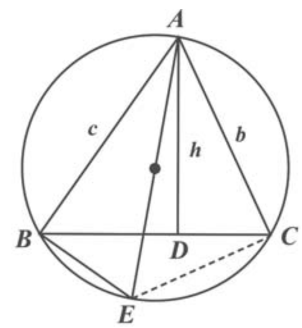

Fig. 2

Therefore, $\dfrac{AE}{AC} = \dfrac{AB}{AD}$ ... (i). Hence, $AE = \dfrac{AC.\,AB}{AD}$ or, $R = \dfrac{bc}{2h}$.

Furthermore, according to the figure, ABC is a triangle and ABEC is a quadrilatera1. As they are inscribed in the same circle, they have the same circumdiameter (AE in the figure).

We can find glimpses of this work in the geometry tought at present in high schools. The rule is illustrated in the following theorem :

If a perpendicular is drawn on the base of a triangle from its vertex, then the rectangle formed by its two other sides will be equal to the rectangle formed by that perpendicular and the circumdiameter of the triangle.

From the aforesaid relation (i), we get AB.AC = AD.AE. Hence the theorem (vide Fig. 2).

Corollary: If $a, b, c,$ be the sides and Δ the area of a triangle having circumradius (R), then $R = \dfrac{abc}{4\Delta}$.

According to the Fig. 2, area of the triangle $ABC(\Delta) = \dfrac{1}{2}BC.AD$ or, $4\Delta = 2BC.AD = 2a.AD$. Further, from the above mentioned relation AE.AD = AB.AC we have 2R.AD = $c.b$ or, $2R = \frac{bc}{AD} = \frac{abc}{a.AD}$ $\therefore$ $R = \frac{abc}{2a.AD}$ $= \frac{abc}{4\Delta}$.

Diagonals of any Cyclic Quadrilateral

Brahmagupta's most important contribution to geometry is the theorem for the length of the diagonals of any cyclic quadrilateral in terms of its sides. It is found to occur in the verse XII.28 of the *Br.Sp.Si.,* which is as follows:

> कर्णाश्रितभुजघातैक्यमुभयथाऽन्योन्य भाजितं गुणयेत्।
>
> योगेन भुज प्रति भुजवधयो: कर्णौ पदे विषमे॥ 28॥
>
> *karṇāśritabhujaghātaikyamubhayathā'nyonya bhājitaṃ guṇayet|*
>
> *yogena bhuja prati bhujavadhayoḥ karṇau pade viṣame* ||28||
>
> ***This means:*** *One should divide the sums of the products of the sides about both the diagonals by each other and multiply the quotients by the sum of the products of the (two pairs of) opposite sides. The square roots of the results (so obtained) are the diagonals of a viṣama quadrilateral (inscribed in a circle).*

That is, if m and n be the diagonals of a cyclic scalene quadrilateral with sides a, b, c, d taken in order (vide Fig.3) then the theorem states :

$$m = \sqrt{\left(\frac{ad+bc}{ab+cd}\right)(ac+bd)}$$

and

$$n = \sqrt{\left(\frac{ab+cd}{ad+bc}\right)(ac+bd)}.$$

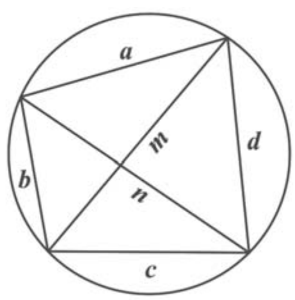

Fig. 3

Brahmagupta gives no proof of the theorem, so we don't know how he derived it. The proof of the theorem is found later in the book 'Mathematics in Ancient and Medieval India' by A. K. Bag.[7] This theorem is now generally known as Brahmagupta's theorem related to the determination of diagonals of any cyclic quadrilateral, and was rediscovered in Europe in 1619 CE by W. Snell (1580–1626), a Dutch astronomer-mathematician.[8]

If we take the product of the diagonals m and n, then we get $m.n = ac + bd$ i.e., the product of the diagonals of a cyclic quadrilateral equals the sum of the products of its two pairs of opposite sides, which is knows as Ptolemy's Theorem, named after C. Ptolemy (c.100 – c.170 CE), a Greek mathematician.

Additionally, it is the first time that we find the following proportion in Brahmagupta's work:

$$\frac{m}{n} = \frac{ad+bc}{ab+cd}.$$

It can be compared with Regiomontanus's (1436 – 1476, German mathematician) efforts in the 15th century to generate a formula concerning the area of an inscribed quadrilateral or with the fact that W.Snell (1580–1626) rediscovered Brahmagupta's formula in Europe in 1615 (?) CE, for which Philip Naude′ the Younger (1684 – 1747, French mathematician) could finally provide a proof in 1727 (!) CE.[9]

Central Altitude-segments of a Cyclic Scalene Quadrilateral

In the verse XII.3I of the *Br.Sp.Si.* , Brahmagupta states the rule for the segments of the altitude passing through the point of intersection of the diagonals of a cyclic quadrilateral in which the diagonals are at right angles. The concerned verse is as below:

> त्रिभुजे भुजौ तु भूमिस्तल्लम्वो लम्बकाधरं खण्डम्।
>
> ऊर्ध्वमवलम्बखण्डं लम्बकयोगार्धमधरोनम्॥ 31॥
>
> *tribhuje bhujau tu bhūmistallamvo lambakādharaṃ khaṇḍam|*
>
> *ūrdhvamavalambakhaṇḍaṃ lambakayogār dhamadharonam* ||31||
>
> ***This means:*** *The lower segments of the two diagonals are taken to be the sides of a triangle whose base is the base of the quadrilateral. Its altitude will be the lower segment of the (central) perpendicular (on the base through the point of intersection of the diagonals). The upper segment of the (central) perpendicular will be half the sum of the two (side) perpendiculars (each) diminished by the lower segment (of the central perpendicular).*

The rule is narrated as follows:

According to the figure (vide Fig.4), the lower segments of the diagonals BD and CA of the cyclic quadrilateral ABCD are BH and CH, where H denotes the point of intersection of the (said) diagonals. The (central) perpendicular on the base BC of the quadrilateral, passing through the point H, is EF and the two side perpendiculars on the same base are AM and DN. Then, in accordance with the aforesaid rule, the upper segment EH of the (central) perpendicular EF is :

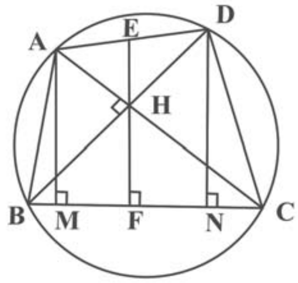

Fig. 4

$$EH = \frac{(AM - HF) + (DN - HF)}{2},$$ from which we get $$EF = \frac{1}{2}(AM + DN).$$

That is, the height of the point E above the base BC is half the sum of the heights of the points A and D above the same base. This implies that E bisects AD, a result now famous as 'Brahmagupta's theorem'[10] and usually stated in the following fashion:

If in a cyclic quadrilateral the diagonals intersect perpendicularly, then the perpendicular on a side of it from the point of intersection of the diagonals always bisects the opposite side.

It is worth mentioning that this theorem and its proof are found to occur in the text books of the present day high school geometry. For a ready reference, the proof has been given in the APPENDIX of the present article.

Construction of Rational Cyclic Scalene Quadrilaterals

We have already noticed Bruhmugupta's rule for finding the circumradius of a triangle and a quadrilateral (verse X11.27. *Br.Sp.Si.*). Here he mentioned the Sanskrit term चतुर्भुजकोणस्पृग्वृत्त (*caturbhujakoṇasprgvṛtta*) which means that he probably considered 'an inscribed quadrilateral whose vertices touch the circle'. His most important contributions to geometry about the cyclic quadrilateral are : finding its area (verse XII.21) and the theorem regarding determination of its diagonals (verse XI1.28). These are also discussed earlier. The problem for constructing cyclic scalene quadrilaterals with rational sides is interesting. Brahmagupta's work, in this context, is indeed a notable achievement. In the verse XI1.38 of the *Br.Sp.Si.*, he gives the method of solution of this remarkable proposition as given below :

जात्यद्वयकोटिभुजा: पर कर्ण गुणा भुजाश्चतुर्विषमे।
अधिको भूर्मुखहीनो बाहुद्वितयं भुजावन्यौ॥ 38॥

jātyadvayakoṭibhujāḥ para karṇa guṇā bhujāścaturviṣame|
adhiko bhūrmukhahīno bāhudvitayaṃ bhujāvanyau ||38||

This means: *The uprights (koṭis) and bases (bhujas) of two rational right-angled triangles (jātyas) multiplied by each other's hypotenuses will give the four sides of a scalene (viṣama) quadrilateral. (Of these four sides) the greatest is the base, the smallest is the face, and the other two sides are the two flanks.*

This states that, if *(a,b,c)* and *(p,q,r)* be the sides of two rational right-angled triangles such that $a^2 + b^2 = c^2$ and $p^2 + q^2 = r^2$ then $c(p,q)$, $r(a,b)$ i.e., cp, cq, ar, br will be the four sides of a scalene quadrilateral.

It is narrated as follows:[ll,12]

Let us construct two right-angled triangles COB and COD with sides *a (p,q,r)* and *p (a,b,c)* respectively. Again, on the other side of BD, we construct two right-angled triangles DOA and BOA with sides *b (p, q, r)* and *q (a, b, c)* respectively. Thus we obtain a scalene quadrilateral ABCD with sides *(cq, ar, cp, br)* in order, and having its diagonals at right angles (vide Fig. 5).

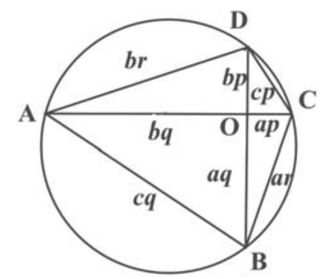

Fig. 5

Now, using the rule previously described in the verse XII.27 of the *Br.Sp.Si.*, we get the circumradius of the triangle $ABC = \frac{cq.ar}{2aq} = \frac{cr}{2}$ and that of the triangle $ABD = \frac{cq.br}{2bq} = \frac{cr}{2}$. This shows that the quadrilateral ABCD is cyclic and its circumradius $= \frac{1}{2}cr$. Since $a^2 + b^2 = c^2$ and $p^2 + q^2 = r^2$, we may write $\frac{1}{2}cr = \frac{1}{2}\sqrt{(ar)^2 + (br)^2} = \frac{1}{2}\sqrt{(cp)^2 + (cq)^2}$ That is, the circumradius is equal to half the square root of the sum of the squares of the opposite sides of ABCD.

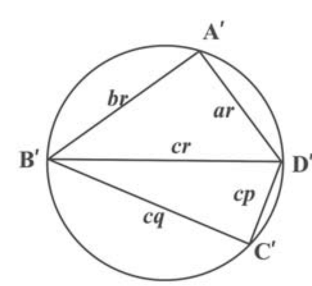

Fig. 6

Again, considering two pairs of opposite sides *(cq, cp)* and *(ar, br)* of ABCD as adjacent sides, we may construct another scalene quadrilateral A′B′C′D′. This quadrilateral so formed, with sides (cq, cp, ar, br) in order will also be cyclic having a pair of opposite angles as right angles (semi-circular angles), and its circumdiameter cr as a diagonal (vide Fig. 6). It is noteworthy that, in these two quadrilaterals (ABCD and A′B′C′D′) the sum of the squares of the greatest and the smallest sides will be equal to that of the remaining two sides.

The two cyclic quadrilaterals with sides (cq, ar, cp, br) and (cq, cp, ar, br), both in order, are called 'Brahmagupta Quadrilaterals'.[13] In particular, if we take two right-angled triangles with sides (3,4,5) and (5,12,13) instead of (a,b,c) and (p,q,r), we obtain Brahmagupta quadrilaterals with sides (60, 39, 25, 52) and (60, 25, 39, 52) both in order. The same example is given by Pṛthūdakasvāmī in his commentary on the *Br.Sp.Si.* (verse XII.38).

It is evident from the cyclic scalene quadrilateral (60, 39, 25, 52) that its diagonals are at right angles, and its greatest and the smallest sides are placed opposite to each other (vide Fig. 5). Besides, the circumradius of the quadrilateral is equal to $\frac{1}{2}\sqrt{(39)^2+(52)^2} = \frac{1}{2}\sqrt{(60)^2+(25)^2} = \frac{65}{2}$.

Conclusion

We have mentioned at the beginning of this article that Brahmagupta makes a remarkable extension of "Heron's formula" by giving its quadrilateral version. Perhaps it is the most beautiful result in Brahmagupta's works; but the glory of his achievement is somewhat dimmed by failure to mention that the formula is correct only in the case of a cyclic quadrilateral. However, the rules for finding the circumradius of an isosceles trapezium and a scalene quadrilateral, as given by him, are mostly correct. Besides, the rule for determining the circumradius of a triangle and a quadrilateral, contributed by him, is also found correct. Needful to say, we can find glimpses of this work in the geometry tought at present in high schools. Brahmagupta's most important contribution to geometry is the theorem for the length of the diagonals of any cyclic quadrilateral in terms of its sides, which was rediscovered in Europe in 1619 CE by W. Snell.

It is worth mentioning here that, in geometry, the theorem as stated by Brahmagupta (in the verse XII.31), is now famous as 'Brahmagupta's theorem' and is found to occur in the text books of the present day high school geometry. Again, the method for the construction of rational cyclic scalene quadrilaterals (verse XII.38) is indeed a notable achievement of Indian geometry in the early medieval period. In this context, it must be noted here that Brahmagupta gives two types of cyclic quadrilaterals as solution for constructing the same and these two types of quadrilaterals are called 'Brahmagupta Quadrilaterals'. Hence, we may conclude that the geometrical contributions regarding cyclic quadrilaterals made by Brahmagupta of course have some relevance in the field of the present day high school geometry.

An earnest effort has been made to make the translalions of the Sanskrit verses as literal as possible. While translating the verses, interpretation of the geometrical contents in modern notation are given wherever felt necessary.

This article is a humble tribute to my revered teacher Dr. Bandana Barua, former Head, Department of Pure Mathematics, University of Calcutta, whose blessing has been a beacon light for me over the years.

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Abbreviation used

Br.Sp.Si. = *Brāhmasphuṭasiddhānta* ❐

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Appendix

Proof of the 'Brahmagupta's theorem'

Let ABCD be a cyclic quadrilateral whose diagonals BD and CA intersect perpendicularly at the point H. The perpendicular HF is drawn from the point H on the side BC of it. If E be the point of intersection of the extended FH and the side AD, then the theorem states that E is the midpoint of AD (vide Fig. 7).

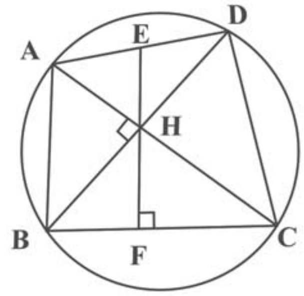

Fig. 7

In the figure, $\angle DAC = \angle DBC$ (angles in the same segment) i.e., $\angle EAH = \angle HBF$. Since $\angle HFB = 90^o$, $\therefore \angle HBF + \angle BHF = 90^o$ or, $\angle BHF = 90^o - \angle HBF = 90^o - \angle EAH$ ...... (i)

Again, $\angle BHF = \angle EHD$ (opposite angles), but since $\angle AHD = 90^o$, $\therefore \angle EHD = 90^o - \angle EHA$ i.e., $\angle BHF = 90^o - \angle EHA$ ...... (ii)

Now, from (i) and (ii) we get $\angle EAH = \angle EHA$. Hence ΔAEH is an isosceles triangles, and thus EA = EH ..... (iii)

Similarly, it may be shown that $\angle EHD = \angle EDH$; hence, ΔDEH is an isosceles triangle, and so EH = ED .... (iv). Finally, from (iii) and (iv) it follows that EA = ED, as the theorem claims.