KOLKATA PAISE RESTAURANT (KPR) PROBLEM

ASIM GHOSH[1,*] AND SUDIP MUKHERJEE[2,3,†]

The Kolkata Paise Restaurant (KPR) problem is a repeated game, played between a large number N of agents having no interaction with each other. In the KPR problem, the customers or agents choose from N restaurants each day simultaneously in parallel decision mode. The problem was created in 2007 by B. K. Chakrabarti. In this review article, we will briefly discuss the strategies developed for KPR problem and also the problems where KPR strategies were applied. In the appendix section, we display the articles and books where Kolkata Paise Restaurant problem was appeared in their citation lists by the scientists outside from Kolkata.

Introduction

The Kolkata Paise Restaurant (KPR) problem is repeatedly played among a large number N of agents or players having no interaction amongst themselves. The agents or players choose from N' restaurants each evening independently ($N' \leq N$). In the problem the prospective customers or players each have the same set of data regarding the success or failure of the various restaurants: the data set gives the number of prospective customers arriving at each restaurant for the past evenings. Let us assume that the price for the meal to be the same for all the restaurants though the customers can have a ranking of preference for each restaurant (agreed upon by all customers). For simplicity we also assume that each restaurant can serve only one customer any evening. As already mentioned, information about the customer distributions for earlier evenings is available to everyone.

Each customer will try to go to the restaurant with the highest possible rank while avoiding the crowd so as to be able to get dinner there. If any restaurant is chosen by more than one customer on any evening, one of them will be randomly chosen (each of them is anonymously treated) and will be served. The rest will not get dinner that evening. The customers collectively learn from their attempts in the past, how to avoid the crowd to get the meal from a high ranking restaurant.

Many years ago, in Kolkata, there were very popular, cheap and fixed rate "Paise Hotel" that were mostly visited by the daily workers or laborers coming to the city for construction works etc. They used to walk (to save the transport costs) to one of these restaurants for their lunches during the *tiffin time* and would miss lunch if they got to a crowded restaurant. Searching for the next restaurant would mean failing to report back to work on time! Paise is the smallest-value Indian coin. There were indeed some well-known rankings of these restaurants, as some of the restaurants would offer tastier food items compared to the others.

A more general example of such a problem can be when the public administration provides hospitals (and beds) in every locality but the locals prefer better ranked hospitals (commonly agreed by everyone) elsewhere. They would then be competing with other 'outsiders' as well as with the local patients of that locality. Unavailability of treatment in the over-crowed hospitals may be considered as lack of the service for those people and consequently as (social) wastage of service by those unattended hospitals.

1 Department of Physics, Raghunathpur College, Raghunathpur, Purulia, West Bengal 723133, India

2 Barasat Government College, Barasat, Kolkata 700124, India.

3 Condensed Matter Physics Division, Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata 700064, India.

* Email: asimghosh066@gmail.com

† Email: sudip.mukherjee@saha.ac.in

One (trivial or dictator's) solution to the KPR problem may be the following: planner (or dictator) requests (or orders) everyone to form a que and each one is assigned a restaurant with rank matching the sequence of the person in the que on the first evening. Then each person will be told to go to the next ranked restaurant in the following evening (for the person in the last ranked restaurant will go to the first ranked restaurant). This shifting process (with periodic boundary condition) will continue for successive evenings. We call this dictator's solution. This is one of the most efficient solution (with utilization fraction of the services by the restaurants equal to unity) and the system achieves this efficiency immediately (from the first evening itself). However, this cannot be any acceptable solution of the KPR problem in reality, where each agent takes his or her own decision (in parallel or democratically) every evening, based on commonly shared information about past events. In KPR problem, the prospective customers try to evolve a learning strategy to get dinners eventually at the best possible ranked restaurant, avoiding the crowd. Generally the evolution of these strategies take considerable time to converge and even then the eventual utilization fraction is far below unity.

The Kolkata Paise Restaurant (KPR) was first conceived in an earlier form in 2007[1] and the present formation of the problem was made[2,3]. After that, several developments were made (including quantum version of the KPR problem)[4-21]. Several reviews and books covering the KPR problems can be found in the[22-25,25-32]. A list of papers, extending the KPR problem to different social situations can be found in the[33-44]. In this review article, we will briefly discuss the strategies developed for KPR problem and also the problems where KPR strategies were applied. In the appendix section, we display the articles and books where Kolkata Paise Restaurant problem was appeared in their citation lists by the scientists outside from Kolkata.

Stochastic Learning Strategies

Here we are going to discuss the dynamics of a few (classical) stochastic learning strategies for the KPR problem, where N agents choose among N' (let $N' = N$) equally priced but differently ranked restaurants every evening such that each agent tries to get dinner in the best restaurant (each serving only one customer and the rest arriving there going without dinner that evening). All agents are taking similar (but not the same) learning strategies and assume that each follow the same probabilistic or stochastic strategy dependent on the information of the past in the game. We will show that a few of these strategies lead to much better utilization of the services than most others.

Suppose an agent chooses the k-th restaurant having rank r_k on any day (t) with the probability $p_k(t)$ given by

$$p_k(t) = \frac{1}{Z}\left[r_k^{\xi} \exp\left(-\frac{n_k(t-1)}{T} \right) \right],$$

$$Z = \sum_{k=1}^{N}\left[r_k^{\xi} \exp\left(-\frac{n_k(t-1)}{T} \right) \right], \quad (1)$$

where $n_k(t)$ is the number of agents arriving at the r_k-th ranked restaurant on the t-th day where $T > 0$ is a scaling (noise) factor and $\xi \geq 0$ is an exponent. Therefore, the probability of selecting a particular restaurant increases with its rank r_k and decreases with its popularity in the previous day (given by the number $n_k(t-1)$). Few properties of the strategies leading to the above probability are the following:

1. For $\xi = 0$ and $T \to \infty$, $p_k(t) = \frac{1}{N}$ corresponds to the purely random choice case for which the average utilization fraction is around 0.63, i.e., on an average the utilization of the restaurants is 63%.
2. For $\xi = 0$ and $T \to 0$, the agents still choose randomly but avoid completely those restaurants which had been visited in the last evening or day ($n(t-1)$) is non-zero). Thus choose (again randomly) from the remaining restaurants. Both analytically and in numerical simulations it seen that the average utilization fraction $\bar{f}$ is around 0.46.

These limiting and also some intermediate cases are given below.

A. Random Choice Strategies : Let us consider the case with $r_k = 1$ for all k (restaurants). Suppose there are λN agents and N restaurants. An agent can select any restaurant with equal probability. Therefore, the probability that a single restaurant is chosen by m agents is given by

$$\Delta(m) = \binom{\lambda N}{m} p^m (1-p)^{\lambda N - m}; \quad p = \frac{1}{N}$$

$$= \frac{\lambda^m}{m!} \exp(-\lambda) \text{ as } N \to \infty. \quad (2)$$

So, the fraction of restaurants not chosen by any agents is given by $\Delta(m = 0) = \exp(-\lambda)$ which implies that average fraction of restaurants occupied on any evening is given by

$$\bar{f} = 1 - \exp(-\lambda) \simeq 0.63 \text{ for } \lambda = 1 \tag{3}$$

for random choice case in the KPR problem[2]. It may be noted this value of the resource utilization factor $\bar{f}$ is obtained at the very rst evening. The convergence time τ here is therefore zero convergence time.

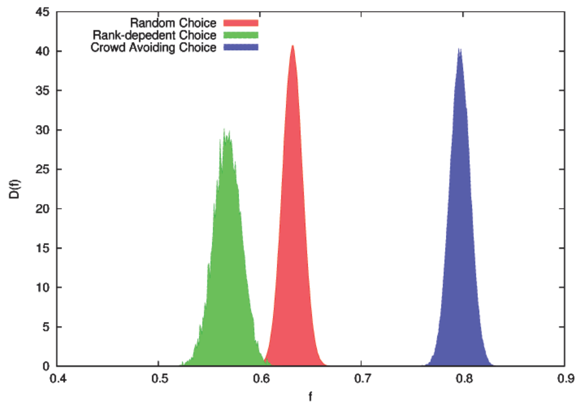

Fig. 1: Figure shows the probability distributions of every day utilization f (f = 1 denotes 100% utilization) for dierent strategies. All distributions are Gaussian shape with peaks at f = 0.63 (random choice), f = 0.58 (simple rank dependent choice) and f = 0.80 (crowed avoiding choice).

B. Rank Dependent Strategies: Here r_k is not a constant (but dependent of k). For any real ξ and $T \to \infty$, an agent goes to the k-th restaurant with probability $p_k(t)$ $= r_k^\xi / \sum r_k^\xi$. The results for such a strategy can then be derived as follows:

If an agent selects any restaurant with probability p then probability finding a single restaurant chosen by m agents is given by

$$\Delta(m) = \binom{N}{m} p^m (1-p)^{N-m}. \tag{4}$$

Therefore, the probability that any restaurant with rank k is not chosen by any of the agents will be given by

$$\Delta_k(m=0) = \binom{N}{0} (1-p_k)^N \; ; \; p_k = \frac{r_k^\xi}{\sum r_k^\xi}$$

$$\simeq \exp\left(\frac{-k^\xi N}{\tilde{N}}\right) \text{as } N \to \infty, \tag{5}$$

where r_k is set equal to k and $\tilde{N} = \sum_{k=1}^{N} r_k^\xi = \int_0^N k^\xi dk$ $= \frac{N^{\xi+1}}{(\xi+1)}$.

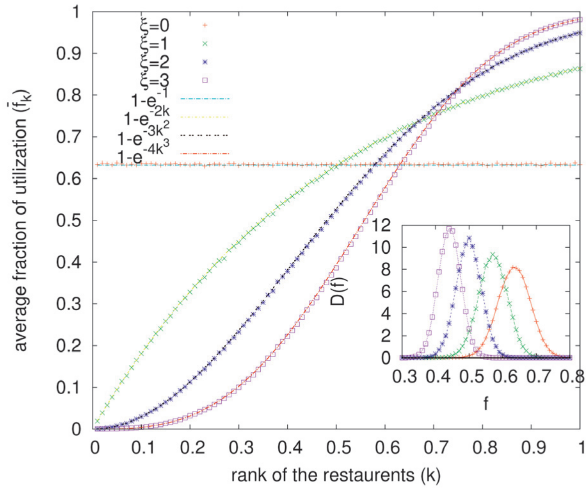

Fig. 2: The main figure shows average fraction of utilization ($\bar{f}_k$) versus rank of the restaurants (k) for different ξ values. The inset shows the distribution $D(f = \sum \bar{f}_k / N)$ of the fraction f agent getting dinner any evening for different ξ values.

Hence

$$\Delta_k(m=0) = \exp\left(-\frac{r_k^\xi(\xi+1)}{N^\xi}\right). \tag{6}$$

Therefore, the average fraction of agents getting food any evening (day) in the k-th ranked restaurant is given by

$$\bar{f}_k = 1 - \Delta_k(m=0). \tag{7}$$

Fig. (2) shows the numerical estimates of $\bar{f}_k$. For ξ = 0, the problem reduces to the random choice case (as considerd in 2:2:1) and one gets $\bar{f}_k = 1 - e^{-1}$, giving $\bar{f} = \sum \bar{f}_k / N \simeq 0.63$. For ξ = 1, we get $\bar{f}_k = 1 - e^{-2k/N}$, giving $\bar{f} = \sum \bar{f}_k / N \simeq 0.58$ [2].

C. Strict Crowd-avoiding Case : We consider here the case where each agent chooses on any evening (t) randomly among the restaurants in which nobody had visited in the last evening (t – 1). This is the case where ξ = 0 and T → 0 in Eq. (1). Numerical simulation results for the distribution $D(f)$ of the fraction f of utilized restaurants is Gaussian with a most probable value at $\bar{f} \simeq 0.46$. This can be explained in the following way: As the fraction $\bar{f}$ of restaurants visited by the agents in the last evening is completely avoided by the agents this evening, so the number of available restaurants is $N(1 - \bar{f})$ for this evening and is chosen randomly by all the agents. Hence, when fitted to Eq. (2) with λ = 1/(1 – $\bar{f}$). Therefore, following Eq. (2), the equation for $\bar{f}$ can be written as

$$(1-\bar{f})\left[1 - \exp\left(-\frac{1}{1-\bar{f}}\right)\right] = \bar{f}.$$

By solving above equation, we get $\bar{f} \simeq 0.46$. This result is well fitted with the numerical results for this limit (xi = 0, T → 0)[2].

D. Stochastic Crowd Avoiding Case : Let the strategy be the following: if an agent goes to restaurant k in the earlier day (t – 1) then the agent will go to the same restaurant in the next day with probability $p_k(t) = \frac{1}{n_k(t-1)}$ and to any other restaurant $k'(\neq k)$ with probability $pk'(t) = \frac{(1-p_k(t))}{(N-1)}$. Numerical results for this stochastic strategy show the average utilization fraction $\bar{f}$ to be around 0.80 and the distribution $D(f)$ to be Gaussian peaked around $f \simeq 0.8$ as shown in Fig. II.[3].

An approximate estimate of the average utilization ratio $\bar{f}$ for this strategy in steady state may proceed as follows: Let $a_i(t)$ denote the fraction of restaurants having exactly i agents (i = 0; ... ; N) visiting on any evening (t) and assume that $a_i(t) = 0$ for $i \geq 3$ at any (large enough) t, as the dynamics stabilizes in steady state. So, $a_0(t)+a_1(t)+a_2(t) = 1$, $a_1(t)+2a_2(t) = 1$ for any (large enough) t. Hence $a_0(t) = a_2(t)$. Now $a_2(t)$ fraction of agents will make attempts to leave (each with probability 1/2) their respective restaurants in the next evening (t + 1), while no activity will occur on the restaurants where, only one came (a_1) in the previous evening (t). These $a_2(t)$ fraction of agents will get equally divided (each in the remaining N – 1 restaurants). Of these $a_2(t)$, the fraction going to the vacant restaurants (a_0 in the earlier evening) is now $a_0(t)a_2(t)$. Hence the new fraction of vacant restaurants at this stage of consideration will be $a_0(t) - a_0(t)a_2(t)$. In the restaurants having exactly two agents (a_2 fraction in the last evening), some vacancy will be created due to this process in steady state, and this fraction will be equal to $\frac{a_2(t)}{4} - a_2(t)\frac{a_2(t)}{4}$. In the steady state, where $a_i(t+1)=a_i(t)$ = a_i for all i and t, we get $a_0 - a_0 a_2 + \frac{a_2}{4} - a_2\frac{a_2}{4} = a_0$. Hence using $a_0 = a_2$ we get $a_0 = a_2 = 0.2$, giving a_1 = 0:6 and $\bar{f} = a_1 + a_2 = 0.8$ in the steady state. The above calculation is approximate as none of the restaurant is assumed to get more than two costumers on any day (a_i = 0 for $i \geq 3$). The advantage in assuming only a_1 and a_2 to be non-vanishing on any evening is that the activity of redistribution on the next evening starts from a_2 fraction of the restaurants only. This of course affects a_0 and a_1 for the next day and for steady state these changes will balance. Numerically we checked that $a_i \leq$ 0:03 for $i \geq 3$ and hence the above approximation does not lead any serious error[3].

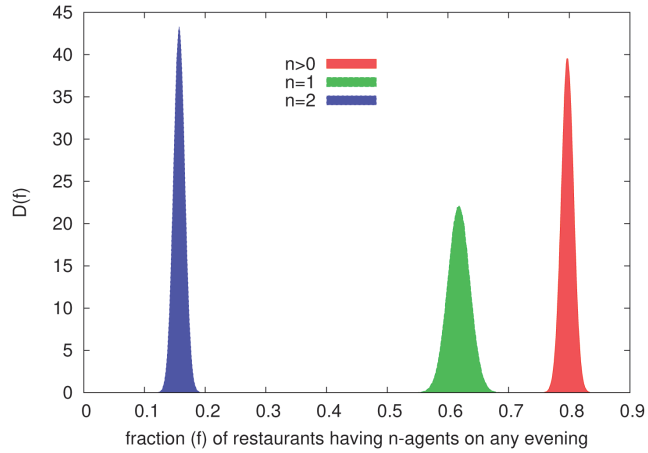

FIG. 3: Plot shows the numerical simulation results for a typical prospective customer distribution on any evening.

Phase Transition in KPR Problem

Here we will discuss the general problem with N restaurants and gN agents, where the fractional density g is a fixed external parameter of the dynamics. Recasting the problem in terms of zero-range interacting particles, we observe a phase transition from a frozen phase with all satisfied agents (and therefore not moving away from the earlier choice of the respective restaurants) to an active phase with unsatisfied ones at a critical density g_c[3]. Extensive numerical simulations as well as some analytical calculations were performed to understand its features, finding a good agreement with the exponents of stochastic fixed-energy sandpiles. The behavior of the relaxation properties of the frozen phase reveals an interesting faster-is-slower effect. The study consists of the general observation that a high level of coordination can arise spontaneously from strategies which involve rather slower dynamics, which however speed up the approach to overall optimization or utilization and individual performance.

The Models : Inspired by the results for the problem with N agents competing for N restaurants (each can serve food to one person per day), we will discuss a more generic and generalized stochastic occupation problem with exclusion.

The rank ordering among the restaurants is disregarded and we consider in general gN agents, where the density g (the ratio between total number of agents and total number of restaurants), can be taken as an external parameter of the dynamics. For brevity, we refer to individuals as particles and restaurants as the sites or nodes of an underlying network. In these terms the original problem is defined on a fully connected graph.

A particle (or agent) moves from the site (restaurant) i to a randomly chosen neighboring site (or restaurant) j with a rate $v(n_i)$ that depends on the number (n_i) of particles that are present at it. This can be mapped to a *zero range process*[8] which allows us to say that the stationary probability distribution of the number of particles per site can factorize in terms of single site functions. Given the nature of the problem, we will discuss models having the jumping rate $v(1) = 0$ for single occupancy, i.e., agents are happy while alone, and $v(n + 1) \geq v(n)$, i.e. the particles repel each other (crowd avoiding). Given this definition for the rates, at low densities ($g < 1$) there are sites filled by single particles. But for high densities ($g > 1$) a finite fraction of sites – so called 'active' sites – have multiple occupancy. It has been shown[8] that there is a transition between these two phases that appears at a certain density $g_c \leq 1$. Note that, in principle, this state is ergodic and hence every configuration of it is accessible. Therefore, for $g \leq 1$, the process sooner or later visit a state where $n_i \leq 1$ for all sites and the dynamics stops (absorbing state). When N is sufficiently large and $g > g_c$, most sites become active ($n > 1$). The order parameter is defined as the steady state density of active sites ρ_a (density of sites having $n > 1$). Therefore the absorbing phase conforms to $\rho_a = 0$, whereas above some density g_c the steady state appears with a non-zero value of the order parameter ($\rho_a > 0$).

We will discuss in particular two models:

(A) $v(n) = 1 - \frac{1}{n}$

(B) $v(n) = (1 - p)\theta(n - 1)$.

We use the parallel dynamics having a simultaneous update of the sites (or restaurants) at each time step, i.e. agents' actions are simultaneous akin to a repeated game problem. Similarly for a sequential update, in which at each time step, a randomly chosen particle jumps with some probability.

The model A is implementation of the stochastic crowd avoiding strategy of the original KPR problem. If the site k has $n_k \geq 1$ particles, each particle stay back with probability $1/n_k$ in the next time step, otherwise it jumps to any of the neighboring sites randomly (see numerical results shown in Figs. III.4 & III.5).

In the model B an external parameter p is proposed that represents the "patience" of costumers to overcrowded conditions. The dynamic is following if the site k has $n_k \geq 1$ particles, each particle stays with probability p in the next time step, otherwise it will jump to any of the neighboring sites randomly.

The model B is similar to a kind of fixed energy sandpile, but the study of its dynamics as a function of the parameter p will acknowledge an interesting faster-is slower effect related to the relaxation time of the frozen state.

Finally a waiting choice will be pointed out that can be *rational* from the point of view of game theory: the agents in overcrowded restaurants could wait simply because they expect that others are leaving them alone.

Results from Numerical Simulations : The times required to reach the steady state below and above g_c are measured. The order parameter ρ_a for below g_c reaches a value $\rho_a = 0$ in the steady state. For $g > g_c$, order parameter ρ_a gets to a stationary state and fluctuates around a mean value $\rho_a^0 (> 0)$. The system has persistent dynamics in this case. The growth of the order parameter is exponential away from g_c, and can be asserted as

$$\rho_a(t) = \rho_a^0 \left[1 - e^{-t/\tau}\right] \quad (8)$$

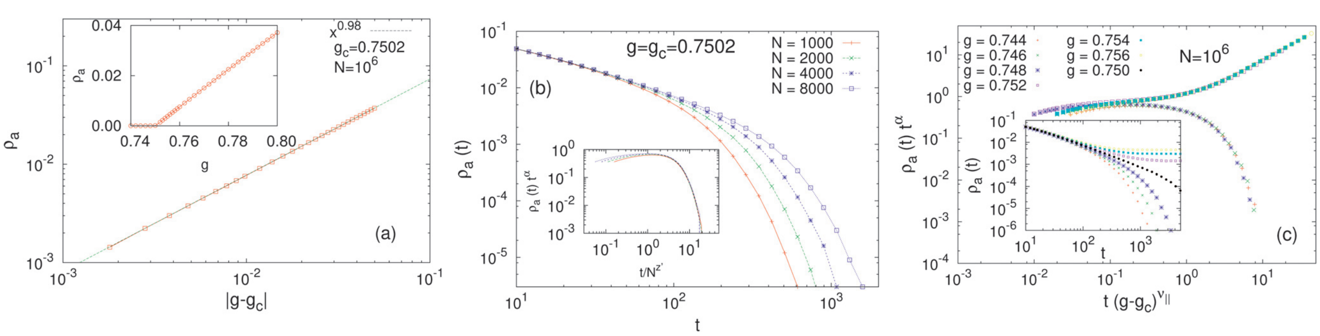

Fig. 4: Simulation results for model (A) in mean field case with estimated $g_c = 0.7502 \pm 0{:}0002$. (a) Variation of steady state density ρ_a of active sites versus $g - g_c$, fitting to $\beta = 0.98 \pm 0.02$. The inset shows the variation of ρ_a with density g. (b) relaxation to absorbing state near critical point for different system sizes, the inset showing the scaling collapse giving estimates of critical exponents $\alpha = 1.00 \pm 0.01$ and $z' = 0.50 \pm 0.01$. (c) Scaling collapse of $\rho_a(t)$. The inset shows the variation of $\rho_a(t)$ versus time t for different densities g. The estimated critical exponent is $\nu_\parallel = 1{:}00$ $0{:}01$. The system sizes N are mentioned. Taken from[8]. (Permission to use the figure from the paper is given by American Physical Society)

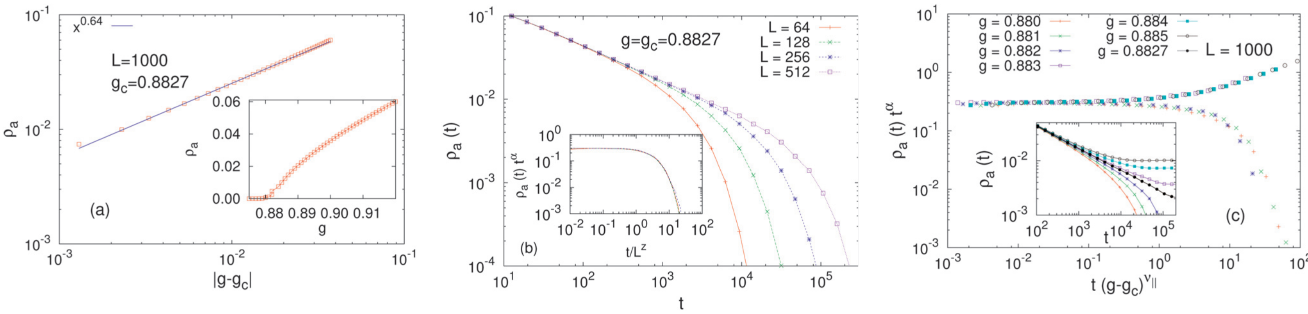

Fig. 5: Simulation results for 2-d case in model (A) with estimated g_c = 0.8827 ± 0.0002. (a) Variation of steady state density ρ_a of active sites versus $g - g_c$, fitting to β = 0.68 ± 0.01. The inset shows the variation of ρ_a with density g. (b) relaxation to absorbing state near critical point for different system sizes, the inset showing the scaling collapse giving estimates of critical exponents α = 0.42 ± 0.01 and z = 1.65 ± 0.02. (c) Scaling collapse of ρ_a(t). The inset shows the variation of $\rho_a(t)$ versus time t for different densities g. The estimated critical exponent is $\nu_{||}$ = 1.24 ± 0.01. The simulations are done for square lattices of linear size L ($N = L^2$). Taken from[8]. (Permission to use the figure from the paper is given by American Physical Society)

for g > g_c, and

$$\rho_a(t) \propto e^{-t/\tau} \tag{9}$$

for $g < g_c$, where τ is time scale of the relaxation. We are going to denote the asymptotic value of the order parameter as ρ_a hereafter. Close to the critical point ($g - g_c \to 0_+$), find $\rho_a \sim (g-g_c)^\beta$ where β is the exponent of order parameter, and $\tau \sim (g-g_c)^{-\nu_{||}}$. Typically $\rho_a(t)$ obeys a scaling form

$$\rho_a(t) \sim t^{-\alpha} F\left(\frac{t}{\tau}\right); \tau \sim (g-g_c)^{-\nu_{||}} \sim L^z, \tag{10}$$

where α and z are the dynamic exponents and L stands for size of the system. Then we get $\beta = \nu_{||}\alpha$ by comparing Eq. (8), Eq. (9) and Eq. (10) when t/τ is a constant for $t \to \infty$. Numerically the time variation of $\rho_a(t)$ is studied and measure the exponents by fitting with above scaling relation.

Model A : *Mean Field case :* For the mean field case, a systems of $N = 10^6$ sites is considered, averaging over 10^3 initial conditions. It is found g_c = 0.7502 ± 0002. Using scaling fitting of $\rho_a(t)$ for different g values (see Fig. 4) give β = 0.98 ± 0.02, z' = 0.50 ± 0.01 (assuming $N = L^4$ and using Eq. (10), a relation $z = 4z'$ is got and therefore z = 2.0 ± 0.04), $\nu_{||}$ = 1.00 ± 0.01, α = 1.00 ± 0.01. And these independently estimated exponent values satisfy the scaling relation $\beta = \nu_{||}\alpha$ well.

Lattice cases : The same dynamics in 1-d and 2-d are studied. For a linear chain in 1-d, $N = L = 10^4$ is taken and averaged over 10^3 initial conditions. For 2-d a square lattice ($N = L^2$) with L = 1000 is considered and averaging over 10^3 initial conditions. Periodic boundary condition are applied in both cases.

(a) The 1-d model is following: The particles can hop only to their nearest neighbor sites, and each particle will choose either left or right neighbor randomly. Here g_c = 1 is found and hence the phase transition is not interesting.

(b) In the 2-d version of the model, a square lattices is considered and the particles choose one of the 4 nearest neighbors randomly. For N = 1000 × 1000, g_c = 0.88 ± 0.01, β = 0.68 ± 0.01, z = 1.65 ± 0.02, $\nu_{||}$ = 1.24 ± 0.01 and α = 0.42 ± 0.01 (Fig. III.5). However these independently estimated exponent values do not fit very well with the scaling relation $\beta = \nu_{||}\alpha$ but this type of scaling violation was also observed in many active-absorbing transition cases.

Model B : *Mean field case :* For the mean field case, $N = 10^6$ is taken, averaging over 10^3 initial condition. Numerically the phase diagram is investigated and the universality classes of the transition. In mean field case, the phase boundary looks to be linear starting g_c = 1/2 for p = 0 and ending at g_c = 1 for p = 1 (Fig. 6), obeying $g_c = \frac{1}{2}(1+p)$. In this case, for p = 0, it is found the critical point to be g_c = 1/2, and this is similar to the fixed energy sandpiles. Along the phase boundary, the critical exponents are the same and they are matching with those of model A.

Lattice cases : The same dynamics is studied in 1-d and 2-d. For a linear chain in 1-d, here also $N = L = 10^4$ is taken and average over 10^3 initial condition. For 2-d, 1000 × 1000 square lattice with L = 1000 is considered and averaging over 10^3 different initial conditions.

(a) For 1-d, for the case p = 0, it is observed g_c = 0.89 ± 0.01, with β = 0.42 ± 0.01, z = 1.55 ± 0.02, $\nu_{||}$ = 1.90 ± 0.02 and α = 0.16 ± 0.01 (Fig. 7). The phase boundary in (g, p) is nonlinear

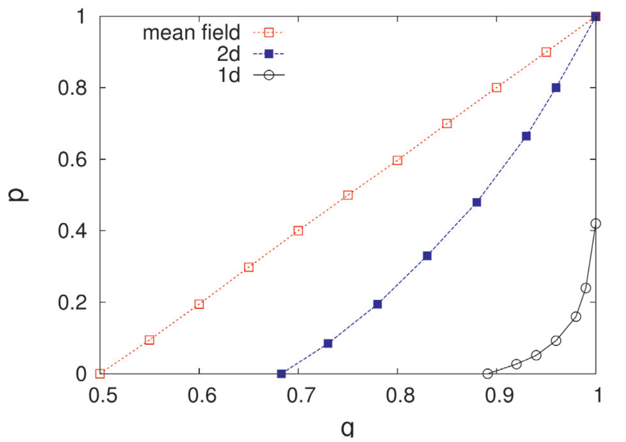

Fig. 6: Phase diagram for the generalized model in the (g, p) plane, showing the phase boundaries separating the active and absorbing phases in 1-d, 2-d and mean field cases. The active phases are on the right of the phase boundaries while the absorbing phases are on the left in the respective cases. The system sizes are $N = 10^5$ for mean field, 1000 × 1000 for 2-d, and 10^4 for 1-d. Taken from[8]. (Permission to use the figure from the paper is given by American Physical Society)

starting from $g_c = 0.89 \pm 0.01$ at $p = 0$ (Fig. 7) to $p = 0.43 \pm 0{:}03$ at $g = 1$ (Fig. 6). Therefore, we can independently define a model at unit density ($g = 1$) and determine the critical probability p_c for which the system goes from an active to an absorbing state.

(b) For 2-d, for the case $p = 0$, it is observed $g_c = 0.683 \pm 0.002$, with $\beta = 0.67 \pm 0.02$, z = 1.55 ± 0.02, $\nu_\parallel = 1.20 \pm 0.03$ and $\alpha = 0.42 \pm 0.01$. The phase boundary looks nonlinear, from $g_c = 0.683 \pm 0.002$ for $p = 0$ (Fig. 6) extending to $g_c = 1$ at $p = 1$.

KPR Strategies on City Size Distribution Modeling

The KPR problem can serve as a model for city growth and organization, where the cities correspond to restaurants and the city population to the customers, who choose to stay or migrate according to the fitness of the cities[15].

Model : In the usual KPR framework of N agents and R restaurants, we take here in the following $R = N$ for the sake of simplicity. We assume that each restaurant i has a characteristic *fitness* p_i drawn from a distribution $\Pi(p)$. The entire dynamics of the agents is defined by p. The concept of time is similar in the case of cities in the sense that people make choices at a certain time scale. Agents visiting a restaurant i on a particular evening t return on the next evening t +1 with probability p_i, or otherwise go to any other randomly chosen restaurant. We consider the dynamics of the agents to be simultaneous.

In terms of cities, we can re-cast the model as follows: every city has some *fitness* and initially people are randomly distributed among the cities. At any point of time, some people will be satisfied in a city and others will not be satisfied by its services. According to our model, the unsatisfied people will shift randomly to any other cities. The same dynamics happens for other cities too. Therefore at every time step (which can of the order of days or months) cities may lose some people and may also gain some people. We consider different types of *fitness* distribution and observe the population distribution for the cities.

The *fitness* parameter above is a proxy for a generic city index, which can be any intrinsic property such as the measure of wealth, economic power, competitiveness, resources, infrastructure etc. or a combination of many of these. It is important to note at this point that we are using the restaurant model (KPR) paradigm to model the distribution of sizes of urban agglomerations (cities), where migration between cities is modeled by the movement of agents across restaurants.

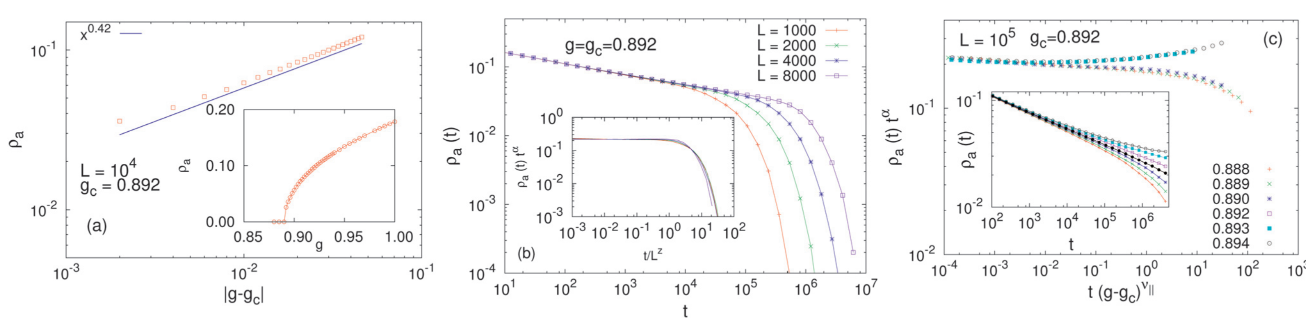

Fig. 7: Simulation results for the case $p = 0$ in 1-d, $g_c = 0.892 \pm 0.001$. (a) Variation of steady state density ρ_a of active sites versus $g - g_c$, fitting to $\beta = 0.42 \pm 0.01$. The inset shows the variation of ρ_a with density g. (b) relaxation to absorbing state near critical point for different system sizes L, the inset showing the scaling collapse giving estimates of critical exponents $\alpha = 0.15 \pm 0.01$ and z = 1.40 ± 0.02. (c) Scaling collapse of $\rho_a(t)$. The inset shows the variation of $\rho_a(t)$ versus time t for different densities g. The estimated critical exponent is $\nu_\parallel = 1.90 \pm 0.02$. The simulations are done for linear chains of size L (= N). Taken from[8]. (Permission to use the figure from the paper is given by American Physical Society).

Results : *Distribution of sizes :* Let us consider the case when p_i is uniformly distributed in [0, 1), i.e, $\Pi(p) = 1$. In practice, we use a natural cutoff for p as $1 - 1/N$. The probability density of the number of agents s at a particular restaurant $P(s)$ has a broad distribution, and in fact is a power law for most of its range, but has an exponential cutoff:

$$P(s) \sim s^{-\nu} \exp\left(-s/S\right) \quad (11)$$

where S is a constant which determines the scale of the cutoff. The exponential cutoff is an artifact of the upper cutoff in $\Pi(p)$. The power law exponent is $\nu = 2.00(1)$ as measured directly from the fit of the numerical simulation data (Fig. IV 8).

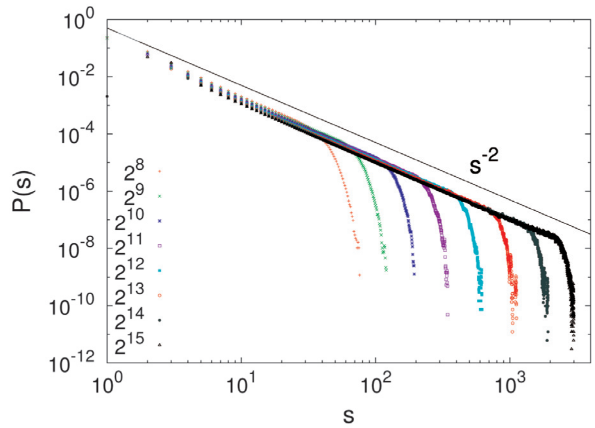

Fig. 8: The probability density $P(s)$ for fraction of restaurants with s agents. The data is shown for dierent system sizes $N = 2^8; 2^9; 2^{10}; 2^{11}; 2^{12}; 2^{13}; 2^{14}; 2^{15}$. The power law exponent is compared with s^{-2}. Taken from[3]. (Permission to use the gure from the paper is given by American Physical Society)

Let $a_i(t)$ denote the number of customers on the evening t in the restaurant i characterized by fitness p_i in the steady state. So, $\sum_i a_i(t) = N$. Let n' denote the average number of agents on any evening who are choosing restaurants randomly. Then, for a restaurant i, $a_i(t)p_i$ agents are returning to restaurant i on the next evening, and an additional n'/N agents on the average additionally come to that restaurant. This gives

$$\overline{a_i(t+1)} = \overline{a_i(t)}p_i + n'/N, \quad (12)$$

where $\overline{a_i}$ would now denote the average quantity. In the steady state, we have $\overline{a_i(t+1)} = \overline{a_i(t)} = \overline{a_i}$ and hence

$$\overline{a_i}(1-p_i) = \frac{n'}{N} \quad (13)$$

giving

$$\overline{a_i} = \frac{n'}{N}\frac{1}{1-p_i}, \quad (14)$$

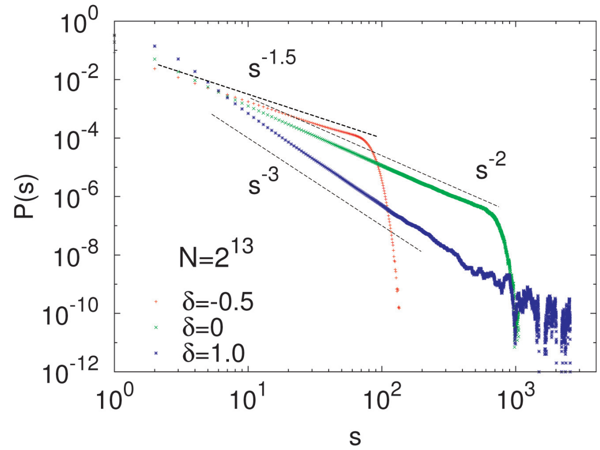

Fig. 9: The probability density $P(s)$ for fraction of restaurants with s agents, for different distributions $\Pi(p) = (1 + \delta)(1 - p)^{\delta}$, with $\delta = -0.5$, 0, 1.0. The power law exponents agree with $\nu = 2 + \delta$. The data are shown for $N = 2^{13}$. Taken from[15]. (Permission to use the figure from the paper is given by American Physical Society)

These calculations hold for large p_i (close to 1) which give large values of a_i close to $\overline{a_i}$. Thus, for all restaurants,

$$\sum_i \overline{a_i} = N = \frac{n'}{N}\sum_i \frac{1}{1-p_i}$$

$$\Rightarrow n' = \frac{N^2}{\sum_i \frac{1}{1-p_i}} \quad . \quad (15)$$

Now, let us consider a case of $\Pi(p) = 1$, where $p_i = 1 - i/N$ for $i = 1, 2, \ldots, N$. Thus,

$$n' = \frac{N}{\sum_i \frac{1}{i}} \approx \frac{N}{\ln(N+1)} \quad (16)$$

for large N. One can numerically compute $P(s)$ for this particular case and the computed value of the cutoff in $P(s)$ which comes from the largest value of p_i which is $p_1 = 1 - 1/N$, and it agrees nicely with the estimate, Eq. 16.

One can derive the form of the size distribution $P(s)$ easily. Since, R.H.S. of Eq. (13) is a constant (= C, say), $dp = da/a^2 = ds/s^2$, since a_i being the number of agents in restaurant i denotes nothing but the size s. An agent with a particular fitness p ends up in a restaurant of characteristic size s given by Eq. (13), so that one can relate $\Pi(p)dp = P(s)ds$. Thus,

$$P(s) = \Pi(p)\frac{dp}{ds} = \frac{\Pi\left(1-\frac{C}{s}\right)}{s^2}. \quad (17)$$

Thus, for an uniform distribution $\Pi(p) = 1$, $P(s) \sim s^{-2}$ for large s. It also follows that for $\Pi(p) = (1 + \delta)(1 - p)^{\delta}$, one should get

$$P(s) \sim s^{-(2+\delta)}, \text{ with } -1 < \delta < \infty. \tag{18}$$

Thus ν does not depend on any feature of (p) except on the nature of this function near $p = 1$, i.e., the value of δ, giving $\nu = 2 + \delta$. Fig. 9 compares the numerical simulation results for $\Pi(p) = (1 + \delta)(1 - p)^{\delta}$ and there is indeed an agreement with $\nu = 2 + \delta$ (for more details[15]).

KPR Strategies on Minority Game Problem

The minority game (MG) is a simple two choice game played between N players, where the players are required to make a choice between two options at each step. The players ending up in the minority, i,e, choice with fewer people, receive a fixed positive pay off. The number of agents is an odd number, so that at all steps one group belong to the minority. This is a variant of the El Farol bar problem. Like the El Farol bar problem, the agents are required to make independent and parallel decisions. The pay-off in the MG received by the minority population, does not depend on the number of people in the minority. Hence a 'socially effcient' system is the one where the populations are divided among the two choices almost equally, i.e. sufficiently close to $(N - 1)/2$. It is also important, however, that such a division is reached in a finite time (as opposed to, say, the 2^N order scale, which will sample eventually all configurations). A random choice at each step will get rid of the convergence time problem, which will be effectively 0, but the fluctuation in the population in each choices will scale as $\sqrt{(N)}$. This is a highly ineffcient strategy in terms of resource utilization, since a considerable number of agent could still be accommodated in the minority. Several adaptive strategies have been studied[45] in order to reduce this fluctuation and to make the system more effcient. However, the most complex strategies could not change the scaling of the fluctuation, but could only reduce the pre-factor in the scaling. Therefore, a significant resource misuse is likely in these strategies.

In a similar way, stochastic strategies were also used in the MG problem in[7]. Here the stochastic crowd avoiding strategy of the KPR problem was used for the MG. The fluctuation could be made arbitrarily small and this could be achieved in log log N time. In terms of resource utilization, this strategy performs best. However, there are some significant differences with the classical MG problem and this case. Particularly, in the classical MG the agents know only if they were in the majority or minority at each step. In this case, however, they are supplied with the information regarding the difference of population among the two choices as well.

In this section we will deal with question if this additional information regarding the excess population in the majority is indeed essential in reaching a low fluctuation state in the MG problem within a small time[26]. As a first step, the excess crowd size is guessed by the individual agents and are not supplied to them exactly. It can be shown that as long as the guess value is not too far from the actual value, the strategy still works. When the guess values are different among individual agents and they also vary in each time step, the minimum fluctuation is still reached as long as the average value of the guess is not far from the actual value. In fact, a continuous transition can be seen in the resource utilization depending on the accuracy of the guess of the crowd. In the end we will also discuss the more realistic case of incorporating some random traders as well.

Strategy of the Agents : In the case of the KPR strategy being applied to the MG problem in[7], the agents in the majority shift with the probability

$$p_+ = \frac{\Delta(t)}{M + \Delta(t) + 1}, \tag{19}$$

and the agents in the minority remain with their choice ($p_- = 0$). The total population ($N = 2M + 1$) is divided between the two choices as $M + \Delta(t) + 1$ and $M - \Delta(t)$ with $\Delta(t) = (|N_A(t) - N_B(t)| - 1)/2$, where $N_A(t)$ and $N_B(t)$ are the populations in the two choices at time t. Following this strategy, the agents can reach the zero fluctuation limit in log log N time[7]. Therefore, the resource utilization is maximum in that case. However, its distribution is highly asymmetric in the sense that after the dynamics stops in the $\Delta(t) = 0$ limit, the agents in the minority (majority) stay in their place for ever; hence, only one group always benefits. Other than that, in this strategy the knowledge of $\Delta(t)$ is made available to all the agents, which is not in general the case for the classical version of the MG.

In the following discussions, we will go through several variants of the above mentioned strategy. Primarily we will discuss the possibilities to avoid the freezing of the dynamics while keeping the fluctuation as low as possible. We then discuss if it is possible to achieve the small fluctuation states without knowing the magnitude of $\Delta(t)$.

Uniform Approximation in Guessing the Excess Crowd : We consider the case where the agents know the value of $\Delta(t)$. Our intention here is to find a strategy where the dynamics of the game does not stop and the fluctuation can be made as small as required.

To do that consider the following strategy: The shifting probability of the agents in majority is

$$p_+(t) = \frac{\Delta'(t)}{M + \Delta'(t) + 1}, \qquad (20)$$

[where $\Delta'(t) = G\Delta(t)$ and G is a constant] and as before the minority remains with their choice in the following step. A steady state is reached in this model where the fluctuation is arbitrarily small.

Steady-state behavior : To understand when such a steady state value is possible, note that when the transfer of the crowd from majority to minority is twice the difference of the crowd, the minority then will become the majority and will have the same amount of excess people as before. Quantitatively, if the initial populations were $M + \Delta$ and $M - \Delta$ roughly, and if 2Δ people are shifted from majority to minority, then the situation would continue to repeat itself, as the transfer probability solely depends on the excess crowd. Clearly, this is possible only when $G > 1$. Formally, if the steady-state value of $\Delta(t)$ is Δ_s, then the steady state condition requires

$$\left(M + \Delta_s + 1\right) \frac{G\Delta_s}{M + G\Delta_s + 1} = 2\Delta_s \, . \qquad (21)$$

Simplifying this, one gets either $\Delta_s = 0$ or

$$\Delta_s = \frac{G-2}{G}(M+1). \qquad (22)$$

For $G < 2(= G_c)$, $\Delta_s = 0$ would be the valid solution, since the above equation predicts a negative value for Δ_s, which indicated no steady-state saturation. Therefore, there is an active-absorbing type phase transition[46] by tuning the value of G. When $0 < G < 2$, the system reaches the minimum fluctuation state where $\Delta(t) = 0$ and the dynamics stops (the dynamics will differ qualitatively for $G < 1$ and $G > 1$). For $G > 2$, however, a residual fluctuation remains in the system, keeping it in the active state. This could be interpreted as, until the guessed value of the crowd is not too incorrect (twice as large), the agents can still find the minimum fluctuation state. However, when the guess becomes too far away from the actual value, a fluctuation remains in the system.

For this phase transition, it is now possible to define an order parameter for the problem as $O(t) = \Delta(t)/M$ and its saturation values behaves as $O_s = 0$ when $G < 2$ and when $M \gg 1$, for $G > 2$, with $O_s = (G - G_c)/G$ giving the order parameter exponent $\beta = 1$ for this continuous transition. In Fig. V.10 the results of the numerical simulation ($M = 10^5$) as well as the analytical expression for the order parameter are shown.

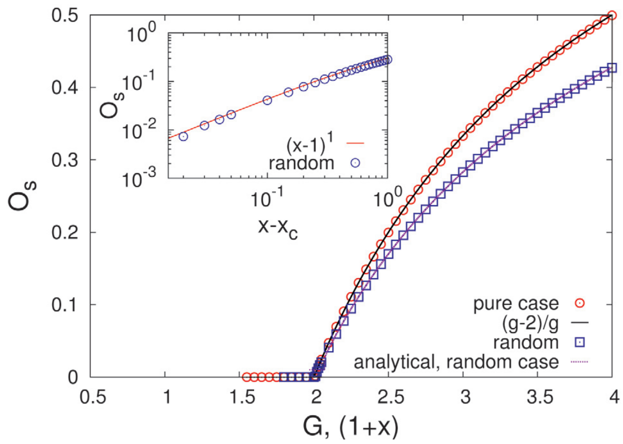

Fig. 10: Steady state values of the order parameter O_s are shown for different values of G and x. The solid lines show the analytical results for the pure and annealed disordered cases. Both match very well with the simulation points. Inset shows the log-log plot near the critical point for the disordered case, confirming $\beta = 1{:}00 \pm 0{:}01$. All simulation data are shown for $M = 10^5$. Taken from[9]. (Permission to use the gure from the paper is given by American Physical Society)

Dynamics of the system : When the excess crowd is known to each agents, it is possible to calculate the time dependent behavior of the order parameter both at and above the critical point. Let at an instant t, the populations in the two choices A and B are $N_A(t)$ and $N_B(t)$ respectively with $N_A(t) > N_B(t)$. Therefore, by definition

$$\Delta(t) = \frac{N_A(t) - N_B(t) - 1}{2}. \qquad (23)$$

The amount of the population to be shifted from A to B using this strategy would be

$$S(t) = \frac{G\Delta(t)}{M + G\Delta(t) + 1}(M + \Delta(t) + 1) \qquad (23)$$

$$\approx G\Delta(t)\,, \qquad (24)$$

when $\Delta(t)$ is small compared to M, i.e., when G is close to G_c or for large time if $G \le G_c$.

Clearly, $N_A(t+1) = N_A(t) - S(t)$ and $N_B(t+1) = N_B(t) + S(t)$, giving (where we assume population inversion)

$$\Delta(t+1) = \frac{N_B(t+1) - N_A(t+1) - 1}{2}$$

$$\approx G\Delta(t) - \Delta(t) - 1 . \quad (25)$$

Therefore, the time evolution of the order parameter can be written as

$$\frac{dO(t)}{dt} = -(2-G)O(t) - \frac{1}{M} . \quad (26)$$

Neglecting the last term and integrating,

$$O(t) = O(0)\exp\left[-(2-G)t\right]. \quad (27)$$

The above equation signifies an exponential decay of the order parameter for the subcritical region ($1 < G < 2$). It also gives a time scale $\tau \sim (G_c - G)^{-1}$ which diverges as the critical point is approached. These are also confirmed by the numerical simulations.

In Eq. (24), the leading order term was kept only. If, however, the next term is kept, the expression becomes

$$S(t) \approx G\Delta(t) - \frac{1}{M}\left(G^2\Delta^2(t) - G\Delta^2(t)\right). \quad (28)$$

The time evolution equation of the order parameter then reads

$$\frac{dO(t)}{dt} = -(2-G)O(t) - G(G-1)O^2(t) - \frac{1}{M} . \quad (29)$$

Now, for the dynamics exactly at the critical point, i.e., $G = 2$, the first term in the right-hand-side is zero. The last term can be neglected, giving the order parameter as

$$O(t) = \frac{O(0)}{2O(0)t + 1} . \quad (30)$$

In the long time limit $O(t) \sim t^{-1}$, giving $\delta = 1$.

Therefore we see that under this approximation, the usual mean field active-absorbing transition exponents are derived. These exponents are also obtained using the numerical simulations.

Effect of Random Traders : According to the strategies mentioned above, if the excess population is known to the agents (which in this case is in fact a measure of the stock's price) the fluctuations can have arbitrarily small value. However, in real markets, there are agents who follow certain strategies depending on the market signal (chartists) and also some agents who decide completely randomly (random traders). Here we discuss the effect of having random traders in the market, while the rest of the populations follow the strategies mentioned above.

Single Random Trader : When a single random trader is present, even when $\Delta(t) = 0$, that trader would choose randomly between the two choices for the following steps irrespective of whether he or she is in the minority or majority. This will create a changeover between majority and minority with an average time of two time steps. In this way, the asymmetry in the resource distribution can be avoided completely. However, that single agent will always be in the majority.

More than One Random Traders : As is discussed before, when all agents follow the strategy described by Eq. (19), after some initial dynamics, $\Delta(t) = 0$ implying that they do not change side at all. However, with a single random trader, in an average time period 2, as he or she selects alternatively between the two choices, the rest of the population is divided equally between the two choices and it is the random trader who creates the majority. However that trader is always a loser. This situation can be avoided when there is more than one random trader. In that case, it is not possible always to have all of them in the majority. There will be some configurations where some of the random traders are in the minority, making their time period of wining to be 2 (due to the symmetry of the two choices). The absorbing state (for $G < G_c$), therefore, never appears with random traders, though the fluctuation becomes non-zero for more than one random traders. However, if the number of random traders (= pN, where p is the fraction of random traders) is increased, the fluctuation in the excess population will also grow eventually to $N^{1/2}$(see Fig. V 11). Therefore, the most effective strategy could be the one in which (i) the fluctuation is minimum and (ii) the average time period of gain will be 2 for all the agents, irrespective of the fact whether they are random traders or chartists. These two are satisfied when the number of random traders is 2. Furthermore, if one incorporates the random traders in the strategy with partial knowledge of the excess crowd, a state of very small fluctuations can still be reached.

Conclusion and Discussion

In KPR problem each agent makes decision in each day t independently and is based on the information about the rank k of the restaurants and their previous day prospective customer crowd size given by the numbers $n_k(t - 1) \ldots n_k(0)$. Here we discussed the several stochastic strategies where each agent chooses the k-th ranked restaurant with probability $p_k(t)$ described by Eq. (1). The utilization fraction f_k of the k-th ranked restaurants on every evening is found and their average (over k) distributions $D(f)$ are shown in Fig. II.2 for some special cases.

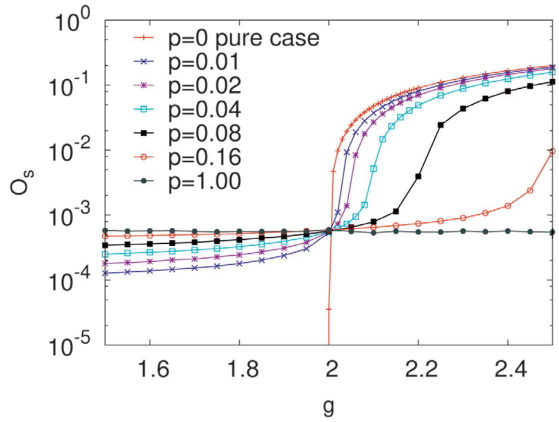

Fig. 11: The saturation values of O_s are plotted against G for different fractions p of the random traders. $M = 10^6$ for the simulations. Taken from[9]. (Permission to use the figure from the paper is given by American Physical Society)

Numerically we find their distributions to be Gaussian with the most probable utilization fraction $\bar{f} \simeq 0.63$, 0.58 and 0.46 for the cases with $\alpha = 0$, $T \to \infty$; $\alpha = 1$, $T \to \infty$; and $\alpha = 0$, $T \to 0$ respectively. For the stochastic crowd-avoiding strategy, we get the best utilization fraction $\bar{f} \simeq 0.8$. The analytical estimates for $\bar{f}$ for the stochastic crowd-avoiding strategy agree very well with the numerical observations. In all these cases, we assume $N' = N$, that is the number of choices for each of the N agents is the same as the number of agents or players. All the stochastic strategies, being parallel in computational mode, converge to solution at smaller time steps ($\sim \sqrt{N}$ or weakly dependent on N) while for deterministic strategies the convergence time is typically of order of N, which is useless in the truly macroscopic ($N \to \infty$) limits. However, deterministic strategies are useful for small N and rational agents can design appropriate punishment schemes for the deviators.

The KPR problem has a dictated solution that leads to one of the best possible solution to the problem, with each agent getting his dinner at the best ranked restaurant with a period of N days, and with best possible value of $\bar{f}$ (= 1) starting from the first evening itself. However the parallel decision strategies (employing evolving algorithms by the agents, and past informations, e.g., of n(t)), which are necessarily parallel among the agents and stochastic (as in democracy), are less efficient ($\bar{f} \ll 1$; the best one the stochastic crowd-avoiding strategy, giving $\bar{f} \simeq 0.8$ only). We note that most of the "smarter" strategies lead to much lower efficiency or less utilization. Next we have discussed how a KPR strategy gives rise to a phase transition from an active to a frozen phase, as the density varies. We have considered that g_N agents are competing among themselves to get the best service from N equally ranked restaurants. In the original KPR problem, where density $g = 1$ is far from its critical value g_c, the relaxation time τ, given by Eq. (10), never showed any system size $L = N^{1/d}$ dependence. These models are recast in terms of zero-range interacting particles in order to have analytical insights on the systems' behavior. For $g \leq 1$, absorbing configurations are present, and that can be reachable or not, depends on the basic dynamics. The existence of a critical point g_c is found above which the system is unable to reach frozen configurations. When the agents are moving if and only if they are competing with other agents (model B) with $p = 0$, they could not reach satisfactory configurations if the density is above $g_c = 1/2$. When the agents wait longer (higher p) speed up the convergence, increasing g_c and decreasing the time to reach steady configurations (faster-is-slower effect). The phase transition is numerically investigated in finite dimensions finding a good agreement with the exponents of stochastic fixed-energy sandpile.

We model city growth as a Kolkata Paise Restaurant Problem problem, specifically in the context of city size distributions. Zipf law for city size distribution can be thought to be a consequence of the variation in the quality of available services, which can be measured in terms of various amenities. We argue that this measure can be characterized by an intrinsic fitness. We make a correspondence from the population in cities to the number of customers in restaurants in the framework of the Kolkata Paise Restaurant problem, where each restaurant is characterized by an intrinsic fitness p similar to the difference in the quality of services in different cities. We showed the size distributions, and the exact value of the utilization fraction for the case when choices are made independent of fitness. Results for the case with uniform fitness are also reported. When fitness is uniformly distributed, it can give rise to a power (Zipf) law for the number of customers in each restaurant.

In the stochastic strategy minority game, a very efficient strategy is the one described by Eq. (19), where the agents very quickly (in log log N time) get divided almost equally (M and $M + 1$) between the two choices. This strategy guarantees that a single cheater, who does not follow this strategy, will always be a loser[7]. However, the dynamics in the system stops very quickly, making the resource distribution highly asymmetric (people in the majority stays there for all subsequent choices) thereby making this strategy socially unacceptable. We then discussed several modications in the above mentioned strategy to avoid this absorbing state. The presence of a single random trader (who picks between the two choices completely randomly) will avoid this absorbing state and the asymmetric distribution will also vanish. However, this will always make that particular trader a loser. But the presence of more than one random trader will avoid such a situation too, making the average time period of switching between majority and minority for all the traders (irrespective of whether they are chartists or random traders) to be 2. We also show that by varying a parameter, the agents can achieve any value of the fluctuation. This is an active-absorbing type phase transition for which the critical exponents can also be found analytically, which are well supported by numerical simulations.

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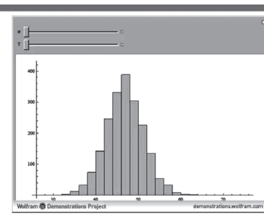
Appendix A

In this appendix, [A.1-A.14] present the excerpts from Wolfram Demonstrations, Wikipedia, articles and books, where Kolkata Paise Restaurant Problem has been discussed and developed by researches from different international institutions in social and natural sciences.

Wolfram Demonstrations Project



The KPR problem is a repeated game, played between a large number of agents having no interaction among themselves. In KPR, N prospective customers (the agents) choose from N restaurants each evening (time t) in parallel decision mode. Each restaurant has the same price for a meal but a different rank k (agreed upon by all the customers) and can serve only one customer. If more than one customer arrives at any restaurant on any evening, one of them is randomly chosen and is served and the rest do not get dinner that evening. Information regarding the customer distributions for earlier evenings is available to everyone. Each evening, each customer chooses a restaurant independent of the others. Each customer wants to go to the restaurant with the highest possible rank while avoiding a crowd so as to be able to get dinner there.

In Kolkata, there were very cheap and fixed rate "Paise Restaurants" (also called "Paise Hotels") that were popular among the daily laborers in the city. During lunch hours, the laborers used to walk (to save the transport costs) to one of these restaurants and would miss lunch if they got to a restaurant where there were too many customers. Walking down to the next restaurant would mean failing to report back to work on time! Paise is the smallest Indian coin and there were indeed some well-known rankings of these restaurants, as some of them would offer tastier items compared to the others.

The KPR problem seems to have a trivial solution: suppose that somebody assigns a restaurant to each person and rotates them on successive evenings—the fairest way; call that the dictated solution. This, however, is NOT a true solution of the KPR problem, where each agent decides on his own every evening, based on complete information about past events. In KPR, the customers try to evolve a learning strategy to eventually get to something like the dictated solution.

Let the strategy chosen by each customer in the KPR game be such that, at any time t, the probability $p_k(t)$ of arriving at the k^{th} ranked restaurant is given by

$$p_k(t) = \frac{1}{z}\left[k^{\alpha} \exp\left(-\frac{n_k(t-1)}{T}\right)\right] \qquad z = \sum_{k=1}^{N}\left[k^{\alpha} \exp\left(-\frac{n_k(t-1)}{T}\right)\right]$$

where $n_k(t)$ denotes the number of customers arriving at the k^{th} ranked restaurant on the t^{th} evening and , α,T denote two parameters.

Let f be the utilization fraction, which is the percentage of people getting food on any evening. In this Demonstration, the histogram gives the distribution f of against the fraction itself for different values of the parameters α and t in the expression for $p_k(t)$ for N=50, the number of agents and of restaurants, when the data is averaged over 2000 time steps or evenings. You can change the strategy for probabilistic choices of differently ranked restaurants by changing the values of the parameters α and T. For example, the random choice of restaurants by the customers (independent of rank) corresponds to α=0 and T-->infinity.

On a collective level, we look for the fraction f of customers getting dinner in any evening and also its distribution for various strategies of the game. The distribution f will be Gaussian with most probable utility fraction $f=f_0$. We get f_0--->0.63 for α=0, T--->infinity (pure random choice);f_0--->0.57 for α=1, T---->infinity (strict rank-dependent choice); f_0=0.46 and α=0 for T--->0, (avoiding-past-crowd choice).

FIG. A.1: The entry on Kolkata Paise Restaurant (KPR) Problem in Wolfram Demonstrations Project (website: http://demonstrations.wolfram.com/ Kolkata Paise Restaurant KPR Problem)

Kolkata Paise Restaurant Problem

Taken from WIKIPEDIA

Another variant of the El Farol Bar problem is the Kolkata Paise Restaurant Problem [5][6] [7] [8] [9] [10] [11] where the number of choices (n) as well as the number of players (N) are (macroscopically) large; typically n = N (while in the El Farol Bar Problem n = 2, N is macroscopically large). Both are repetitive and information regarding the history of choices made by different players for different restaurants are available to every one. For the choices for a single restaurant on any evening by more than one player, one is randomly selected from them and served food (payoff = 1) while others lose (payoff = 0). Hence, while each player gains a point (payoff) if her choice of the restaurant any evening is unique (not made by other players on the same evening), the resource utilization is maximised when each restaurant is chosen by at least one player.

In Kolkata there were very cheap and fixed rate “Paise Restaurants” that were popular among the daily labourers in the city. During lunch hours, the labourers used to walk (to save the transport costs) to one of these restaurants and would miss lunch if they got to a restaurant where there were too many customers. Walking down to the next restaurant would mean failing to report back to work on time! Paise is the smallest Indian coin and there were indeed some well-known rankings of these restaurants, as some of them would offer tastier items compared to the others. A more general example of such a problem would be when the society provides hospitals (and beds) in every locality but the local patients go to hospitals of better rank (commonly perceived) elsewhere, thereby competing with the local patients of those hospitals. Unavailability of treatment in time may be considered as lack of the service for those people and consequently as (social) wastage of service by those unattended hospitals.

The statistics of individual pay-offs for the adopted strategies and the statistics for the social utilization (ratio of the attended restaurants on any evening and N) of course depends on n/N and has an average value dependent of the strategies adopted by the players. It is seen that a stochastic strategy with probability of choosing the same restaurant(as the one chosen last evening) going inversely with the number of players who made the same choice last evening, and choosing others with equal probability, gives better result (giving utilization fraction about 0.79) than deterministic or simple random choice (noise trader) (with utilization fraction = 1 - exp[-1] ~ 0.63) strategies.

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FIG. A.2: Part of the entry on Kolkata Paise Restaurant (KPR) Problem in Wikipedia (as in December 2017; website: https://en.wikipedia.org/wiki/El Farol Bar problem#Kolkata Paise Restaurant Problem

Physica A 490 (2018) 745–753

Econophysics of a ranked demand and supply resource allocation problem

Avner Priel [a,b] , Boaz Tamir [c],
[a]Department of Physics, University of Alberta, Canada
[b]The Mina and Everard Goodman Faculty of Life-Sciences, Bar Ilan University, Israel
[c]Faculty of interdisciplinary studies, Bar Ilan University, Israel

Abstract: We present a two sided resource allocation problem, between demands and supplies, where both parties are ranked. For example, in Big Data problems where a set of different computational tasks is divided between a set of computers each with its own resources, or between employees and employers where both parties are ranked, the employees by their fitness and the employers by their package benefits. The allocation process can be viewed as a repeated game where in each iteration the strategy is decided by a meta-rule, based on the ranks of both parties and the results of the previous games. We show the existence of a phase transition between an absorbing state, where all demands are satisfied, and an active one where part of the demands are always left unsatisfied. The phase transition is governed by the ratio between supplies and demand. In a job allocation problem we find positive correlation between the rank of the workers and the rank of the factories; higher rank workers are usually allocated to higher ranked factories. These all suggest global emergent properties stemming from local variables. To demonstrate the global versus local relations, we introduce a local inertial force that increases the rank of employees in proportion to their persistence time in the same factory. We show that such a local force induces non trivial global effects, mostly to benefit the lower ranked employees.

Indeed, terms like non-equilibrium phase transition could be applied to social phenomena and are used in computational sociology [3,4]. Some generative examples are El Farole Bar problem, the Kolkata Paise Restaurant problem (KPR), and Minority Games [5–11]. The KPR problem is a resource allocation problem defined as follows: each day, a set of agents are looking for restaurants for their lunch...

The KPR problem is a repeated game in the language of game theory, having many versions. In one version, the restaurants are ranked [6], and the agents pick the restaurants according to their rank. Other versions suggest different meta-rules, for example a fixed probability to avoid a restaurant which was over crowded....

we study the properties of the convergence to a steady-state, as well as the phase transition observed w.r.t. the parameter g, the density of employees. For the general theory of phase transition see [13], and for some examples of phase transition in resource allocation problems see [14].

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FIG. A.3: Title, abstract and some excerpts are shown from the[44].

Quantum Theory: Reconsideration of Foundations 6
AIP Conf. Proc. 1508, 492-496 (2012)

Strategies in a Symmetric Quantum Kolkata Restaurant Problem

Puya Sharif and Hoshang Heydari
Physics Department, Stockholm University 10691 Stockholm, Sweden
E-mail:ps@puyasharif.net

Abstract: *The Quantum Kolkata restaurant problem is a multiple-choice version of the quantum minority game, where a set of n non-communicating players have to chose between one of m choices. A payoff is granted to the players that make a unique choice. It has previously been shown that shared entanglement and quantum operations can aid the players to coordinate their actions and acquire higher payoffs than is possible with classical randomization. In this paper the initial quantum state is expanded to a family of GHZ-type states and strategies are discussed in terms of possible final outcomes. It is shown that the players individually seek outcomes that maximize the collective good.*

QUANTUM KOLKATA RESTAURANT PROBLEM:-
This is a general form of a minority game [5, 6, 7], where n non-communicating agents (players), have to choose between m choices. A payoff of $ = 1 is payed out to the players that make unique choices. Players making the same choice receive $ = 0. The challenge is to come up with a strategy profile that maximizes the expected payoffs E i ($) of all players i, and has the property of being a Nash equilibrium. In the absence of communication, in a classical framework, there is nothing else to do, but to randomize.

References (partial):
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FIG. A.4: Title, abstract and some excerpts are shown from the[11].

Systems Engineering Essay Competition 2015

A Holistic Approach to Crowd Congestion Management in Singapore's Mass Rapid Transit System

Mithila Harish
Student Category
NUS Graduate School for Integrative Sciences and Engineering (NGS), NUS
Contact number: 90178530, Email address: mithila.harish@u.nus.edu
Published and used with permission by Temasek Defence Systems Institute and
Department of Industrial and Systems Engineering, Faculty of Engineering, NUS

Abstract: *As the population in Singapore continues to increase, it is extremely important that an increased focus is given to crowd control and management. In this regard, this paper gives an overview of traditional crowd modelling strategies and proposes some new techniques to potentially improve upon existing models, as well as examining the ease with which they can be implemented. It is hoped that these models will provide a better realization of crowd behavior and how these systems can help in building a smart city. The algorithm suggested is targeted at the Mass Rapid Transit System but it is hoped that it can be applied to crowded environments in general, with suitable modifications.*

6.1 **Sandpile model in General**
An interesting paper by Ghosh et al [23] describes an experiment modelled on the Kolkata Paise Restaurant Problem [KPR] [24] - which is in turn similar to the El Farol Bar Problem proposed by Arthur [25] to analyze whether the system moves to an absorbing from an active state, i.e. from multiple to single occupancy which illustrates Self Organized Criticality. The KPR Problem analyzes the case when gN agents wish to go to N restaurants wish the assumption that agents will not prefer to go to crowded restaurants. The result of this analysis is to observe how efficiently resources are utilized, given such constraints.

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FIG. A.5: Title, abstract and some excerpts are shown from the[47].

Late-Breaking Developments in the Field of Artificial Intelligence
Papers Presented at the Twenty-Seventh AAAI Conference on Artificial Intelligence

The Value of Ignorance about the Number of Players

Noga Alon, Reshef Meir and Moshe Tennenholtz
Microsoft Research
nogaa@tau.ac.il, reshef.meir@mail.huji.ac.il, moshet@microsoft.com

Minority games applied the El Farol Bar as a metaphor for various economic situations (Challet, Marsili, and Zhang 2001). Our variant above and the Kolkata Paise Restaurant problem (Chakrabarti 2007), where players choose from multiple restaurants, are useful analogies to real problems like congestion of roads and of service providers—typically modeled as congestion games (Rosenthal 1973).

Referece (partial):
Chakrabarti, B. K. 2007. Kolkata restaurant problem as a generalised el farol bar problem. In Econophysics of Markets and Business Networks. Springer. 239–246.

FIG. A.6: Title, abstract and some excerpts are shown from the[35].

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A Theatre Attendance Model

Michele Bisceglia, University of Bergamo, Bergamo, Italy

Abstract: *In this manuscript, the author proposes a model that constitutes a generalization of the El Farol Bar problem. In this model, in each period, each one of the n agents decides the arrival time at a theatre with free entry in which there are k (k<n) seats. Each individual wants to minimize the waiting time (before the beginning of the show) but prefers to assist to the show comfortably seated. The author introduces a utility function that takes into account these aspects, in which also agents' heterogeneity, in terms of different patience or comfort preferences, is considered. The author examines some possible approaches to this problem, and provides a new inductive reasoning modeling for a simplified version of this Theatre Attendance model, according to which each agent decides the arrival time at the theatre in a certain period by looking at the outcome of the previous round.*

Lastly, an interesting generalization of the Bar Attendance model is the Kolkata Paise Restaurant problem. In this model, in each period, N people has to choose between n restaurants. This model has been proposed by Chakrabarti (2007) and deeply investigated (in the case n = N) firstly by Chakrabarti et al. (2009).

References (partial):

Chakrabarti, B. K. (2007). Kolkata Restaurant problem as a generalised El Farol Bar problem. In B. K. Chakrabarti & A. Chatterjee (Eds.), Econophysics of Markets and Business Networks (pp. 239-246), Springer.

Chakrabarti, A., Chakrabarti, B. K., Chatterjee, A., & Mitra1, M. (2009). The Kolkata Paise Restaurant problem and resource utilization. Physica A: Statistical Mechanics and its Applications, 388(12), 2420-2426.

FIG. A.7: Title, abstract and some excerpts are shown from the[42].

SOCA
Springer-Verlag London 2015

An adaptive agent-based system for deregulated smart grids

Nicola Capodieci[1] · Giuliano Andrea Pagani [2] · Giacomo Cabri [1] · Marco Aiello [2]

[1] Dipartimento di Scienze Fisiche, Informatiche e Matematiche, Università di Modena e Reggio Emilia, Modena, Italy
[2] Johann Bernoulli Institute, University of Groningen, Groningen, The Netherlands

Abstract: *The power grid is undergoing a major change due mainly to the increased penetration of renewables and novel digital instruments in the hands of the end users that help to monitor and shift their loads. Such transformation is only possible with the coupling of an information and communication technology infrastructure to the existing power distribution grid. Given the scale and the interoperability requirements of such future system, service-oriented architectures (SOAs) are seen as one of the reference models and are considered already in many of the proposed standards for the smart grid (e.g., IEC-62325 and OASIS eMIX). Beyond the technical issues of what the service-oriented architectures of the smart grid will look like, there is a pressing question about what the added value for the end user could be. Clearly, the operators need to guarantee availability and security of supply, but why should the end users care? In this paper, we explore a scenario in which the end users can both consume and produce small quantities of energy and can trade these quantities in an open and deregulated market. For the trading, they delegate software agents that can fully interoperate and interact with one another thus taking advantage of the SOA. In particular, the agents have strategies, inspired from game theory, to take advantage of a service-oriented smart grid market and give profit to their delegators, while implicitly helping balancing the power grid. The proposal is implemented with simulated agents and interaction with existing Web services. To show the advantage of the agent with strategies, we compare our approach with the "base" agent one by means of simulations, highlighting the advantages of the proposal.*

The minority game which best fits the smart grid agent negotiation process is the El Farol Bar game. A recent variation of the El Farol Bar problem, called the Kolkata Paise restaurant problem [12], was proposed in order to study minority games characterized by a macroscopically large number of possible strategies for the participating agents: In our case, a future smart grid will have a large number of agents involved, but each agent will have a restricted amount of choices that are related to either trying to stipulate a contract with a prosumer or a Genco. Therefore, the traditional version of the El Farol Bar game better suits our reference model.

Referencs (Partial):
12. Chakrabarti AS, Chakrabarti BK, Chatterjee A, Mitra M (2009) The Kolkata Paise Restaurant problem and resource utilization. Phys A Stat Mech Appl 388(12):2420–2426

FIG. A.8: Title, abstract and some excerpts are shown from the[40].

Quantum Inf Process (2013) 12:577–586

Three-player quantum Kolkata restaurant problem under decoherence

M. Ramzan

Department of Physics, Quaid-i-Azam University,
Islamabad 45320, Pakistan

Abstract: *Effect of quantum decoherence in a three-player quantum Kolkata restaurant problem is investigated using tripartite entangled qutrit states. Different qutrit channels such as, amplitude damping, depolarizing, phase damping, trit-phase flip and phase flip channels are considered to analyze the behaviour of players payoffs. It is seen that Alice's payoff is heavily influenced by the amplitude damping channel as compared to the depolarizing and flipping channels. However, for higher level of decoherence, Alice's payoff is strongly affected by depolarizing noise. Whereas the behaviour of phase damping channel is symmetrical around 50 % decoherence. It is also seen that for maximum decoherence (p = 1), the influence of amplitude damping channel dominates over depolarizing and flipping channels. Whereas, phase damping channel has no effect on the Alice's payoff. Therefore, the problem becomes noiseless at maximum decoherence in case of phase damping channel. Furthermore, the Nash equilibrium of the problem does not change under decoherence.*

More recently, Sharif et al. [12] has proposed the quantum solution to a three-player Kolkata restaurant problem. The Kolkata Paise Restaurant (KPR) [13] is a repeated game similar to the Minority games, played between a large number of agents having no interaction among themselves.

References (partial):

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FIG. A.9: Title, abstract and some excerpts are shown from the[12].

Quantum Inf Process
DOI 10.1007/s11128-015-1144-4

Improving the payoffs of cooperators in three-player cooperative game using weak measurements

Xiang-Ping Liao [1] · Xiang-Zhuo Ding [2] ·Mao-Fa Fang [3]

[1] College of Science, Hunan University of Technology, Zhuzhou 412008, Hunan, China
[2] College of Mechanical and Vehicle Engineering, Hunan University, Changsha 410081, Hunan, China
[3] College of Physics and Information Science, Hunan Normal University, Changsha 410081, Hunan, China

Abstract: *In this paper, an efficient method is proposed to improve the payoffs of cooperators in cooperative three-player quantum game under the action of amplitude damping, bit flip and depolarizing channels using weak measurements. It is shown that the payoffs of cooperators can be enhanced to a great extent in the case of amplitude damping channel, and the payoff sudden death can be avoided in the case of bit flip and depolarizing channels. Moreover, the payoffs of cooperators tend to a constant by changing weak measurement strength in spite of sufficiently strong decoherence.*

It has been shown that being well aware of the dimensionality of the system, a player can achieve a mean payoff equal to almost 1. Sharif et al. [20] proposed the quantum solution to a three-player Kolkata restaurant problem. The Kolkata paise restaurant (KPR) [21] is a repeated game similar to the minority games, played between a large number of agents among whom there are no interactions.

References (partial):

20. Puya, S., Hoshang, H.: Quantum solution to a three player Kolkata restaurant problem using entangled qutrits. arXiv:1111.1962 [quant-ph], (2011)
21. Chakrabarti, A.S., Chakrabarti, B.K., Chatterjee, A., Mitra, M.: The Kolkata paise restaurant problem and resource utilization. Phys. A 388, 2420–2426 (2009)

FIG. A.10: Title, abstract and some excerpts are shown from the[36].

PHYSICAL REVIEW E 93, 042307 (2016)

Law of corresponding states for open collaborations

Marco Gherardi, [1,2,3,*] Federico Bassetti, [4] and Marco Cosentino Lagomarsino [1,5]

[1] Sorbonne Universités, UPMC Univ Paris 06, UMR 7238, Computational and Quantitative Biology, 15 rue de l'École de Médecine Paris, France
[2] Dipartimento di Fisica, Università degli Studi di Milano, via Celoria 16, 20133 Milano, Italy
[3] I.N.F.N. Milano
[4] Dipartimento di Matematica, Università di Pavia, Pavia, Italy
[5] CNRS, UMR 7238, Paris, France

Abstract: *We study the relation between number of contributors and product size in Wikipedia and GitHub. In contrast to traditional production, this is strongly probabilistic, but is characterized by two quantitative nonlinear laws: a power-law bound to product size for increasing number of contributors, and the universal collapse of rescaled distributions. A variant of the random-energy model shows that both laws are due to the heterogeneity of contributors, and displays an intriguing finite-size scaling property with no equivalent in standard systems. The analysis uncovers the right intensive densities, enabling the comparison of projects with different numbers of contributors on equal grounds. We use this property to expose the detrimental effects of conflicting interactions in Wikipedia.*

The marginal distribution of the number of contributors, i.e., the number N(n) of projects with a given number n of contributors, is well described by a power law, $N(n) = N_1 n^{-\beta}$, for both Wikipedia and GitHub (Fig. 2), where N_1 is the number of one-man projects. Such a wide distribution has been already noted, and may reflect preferential-attachment dynamics [34], or the variable intrinsic appeal of projects [35].

References (partial):

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FIG. A.11: Title, abstract and some excerpts are shown from the[41].

Proceeding: WWW '16 Proceedings of the 25th International Conference on World Wide Web, Pages 177-187

Mean Field Equilibria for Competitive Exploration in Resource Sharing Settings

Pu Yang, Krishnamurthy Iyer, Peter I. Frazier

School of Operations Research and Information Engineering, Cornell University Ithaca, NY, 14850

Abstract: *We consider a model of nomadic agents exploring and competing for time-varying location-specific resources, arising in crowdsourced transportation services, online communities, and in traditional location-based economic activity. This model comprises a group of agents, and a set of locations each endowed with a dynamic stochastic resource process. Each agent derives a periodic reward determined by the overall resource level at her location, and the number of other agents there. Each agent is strategic and free to move between locations, and at each time decides whether to stay at the same node or switch to another one. We study the equilibrium behavior of the agents as a function of dynamics of the stochastic resource process and the nature of the externality each agent imposes on others at the same location. In the asymptotic limit with the number of agents and locations increasing proportionally, we show that an equilibrium exists and has a threshold structure, where each agent decides to switch to a different location based only on their current location's resource level and the number of other agents at that location. This result provides insight into how system structure affects the agents' collective ability to explore their domain to find and effectively utilize resource-rich areas. It also allows assessing the impact of changing the reward structure through penalties or subsidies.*

Our model can be seen as an extension of the Kolkata Paise Restaurant Problem [7]. In this game, each agent chooses (simultaneously) a restaurant to visit, and earns a reward that depends both on the restaurant's rank, which is common across agents, and the number of other agents at that restaurant. This reward is inversely proportional to the number of agents visiting the restaurant.

The Kolkata Paise Restaurant Problem is itself a generalization of the El Farol bar problem [3, 8]. The Kolkata Paise Restaurant Problem is studied both in the one-shot and repeated settings, with results on the limiting behavior of myopic [7] and other strategies [12], although we are not aware of existing results on mean-field equilibria in this model. Our model is both more general, in that we allow general reward functions and allow location's resource to vary stochastically, and more specific, in that our locations are homogeneous. Our model also differs in that our agents' decisions are made asynchronously.

References (partial):
[7] A. S. Chakrabarti, B. K. Chakrabarti, A. Chatterjee, and M. Mitra. The kolkata paise restaurant problem and resource utilization. Physica A: Statistical Mechanics and its Applications, 388(12):2420–2426, 2009.
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FIG. A.12: Title, abstract and some excerpts are shown from the[39].

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A Congestion Game Framework for Emergency Department Overcrowding

Elizabeth Verheggen

Abstract: *Hospitals often manage capacity and resource constraints by different strategies implemented at their system access points. Emergency departments are key portals where timely access to care is a crucial quality of service and safety metric. Individuals vying for both urgent and nonurgent care seek these services analogous to the Tragedy of the Commons archetype. In a commons, a resource is used as if it belonged to everyone. Competition for a finite, decentralized, and shared resource risks its depletion as individuals optimize their own objectives while impacting the choices of others. As a result, overall system performance degrades. Ambulance diversion, extensive wait times and patient elopements, referred to as left without being seen, epitomize overutilization and inefficient load balancing. Traditionally, many hospitals were able to build their way out of congestion. Adding capacity, however, is at odds with concerted efforts to reign in the costs of health care. In an effort to break with this tradition, we exploited insights from game theory to inform the development of policies for more effective capacity management related to emergency department use, and to highlight related challenges. We examined emergency department overcrowding within the framework of a congestion game, the El Farol Bar Game and its variants, which illustrate the Tragedy of the Commons. In a series of agent-based simulations of the games, we found no statistically significant difference between the predictions of two games and our empirical observations during our most congested time periods of nonurgent patient attendance. Given the new competitive social context of real-time publicly advertised door-to-doctor wait times, and the implications that burgeoning information technologies have for the strategies invoked by providers and patients, it seems a bar might be the best metaphor to understand emergency department congestion.*

Similarly, the Kolkata Paise Restaurant Problem (KPR) is a variant of the EFBP that promises insights for ongoing work in the catchment area of the hospital ED modeled here [117, 118]. The KPR resource utilization problem swaps Santa Fe for Kolkata and has a similarly interesting storyline. In Kolkata where paise is the smallest Indian coin, these inexpensive and fixed rate restaurants, some ranked better than others, were frequented by laborers. Walking to a restaurant and finding it crowded meant missing lunch. Walking to the next restaurant meant reporting back late from lunch. In both the KPR and EFBP the number of players is macroscopically large, however, the number of choices is also macroscopically large in the KPR problem compared with only two in the EFBP.

References (partial):
[117] Chakraborti A, Challet D, Chatterjee A, Marsili M, Zhang YC, Chakrabarti BK (2013) Statistical mechanics of competitive resource allocation, pp 1–24. arXiv:http://arXiv.org/abs/1305.2121 [physics.soc-ph]
[118] Chakrabarti AS, Chakrabarti BK, Chatterjee M, Mitra M (2009) The Kolkata paise restaurant problem and resource utilization. Phys A 388:2420–2426

FIG. A.13: Title, abstract and some excerpts are shown from the[37].

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Chapter 8
Cooperation: Spontaneous Emergence of the Invisible Hand

However, agents in the real world often have to face competition in the limited resource, which distributes in different places in a biased manner. Examples for such phenomena include companies competing among markets of different sizes [121], drivers selecting different traffic routes [122], people betting on horse racing with the odds of winning a prize, and making decisions on which night to go to which bar [123].

Chapter 10
Partial Information: Equivalent to Complete Information

It is well known in statistical physics that there exist a lot of phase transition phenomena, e.g., the melting of ice (classified as first-order phase transition) and the superfluid transition (classified as second-order phase transition); both second-order and higher order phase transitions are also called continuous phase transitions [38]. In complex adaptive systems, phase transition phenomena can be seen as well [7, 87,146–148].

References (partial):
123. Chakrabarti, B.K.: Kolkata restaurant problem as a generalised el farol bar problem. In: Econophysics of Markets and Business Networks, pp. 239–246. Springer (2007).
146. Biswas, S., Ghosh, A., Chatterjee, A., Naskar, T., Chakrabarti, B.K.: Continuous transition of social efficiencies in the stochastic strategy minority game. Phys. Rev. E 85, 031104 (2012)

FIG. A.14: Title, abstract and some excerpts are shown from the[31].