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A Geometric Interpretation of the Collatz Conjecture

***Abstract*: This paper presents a novel geometric interpretation of the Collatz Conjecture. By analyzing the intersection points of specific linear equations, we demonstrate a relationship between these intersections and the iterative steps defined by the Collatz function. Furthermore, we explore the connection between the conjecture and polynomial expressions, offering a visual representation through a graphical analysis of polynomial coefficients and exponents.**

Keywords: Geometric Interpretation, Linear Equations, Polynomial Representation and Collatz Conjecture.

Linear Equations and Intersection Points

We define two sets of linear equations based on the parity of n:

For odd positive integers n, the straight line defined by the equation $-4x + 3y = 6n - 4$ intersects with the line $-2x + y = -2$ at the point $(3n + 1, 6n)$.

For even positive integers n, the straight line defined by $(n + n/2)x + (n/4)y = 2n + (n/2(n - 1))$ intersects with the line $-2x + y = -2$ at the point $((n/4) + 1, n/2)$.

Connection to the Collatz Function

Let f(x) be a function defined as follows:

$f(x) = 3n + 1$, if n is a positive odd integer.

$f(x) = n/2$, if n is a positive even integer.

This function, f(x), corresponds to the Collatz Conjecture,[1] where the x-coordinate of the intersection point for odd n and the y-coordinate of the intersection point for even n define the respective operations.

Polynomial Representation

$x^n + (-1)^k x^{n-2(1+k)} + ... + (-1)^k x^2 + (-1)^k$, where k is 1 for n as positive odd integer or k is 2 for n as positive even integer, is a polynomial. If we square this polynomial then there we will get two parts[2], initial and trailing parts $x^{2n} + (-1)^k 2x^{2n-2(1+k)}$ and $1 + 2x^2$ respectively.

Geometric Interpretation of Polynomials

If we draw a graph taking coefficients and exponents of x as x coordinates and y coordinates there will have two straight lines. And if we expand these lines, then, these two lines will intersect at $(3n + 1, 6n)$ for n as positive odd integer or $(n/4 + 1, n/2)$ for n as positive even integer.

The spectra of the graph of the figure represents three types of straight lines. A single red colour, which is the tangent of common trailing parts of the square of the polynomials for all odd and even. And the purple and blue coloured straight lines represent the tangent of initial parts of the square of the polynomials with positive odd and even n respectively. And these tangents of the initial parts have their own intersection points with the tangent of common trailing part

Discussion

This geometric approach provides a different perspective on the Collatz Conjecture. The intersection points of the defined linear equations directly relate to the values generated by the Collatz function. The polynomial representation and its associated graphical interpretation

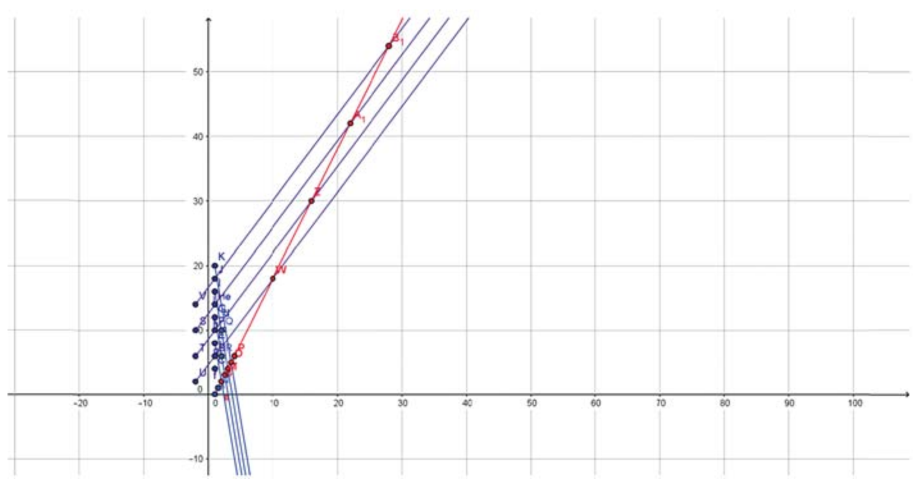

Fig. Description

offer a visual means to analyze the behavior of the conjecture.

Conclusion

This analysis demonstrates a geometric framework for understanding the Collatz Conjecture. The relationships between linear equation intersections, polynomial expressions, and the Collatz function provide avenues for further exploration and potential solutions to this longstanding mathematical problem. ❐

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