THERMAL RADIATION LAWS, BOSE STATISTICS AND ITS IMMEDIATE IMPACT*

P. K. ROY**

The present article is written on the occasion of 50 years of Bose statistics. It seeks to convey to the widest possible circles of readers mainly an historical yet logically connected picture of the theoretical developments leading from Kirchhoff's law to Bose statistics.

One of the growing giants in the generally green garden of physics is statistical physics. It is just about a century and a quarter old. Even in this short span it has dug its own roots deep down into the soil of physics. Its remifications are numerous indeed.

The seed of the sapling was sown by the German physicist Rudolf Julius Emmanuel Clausius [1822-88]. The nascent sprout was nurtured by the English physicist James Clerk Maxwell [1831-79], the Austrian physicist Ludwig Eduard Boltzmann [1844-1906], the American physicist Josiah Willard Gibbs [1839-1903], to name a few. Like every other branch of science, statistical physics too developed in a characteristic way. In its struggle for survival it had to go through various ups and downs. But the signposts of its development are brilliant flashes of some indubitably bold steps. Statistical physics is indissolubly linked with the world of experience. Which is why it has succeeded so well, serious setbacks notwithstanding, in securing the necessary elan vital. .

In the last quarter of the nineteenth century the tender shoot of statistical physics had to withstand a severe strain. Here we are not referring to the seemingly telling blows delivered at the yet-underdeveloped branch by the Austrian physicist Josef Loschmidt and Ernst Zermelo, the young German mathematician. Nor we are referring to the systematic attack by the energeticists. Instead, we wholly refer to the so called problem of distribution of energy in the normal spectrum of radiating heat.

In this connection we shall review, not statistical physics, but the chequered history of the deduction of the laws of thermal, i.e. heat radiation. We shall trace how the problem originated and how it unfolded majestically, how it defied obstinately and for long all attempts at a solution acceptable to the canons of science, and how it finally submitted to the relentless efforts of great physicists enriched by the experience gathered over almost three quarters of a century.

As is well known, a solid body when heated radiates, i.e. emits light whose spectrum is continuous. The color of the visible light is initially a dull cherry red. Then, as temperature goes up, it becomes a brighter orange and finally changes to a blinding white light. The reason for this is not very far to seek. Radiations of different frequencies make up the visible light. At the red end the frequency is low, while it is high at the violet end. The quality of the light emitted is however determined solely by the temperature : at low temperatures the low frequency waves predominate and hence it looks red, whereas the shorter wavelengths (wavelength is inversely proportional to frequency!) appear at higher temperatures and mingle with red to give the white color. The emitted radiation rapidly becomes more and more intensive as temperature rises. Beyond the two extremes extend the so called infrared and ultraviolet regions which are not visible to naked eyes.

* Reprinted from Science and Culture, Vol. 40, No. 7, pp. 271-294.

** Department of Physics, Calcutta University, India, now deceased.

From Kirchhoff to Wien

In the theory of heat and thermal radiation two significant contributions were made in the second half of the sixth decade of the last century. In 1857 Clausius introduced the notion of mean kinetic energy and mean free path in his article picturesquely entitled ‘On the nature of motion we call heat’.

“ . . . .a relation worth-while investigation” – Kirchhoff

This was followed by a series of papers on heat and radiation published by Gustav Robert Kirchhoff [1824-87], a German physicist. In his early papers of 1859-60, Kirchhoff introduced the idea of hohlraum or cavity radiation : it is radiation in a cavity surrounded by impenetrable walls of equal temperature. Then a black body was also defined as a perfect absorber, i.e. as an ideal body which absorbs the whole of the radiant energy falling upon it. If one denotes by absorptive power the fraction which the body absorbs of radiant energy incident upon it, the absorptive power of a black body is evidently unity. With the help of the “law of exchanges” enunciated in 1792 by the Swiss physicist, Pierre Prevost [1751-18391, Kirchhoff deduced the existence of a certain temperature equilibrium for radiation. He then derived that the ratio of the emissive power of any body at a given temperature, to its absorptive power is the same for all bodies and is equal to the emissive power of a black body at that temperature (since the absorptive power is unity) ; here, by emissive power is meant the radiant energy emitted by the body per unit time. Thus the cavity radiation in no way depends upon the properties of the surrounding walls. On the basis of simple thermodynamic considerations was enunciated the principle “that the intensity of radiation from a black body is dependent only upon the wavelength of the radiation and the temperature of the radiating body, a relation worth-while investigation”. A black body or complete radiation is distinguished by the fact that all monochromatic rays contained in it have the same temperature.

The results obtained by Kirchhoff are known in the literature as the Kirchhoff theorem or law. This is one of the most general theorems in radiation theory. It should however be mentioned that the Scottish physicist, Balfour Stewart [1828-87], arrived at similar results a year and a half earlier (1858) on the basis of his exprimental investigations.

The theorem of Stewart and Kirchhoff implied the existence of a universal function, universal for it is independent of the nature of the substances ; it depended upon the wave length or frequency and temperature only. Thus, with the promise of an illuminating insight into the relation between energy and temperature, it led to the search for the spectral distribution of the black body radiation. It is no exaggeration to say that the history of the next forty years of researches in this field of heat and radiation is fundamentally the history of the search for the explicit representation of this universal function.

Two momentous contributions

For the next twenty five years negligible progress was made in the determination of this universal function. Although it was during this period that the English physicist, James Clerk Maxwell, had made two momentous contributions. First, Maxwell gave a great fillip to the kinetic theory of gases ; in 1860 he introduced the now familiar law of the distribution of velocities among the molecules of a gas.

In 1864 Maxwell gave his dynamical theory of the electromagnetic field. Here he provided the most quantitative, that is to say, mathematical formulation to the qualitative concepts of his countryman, Michael Faraday [1791-1867], about the whole complex of the extant electromagnetic phenomena. Thus, for the first time since Isaac Newton [1642-1727], field concepts appeared, hand in hand with the mechanical concepts, as fundamental and independent entities. The space-time laws of these fields were precise, yet simple and clear. Only the mechanistic interpretation of the discoverer and his subsequent followers made them appear clumsy, confusing and contradictory. Maxwell published his famous treatise on electricity and magnetism in 1873 ; therein he talked about radiation pressure also. Thus arose the idea of radiation exerting pressure on the surrounding walls of an enclosure.

In 1876, the Italian physicist, Adolf Bartoli [1851-96], inferred the existence of radiation pressure from a consideration of the validity of the second law of thermodynamics, when the radiant energy is transported from a cold body to a hot one by means of a moving mirror.

Bartoli’s paper stimulated Boltzmann to devise (1884) an ideal or thought experiment. The object of the gedanken experiment was to calculate the energy density of radiation in an isothermal enclosure, i.e., a cavity whose walls and contents are all at a common temperature. He assumed that radiation within a hollow extensible vessel played the part of the working substance of a heat engine. Assuming the radiation to exert pressure on the walls, he got two thermodynamic quantities, temperature and pressure, to describe the situation. From thermodynamic considerations now Boltzmann derived that the total intensity of the black body radiation is proportional to the fourth power of the absolute temperature. This relation had already been guessed, on empirical grounds, by his former teacher, Josef Stefan [1835-93], in 1879 ; which is why this law is known as the Stefan-Boltzmann law. In this way Boltzmann connected the empirical law with the basis of Maxwell's theory. However, in 1897, the above law came to be firmly accepted on the basis of the experiments of the German physicists, F. Paschen, Otto R. Lummer [1860-1925] and Ernst Pringsheim [1859-1917].

Boltzmann connected the empirical law with the basis of Maxwel1's theory

Thus one of the major problems of radiation theory was solved by the Stefan-Boltzmann law; though it touched only total black body radiation. But it was known that the radiation contained rays of different wavelengths and that their intensities altered with the temperature of the body. Therefore the task remained of investigating the manner in which intensity changed with wavelength and temperature.

There were a number of reasons why research along this line made no satisfactory progress for a long time.

A completely black body, according to Kirchhoff, would neither reflect light nor would allow light to pass through. Evidently such a body does not exist in nature. Even substances such as soot, platinum black, etc., reflect part of the light incident upon them. As late as 1895 this difficulty could not be resolved. In this year the German physicists, Wilhelm Wien [1864-1928] and Lummer, stated the principle according to which a completely black body could be realized in practice. They showed how the radiation, emerging from a small opening in the wall of an enclosure maintained at a uniform temperature, could behave in the same manner as the radiation emitted by a completely black body. For, any radiation entering the hole from outside would be partly absorbed and partly reflected by the walls. The reflected radiation had indeed a very small chance of emerging from the hole ; even then, such minute portions as did so would have undergone so many reflexions that its intensity would be negligible. In this sense a hollow body with a small opening in its wall could be regarded as a perfect absorber. Thus if a small hole be made in a hollow metal vessel heated to some given temperature, the radiation in the interior would possess exactly the spectral distribution of intensity characteristic of an ideal black body. The same could be concluded about the radiation emerging from the opening. The distribution of energy amongst various wavelengths could now be experimentally determined by dispersing the black body radiation emerging out. In thus suggesting a concrete realization of the ideal absorber Lummer and Wien based their principle of the arrangement on the views of Kirchhoff and Boltzmann ; it should however be noted that such a principle was partially applied by C. Christiansen in 1884.

.... decisive experimental study .... could not be delayed for long

On the other hand, the American astronomer and airplane pioneer, Samuel Pierpont Langley [1834-1906], invented in 1880 a remarkable instrument in the course of his investigations into the atmospheric absorption of solar radiation. Disgusted with the crudeness of a thermopile for measuring radiation, he devised a new apparatus of much higher sensitivity. This he named bolometer ($\beta O\lambda\eta$ means light). Subsequently, in 1886, Langley employed his spectrobolometer to study the distribution of radiation in the spectrum of a number of heat sources of temperature, both high and low.

Given the spectrobolometer and the actual construction of an is athermal enclosure as a source of black body radiation, decisive experimental study of the spectral distribution of energy could not be delayed for long. Indeed, very soon the German physicists started pouring data.

The results of these classical researches can be summarized as follows. An incandescent body, maintained at a definite temperature, radiates. The emitted radiation depends on its wavelengths in a characteristic way : while, for extremely short waves the intensity is vanishingly small, the same holds for the extremely long waves also. Thus is inferred the existence of a maximum value for some definite wavelength. On changing the temperature of the radiating body, this representation also changes. In particular, the position of the maximum shifted, with rise of temperature, in the direction of shorter waves, It was further found that the product of temperature and the wavelength corresponding to the maximum intensity was a constant depending upon the nature of the bodies.

The belatedness of the experimental developments notwithstanding, the next notable advance, after the Stefan-Boltzmann law, was already achieved by Wien.

In 1893, following Boltzmann, Wien also devised an ingenious thought experiment. He considered a sphere with white, i.e., perfectly reflecting walls. The walls had to be perfectly reflecting so that no exchange of heat took place between the radiation and the walls. Thus the thermal capacity of the walls did not enter into the calculation.

Radiation from a black body was allowed to enter the sphere ; this would, in the final analysis, behave as though the walls were themselves radiant and had the same temperature as the black body. Wien then imagined the black body to be sealed off from the white walls. As a result, he obtained an idealized case where radiation would reciprocate between the reflecting walls for good. The walls of the enclosure were now allowed to vary slowly so that the entire radiation occupied a smaller volume ; this is what is usually referred to as adiabatic contraction of a perfectly reflecting sphere ; for, in completing the process, no heat entered or left the system.

Now, due to this slow contraction, a number of things happen. First, just as in the case of compression of a gas, some work is expended in the process of contraction because of the pressure exerted by radiation striking the walls. If the total energy is to be conserved, this work must be converted into radiation. Thus the density of radiant energy, and hence temperature, increases. Secondly, the light ray undergoes a change in frequency or wavelength as it is reflected by a moving mirror, just as the pitch of a note changes when the source of sound moves relative to the observer. This is the so called Doppler effect (1842), named after the German physicist, Christian Doppler [1803-53].

Lastly, a contraction of the type considered does not destroy the black body property of the enclosed radiation. That is to say, the black body radiation remains black during any slow adiabatic change. For contraction, the energy density of radiation increases and so also the temperature. Thus the spectral composition of the radiation which has been changed as a result of the contraction of the enclosure is the same as it would be if the density of radiation were increased by just raising the temperature. For, if it were not so, the radiation from certain wavelengths would have an energy corresponding to a higher temperature than the radiation in certain other wavelengths. One could thus produce, by means of filters absorbing these wavelengths only, unequal radiation densitie in the two spaces enabIing one to generate work from - heat without compensation. And this would contradict the second law of thermodynamics.

Wien now had to complete the various calculations in connection with the problems stated above. The outcome of all these calculations was the discovery of the theorem named after him. The merit of the theorem lay in that the spectral distribution of the energy density was given by a universal function multiplied by the fifth power of the inverse of the wavelength. But this time, the universal function did not depend upon two variables, namely temperature and wavelength or frequency. Instead, it turned out to be a function of a single variable, namely the product of temperature and wavelength. Equivalently, the argument of the function could be expressed as the ratio of the frequency to the temperature. Thus the theorem of Wien imposed a welcome restriction on the nature of the radiation law : any radiation law must satisfy the theorem in as much as the latter, as a deduction from thermodynamics, must hold in all cases.

The Wien theorem, of course, included the Stefan-Boltzmann law : one had only to integrate the energy density over the whole spectrum.

This theorem also led Wien to an important testable conclusion : the product of temperature and the wavelength, for the corresponding maximum of intensity, is constant ; and, with increasing temperature, the maximum of radiation is displaced in the direction of the shorter wavelength. This is why it is known as the displacement law in view of its evident link with the experimental representation of the intensity of radiation, about which we have already spoken.

Nothing could be said about the constant, just referred to, without determining the explicit form of the universal function. And this explicit representation still eluded the people who were after it.

But it was not long before Wien realized that, without going beyond thermodynamics, he did not question the validity " ... little more than a conjecture"–Rayleigh of the electromagentic theory of light–further progress could no longer be made. The Wien theorem had exhausted the conclusions that could be drawn from pure thermodynamics with respect to radiation theory. In other words, thermodynamics, even when combined with the electromagnetic theory of light, was unable to throw any more light on the manner in which, at a given temperature, the intensity of radiation is distributed over individual frequencies. Wien was thus led to believe that, for further development, the mechanism of the radiation process must be examined in detail. Though it was clear from thermodynamical considerations that the form of the distribution law could in no way depend upon the mechanism assumed. In this, Wien fruitfully utilized the work of Vladimir A. Michelson who had sought (1887) to derive the radiation law of a black body, assuming Maxwell's law of velocity distribution for its molecules.

Wien now started with a gas of molecules, possessing a finite absorptive power for radiation of all wavelengths; it radiated under purely thermal excitation. The gas, contained in a body with perfectly reflecting walls, was separated from an exhausted cavity by a transparent window. As a result, it was accepted that the radiation in the exhausted vessel was the same as that from a black body at the same temperature as the walls of the vessel.

An essential assumption entered into theory : the gas molecules could only emit radiations of a certain wavelength as determined by velocity and the velocity distribution of the molecules obeyed Maxwell's law. The vibrations sent out by each molecule had thus wavelength and intensity depending solely on the velocity of the molecule. An expression for temperature was obtained from the mean square velocity. The Wien theorem then led its author to an explicit representation of the distribution law. This law is known as the Wien law of radiation. This law prescribed for the radiation intensity, or more correctly for the universal function of Wien, an exponential dependence upon its argument. It is thus that a complete form of the energy spectrum of radiation was found out for the first time by Wien (1896), except for two arbitrary constants.

But soon disturbing reports started coming from the German laboratories. In their early study the displacement law of Wien had been verified fairly accurately by Paschen (1897) and Lummer and Pringsheirn (1897). As reliable data about the long wave region started to pour in definitive deviations from the Wien law of radiation were also reported, first by Lummer and Pringsheim (1899/1900) and finally by their country men Heinrich Rubens [1865-1922] and F. Kurlbaum in 1900-1. Thus Wien's law gave a correct description of the spectrum only in the short wave region; it failed beyond doubt at the long wave end.

From Wien to Planck

The law which was just valid for the region where Wien's law failed did not have to wait for long to be found out. The deviation from the Wien law had hardly been reported, when, in June, 1900, the new law was discovered by the British physicist, Lord Rayleigh [1842-1919], very little known now-a-days by his real name John William Strutt. This law is known as the Rayleigh-Jeans law, due to the subsequent contributions by his countryman, James Hopwood Jeans [1877-1946]. When Rayleigh attempted to derive his law, he was unaware that the unrestricted validity of Wien's law, which he thought to be 'little more than a conjecture', was seriously challenged.

Rayleigh derived his formula for long waves. Later on Jeans (1909), who had already corrected it in 1905, was able to prove its validity for any wavelength.

Rayleigh approached the problem from an entirely different angle. He did not make use of any special mutual interaction between matter and radiation. His object of study was electromagnetic vibrations in an empty cubical box with reflecting walls. The walls reflected the waves back and forth. Finally, a system of standing waves were created : the waves adapted themselves to the distances between the walls. The number of modes of these transverse vibrations per unit volume and per unit frequency range was easily computed; for, almost twentyfive years earlier Rayleigh had solved a similar problem in acoustics where, however, the waves happen to be longitudinal. This number having been computed Rayleigh had now to determine the energy to be allocated to each mode of vibration. For this he had to take recourse to the socalled theorem of equipartition of energy in classical statistical mechanics. This theorem, of questionable universal validity, was established by Maxwell (1860/78) and Boltzmann (1868/71).

Now gas molecules, in a state of statistical equilibrium, execute random movements ; that is to say, a gas molecule does not prefer one motion to any other. Also the position and orientation of a moving body are determined by a set of mutually independent geometrical parameters. These parameters are referred to as the degrees of freedom of a system. As regards kinetic energy, no degree of freedom is preferred to another. Thus, for any system which obeys the laws of dynamics, the total energy is partitioned in such a way that every degree of freedom possesses, on the average, the same kinetic energy. This is given by the absolute temperature multiplied by half of a constant named after Boltzmann. Since each mode of vibration corresponded to a degree of freedom, the law of Rayleigh-Jeans immediately followed.

The Rayleigh-Jeans law states that the emission of radiation of a given wavelength varies directly as absolute temperature and inversely as the fourth power of the wavelength. As such it agrees with Wien's theorem. The experimental intensity distribution of Rubens and Kurlbaum (1900/01) is very well reproduced by this law in so far as the long wave components of the radiation are concerned; for short waves the failure is complete. In addition, this law assigns no maximum ; per contra, the spectral intensity distribution increases as the square of frequency, and the radiation energy tends to run entirely into waves of highest frequency, i.e. of - infinitesimal wavelength. For extremely short waves the spectral intensity distribution becomes infinite. The same is true for the total energy of radiation; this is because the number of degrees of freedom is infinite here. This phenomenon was later picturesquely described by the Russian-born physicist, Paul Ehrenfest as "ultraviolet catastrophe." The Rayleigh-Jeans law could not therefore represent the true law of black body radiation.

Rayleigh's law of radiation was, at first, considered to be just another law of limited validity. But soon it turned out to be otherwise. To appreciate the true significance of the law, a digression into the electromagnetic theory of Maxwell is necessary.

In the last decades of the last century two significant steps enriched the electromagnetic theory. The young German physicist, Heinrich Hertz [1857-941, demonstrated in 1888 the existence of Maxwell's electromagnetic waves. He established that the electromagnetic field could exist independent of the material source. Hertz was able to prove that the electromagnetic waves could be reflected. The waves could be refracted. They could also be diffracted and made to interfere with itself. They could be polarized. Even the velocity of the wave's was shown to be equal to that of light. In short, electromagnetic radiation possessed all the properties of light waves. In every case it behaved as light did, except that it was not visible. The material particles in Newton's mechanics could claim to be no more fundamental than the electromagnetic waves of Maxwell. The investigations of Hertz led him to posit, in the process of emission of light by a molecule, the mechanism of a vibrator characterized by the to and fro oscillation of electricity.

But the most important of the deveIopments since Maxwell was to start four years later. The architect was the Dutch physicist, Hendrik Antoon Lorentz [1853-1928]. By 1892 Lorentz started laying the foundations of his electron theory. In his theory a sharp distinction was drawn between matter and ether : the latter was endowed with characteristic immobility.

The most characteristic and valuable feature of the Lorentz theory was that it denuded matter of electromagnetic properties; simultaneously the electromagnetic theory was stripped of the last vestige of mechanical interpretation. The empty space was taken to be the seat of the field everywhere, not excluding the interior of matter. Matter could interact with the field only as carriers of unalterable charges. The electrons were no longer allowed to act directly on each other ; instead, the interaction could now take place only through the field surrounding them. Matter was thus endowed with the capability of both generating a field and reacting to ponderomotive forces. The material particles, of course, obeyed the Newtonian laws of motion.

In 1902 Lorentz applied his theory of electrons to the study of black body radiation. The electron theory was earlier successfully applied, especially by P. Drude [1858-1903], to explain the thermal, and electrical processes in metals. Now, with the help of the expression for electric conductivity as deduced by Drude, Lorentz first calculated the absorptive power of a thin metallic plate for long wave lengths. The thinness of the plate enabled Lorentz to ignore, in his examination of the emission, the absorption which the rays emitted by the obverse side of the plate underwent in traversing the layers lying infront of it. Further, the long wavelengths were necessary to ensure that a large number of impacts of free electrons occurred ; the impacts against the immobile metallic atoms caused the emission of radiation in accordance with the laws of electrodynamics.

Lorentz then computed the emissive power of the metal. By Kirchhoff's law, the ratio of this quantity to the calculated absorptive power must be the emissive power of a block body, irrespective of the special substances used. As a result of this study, Lorentz was led to the radiation law of Rayleigh. Years later, as already noted, Jeans also arrived at a similar result, when he generalized Rayleigh's method.

The upshot of all these classical investigations was a striking result : if the process or radiation obeyed the general laws of classical mechanics and the electromagnetic theory of Maxwell or of the electron theory of Lorentz, then one would be led inexorably to the radiation law of Rayleigh, in contradiction with the empirically established distribution curve.

Thus, at the turn of the century, there emerged, as a portent of the days ahead, a darkening cloud, as yet no larger than the size of a hand, which was eventually to threaten the most cherished concepts of classical physics. But it was born with a silver-lining : harmoniously hinged to the penetrating prognosis of the theoretical physicists, the ingenious and infallible experiments performed in Germany established incontrovertibly the validity, however limited it may be, of the laws of Wien and Rayleigh. It was this splendid synthesis which provided the golden key to a new world which came to revolutionize the twentieth century physics. Appropriately enough, it fell to the lot of the German physicist, Max Karl Ernst Ludwig Planck [1858-1947] of unlock the door of this wonderland.

From Planck to Bose

In the concluding years of the nineteenth century Planck had engaged himself in the study of irreversible processes involving radiation, with the penultimate object of obtaining Wien's law on a more rigorous basis. Since he "did not quite trust the electron hypothesis", the Lorentz theory had no attraction for him. Instead, the theory of hertzian oscillators he added "seemed to me to be particularly suitable device for the purpose" (1918).

He therefore considered a system of charged oscillators contained in an isothermal spherical enclosure with reflecting walls. The basic mechanism of irreversibility was supposed to be reflected in the scattering process, whereby incident plane waves were converted into outgoing spherical waves. Planck supposed radiation damping to be wholly responsible for this.

As early as 1897 Boltzmann, inspite of Planck's explicit support to Zermelo in his fight against statistical mechanics, had pointed out the fundamental flow in Planck's proposal. With a characteristic completeness Boltzmann established that the equations of Maxwell in no way forbade the inverse of the scattering processes in as such as they did not produce a monotonic, i.e., unidirectional approach to equilibrium. In this respect the electromagnetic equations behaved no differently from those of classical mechanics : in both cases one has to employ appropriate assumptions of statistical nature for the processes concerned.

Planck, however, ignored this advice only to find that his efforts took him no nearer to his objective. His oscillators obstinately refused to discriminate between the processes of emission and absorption in the way he wanted them to do. Planck's resonator reacted only to those rays which it also emitted ; it was not in the slightest bit sensitive to the adjacent spectral region.

With hopes belied, Planck was compelled to renounce his path in favor of what he called "thermodynamic approach". But an as-yet-undercurrent of irreconcilable the aversion to statistical mechanics and his familiarity with the second law of thermodynamics made him employ the thermodynamic concepts in the style of Clausius.

In the thermodynamic theory of a gas, a small number of parameters define its state, nowadays referred to as micro-state. The parameters may be quantities, such as temperature and pressure. It is Clausius, however, who introduced the notion of a quantity called entropy. In Greek this word means transformation or evolution. Like temperature and pressure, entropy of a substance is a definite function of its state. The change in the entropy of a substance depends wholly on its initial and final conditions and not on the particular reversible process by which it passes from one to the other. According to Clausius, entropy is always gained, never lost. That is to say, the entropy of the universe tends towards a maximum.

Planck now investigated the thermal equilibrium of a system of linear electromagnetic oscillators and radiation in the enclosure. A fixed natural frequency was attributed to each of these oscillators. On account of weak radiation damping, an oscillator could respond only to those waves in the enclosure whose frequencies lay in the immediate neighbourhood of its natural frequency.

The oscillator thus acts selectively just as a tuning fork of definite pitch commences to resonate only when its own ‘‘proper" tone (or very near it) is contained in the incident sound. In this process the oscillator, in accordance with classical electrodynamics, also emits energy by its own vibrations.

The mechanism of energy exchange is dualistic in nature. As a resonator the oscillator abstracts energy from, while as a radiator it emits energy to, the surrounding radiation. Thus a dynamical equilibrium is set up between just those waves of the radiation, which have the same frequency. In this state of equilibrium, the radiation of a given frequency acquires an intensity. This is equal, according to Kirchhoff's law, to the intensity of black body radiation at this temperature. On the basis of classical electrodynamics Planck calculated (1898) the relation between the average value of the energy of the oscillator and the intensity. This turned out to be very simple.

Planck was now led to compute the mean energy of an oscillator. It is here that Planck had to resort to his thermodynamic approach. In the paper just referred to above, the statistical assumption had already been introduced in the form of the so called hypothesis of "natural radiation" : the requirement of total incoherence was imposed on the harmonic partial constituents of a ray.

In this connection Planck .considered (1899) a function whose reciprocal was the second derivative of entropy with respect to energy. This function had a direct physical significance in connection with the desired irreversibility of the energy exchanges between the resonators and the radiation.

He calculated this function for the radiation law of Wien, as a first step towards the determination of the connection between the entropy and the energy. The theorem of Wien would then always directly give the dependence upon the wave length. The function was found to be proportional to energy. Having thus determined the entropy of an oscillator, Planck demonstrated that the total entropy of the system increased monotonically to an equilibrium value.

By the time Planck derived the radiation law of Wien the experimental situation had become somewhat fluid. Lummer and Pringsheim (1899) had earlier reported systematic deviations from Wien's law in the long wave region. But Paschen (1899) finally contradicted them ; he claimed universal validity for Wien's law. Lummer and Pringsheim, however, did not surrender on this question. They undertook a more reliable analysis and on a wider range. In the sequel, they established (1900) beyond a shred of doubt that in the infrared region the radiation law of Wien failed miserably : "It has been demostrated that black-body radiation is not represented, in the range of wavelengths measured by us, by the Wien-Planck spectral equation"

Once the adequacy of the Wien law of radiation was challenged a number of formulae were suggested in the spring and autumn of 1900. In this very year a series of refined and reliable experiments were concluded by Rubens and Kurlbaum (1900/01). Their unimpeachable data settled the question in favor of the analysis of Lummer and Pringsheim. What is more significant, they showed that, in the infrared region, the experimental distribution curve was well reproduced not by Wien's law but by the radiation law of Rayleigh.

Before communicating their findings to the German Physical Society, Rubens and Kurlbaum however conveyed them to Planck. Planck was now armed with the experimentally established behavior of the distribution curve in the two extreme regions. He wasted no time in setting upon the problem of rendering a mathematical content to the empirically determined form. Of course, it was no longer a difficult problem to tackle. For, he had just to repeat what he had done with Wien's law : to determine the same reciprocal of the second derivative, about which we have spoken earlier, for the Rayleigh distribution in the long-wave region. This time, however, the energy dependence of this reciprocal turned out to be otherwise: instead of linear dependence upon energy, as in the case of Wien's law, it was now quadratic dependence for the infrared region.

"a happily chosen interpolation formula"–Hanck

To get a quick hunch about the law which behaved in simple yet well-defined ways in two well-separated regions, there was "no better alternative" for Planck-to use his own words (1918)-than to try an interpolation formula. Planck now assumed the above-mentioned reciprocal to be determined by two additive terms. One of the terms was proportional to energy, thus corresponding to Wien's law of radiation, valid in the short wave region. The other term was proportional to the square of energy ; this corresponded to Rayleigh's law, valid in the infrared region. The relation between the energy and entropy of an oscillator was thus determined, truly speaking, empirically. With the aid of the displacement law theorem of the average energy of the oscillator was obtained as an explicit function of a single variable, namely the ratio of frequency to temperature. Thus, as a result of "retrogressive computation", Planck arrived at a new radiation law whose validity was yet to be checked over the whole exprimental range of the spectrum.

Planck presented his work to the German Physical Society on October 19, 1900. Rubens checked the new law against the latest experimental results. Complete agreement was reported. The result was published soon without much explanation.

Planck himself was hardly satisfied with the decidedly dubious derivation of the new formula. He therefore sought to give a "true physical character" to his formula which was so far "a happily chosen interpolation formula", to use Planck's own phrase (1918). To Planck the crux of the question was possibly still the relation between the entropy and energy of an oscillator ; there was no reason to question the relationship between the average energy density of radiation and the average energy of the oscillator. He therefore turned with expectation to those very trends of thought of Boltzmann which he had so far been dodging. And Boltzmann, one of the founders of statistical mechanics, did not fail him.

In contrast with the thermodynamical theory, an immensely large number of parameters define the state of a gas in the kinetic theory. These parameters are the coordinates and velocities of the gas molecules. The thermodynamical quantities, such as temperature and pressure, are the average or mean values of certain functions of the parameters of the kinetic theory. A given thermodynamical or macro-state may correspond to a very large number of kinetic or micro-states called 'complexions' by Gibbs ; for each of these micro-states the average values, referred to above, are the same. Boltzmann associates the probability of occurrence of a chosen thermodynamical state with the number of complexions by which it could be realized. Thus the number of complexions is a measure of the probability of a given macro-state. He further asserts that a given thermodynamical state is more likely to occur the greater the number of complexions by which it can be realized. Boltzmann thus introduced the principle that systems tend to pass towards states of greater probability of occurrence. And this he linked directly to the Clausius principle that a system tends to pass towards states of greater entropy.

For this purpose, Boltzmann (1877) expressed the entropy of a system in some given state as the logarithmic measure of the probability of its occurrence. That is to say, aside from a constant factor, entropy is equivalent to the logarithm of the probability of the state under consideration. The Boltzmann relation is the source of insight into processes which are irreversible in the thermodynamic sense. For, from the point of view of mechanics, all courses of events are reversible.

Planck now applied this principle of Boltzmann to a system consisting of a very large number of oscillators of the same frequency. The thermodynamic state is defined by the total energy of oscillation of all the resonators. The micro-state is obtained by computing the instantaneous energy of each individual oscillator.

Following Boltzmann Planck divided the total energy into a large but finite number of identical energy elements. This would enable him to express the number of micro-states belonging to a macro-state by means of a finite number. He had now to determine the number of ways in which the given energy could be shared among the oscillators. It is the logarithm of this number, which would furnish the entropy and eventually the temperature of the system.

The computational device, is prescribed in the works of Boltzmann. But, in order to arrive at the desired radiation formula, Planck had to depart here from the accepted path. In this, his task was greatly facilitated by his prior knowledge of the energy-entropy relation which was required by the empirically determined distribution law. The passage to the zero value of the magnitude of energy is now forbidden. A non-zero finite value is assigned to the energy elements : the energy is taken to be proportional to the frequency. The constant of proportionality determined by this relation later became commonly known as the Planck constant or the elementary quantum of action. Thus the average energy of a linear oscillator of given frequency must be an integral multiple of the product of Planck's constant and frequency ; the smallest amount of energy that can be emitted or absorbed by it is simply the above-mentioned product. This is known in the literature as the hypothesis of Planck.

The Planck formula, apart from a multiplicative factor of known constants, is given by the ratio of two expressions. The numerator is proportional to the cube of frequency, the constant of proportionality being the Planck constant. The denominator, when added to unity, yields an exponential dependence on the variable characterizing the universal function of Wien; it contains, besides the elementary quantum of action, the second constant of the theory, which is the same as the one associated with the entropy-probability relation. Thus Planck's law, like the radiation law of Wien, depends upon two unknown constants. For a long time the quantum of action could not be ascertained by appealing to experiment. The second constant turned out to be of great importance, for it immediately led to the knowledge of other universal constants. It is this constant which furnished exactly the correct size of the atom. This fact possibly gave Planck added conviction in the correctness of his ad-hoc hypothesis.

This derivation had a serious drawback which Planck, fortunately, did not recognise at first. Had he not done so, "he probably would not have", in the words of Einstein (1949), "made his great discovery, because the foundation would have been withdrawn from pure deductive reasoning."

The hypothesis of Planck evidently bears a strange feature. It is without precedence in the classical theory of waves, where the energy of a wave is related to its amplitude. In classical physics there is no necessary relation between the energy and the frequency ; here we can have waves of low energy and high frequency or of high energy and low frequency.

According to the hypothesis of Planck, the average energy of an oscillator depends on the size of the energy elements which it can take up or give out. The greater the energy element, the smaller the average value of the oscillator. Hence oscillators which strive for large energy get, on the average, the least of it. Thus the "ultraviolet catastrophe'' is avoided even when Rayleigh's law is one limit of the formula. The law gives the desired maximum and also the Stefan-Boltzmann law.

"an act of desperation"–Planck

About thirty years later Planck (1931) described his hypothesis as "an act of desperation", while earlier, in the Nobel Prize address, he said, "until after some weeks of the most strenuous work of my life light came into the darkness and an undreamed-of perspective opened up before me". In the light of the historical development of the theory of quanta the above statements may be considered as valid only within the confines of the radiation law. There is precious little to prove that the hypothesis of energy quantum led, in the immediate period following, to any stir among the contemporary physicists.

Neither can it be said that the "undreamed-of perspective", in any way, refers to the flowering of the idea in the period to follow. The "act of desperation" bore the character of a compelling calculational contrivance till a satisfactory elaboration could be established.

Planck was almost fanatically convinced that his law was the radiation law, securely based as it was on experience. Having unveiled a part of nature, which eluded the pleiades of prominent physicists for about four decades, he had no reason to sacrifice'this to the immediate needs of the extant theory. Per contra, any stop-gap theoretical explanation, whether it conflicted with the accepted concepts or not, was not considered as too big a price to pay till a coherent, consistent theory emerged. In fact, he (1931) considered his hypothesis 'a purely formal assumption, and I did not give it much thought except for this : that I had to obtain a positive result, under any circumstances and at whatever cost."

To be frank the door of the new world was unlocked with such conspicuous silence that it can be confidently asserted it did not even create, for a long time, a ripple in the placid pond of theoretical physics. Never before has an explorer, having earned the right of entry after years of arduous work, refused to open the door of a treasure house. It is one of the rare but remarkable achievements of the dead weight of tradition and conformism.

The subsequent contributions of Planck, relating to his hypothesis of energy quanta, are a clear pointer in this respect. In the first decade of this century his contribution in this field appears to amount to nothing. In his original work it was implied that exchange of energy between radiaton and matter took place only in quanta, whether the process was emission or absorption. Following the formidable but futile attempt of the dutch-born physicist, Peter Joseph Wilhelm Debye [1884-1966], to derive in 1910 the radiation law without the aid of any ponderable oscillator (and hence without the necessity of discontinuous processes), Planck came out with several versions of theory of energy quanta. Each of these versions is in turn marked by progressive and systematic, but unsuccessful, retreats into the realm of the field theory of classical physics (1911/14).

If the creator of the hypothesis of energy quanta failed to gauge the potentiality of his own creation, the unique experience of tasting the fruits forbidden by classical physics fell, appropriately enough, to the lot of the most stormy iconoclast among the twentieth century physicists, namely the German-born Albert Einstein [1879-1955].

"an undreamed-of perspective"–Planck

Soon after Planck's original version, Einstein, with an incisiveness characteristic of his thinking, pierced through the envelope of the mathematical artifice and revealed the contradictions masked by it. An essential assumption is implicit in the Planck theory : an individual oscillator can emit or absorb energy only in quanta. As a consequence, a prescription is laid down for the processes of energy exchange : a mechanical structure or radiation is now constrained to change its energy content only by quanta. It is this fact which contradicts the law of mechanics as well as of electrodynamics.

Einstein considered the conflict with classical mechanics to be more fundamental, although he was equally convinced that the laws of Maxwell were really unable to "account for the micro-structure of radiation and .could, therefore, have no general validity" (1949). He realized that both classical mechanics and the electromagnetic theory "are correct only at the limits, but are otherwise false" (1949).

As a result of this invigorating insight Einstein proposed his hypothesis of light quanta in 1905. It was published in the same volume of the German periodical which also contained his famous paper where he seriously and successfully challenged some time-honored concepts of classical physics. In this paper he showed how the radiation, within the limits of validity of Wien's law, behaves as if it were made up of an assembly of independent complexes of energy, each of magnitude determined by the energy-frequency relation. He put forward the hypothesis that parcels of radiation energy of a given frequency occur not only in emission and absorption, but that they have an independent existence in space. All radiation, therefore, consists of indivisible energy quanta. In the process of the propagation of the energy from the center of excitation, the energy is not distributed evenly in the form of ever-increasing volumes of space ; it remains concentrated in a finite number of energy-complexes. These quanta can be emitted and absorbed as whole individuals The energy of a light quantum of a given frequency is determined by the product of frequency and the Planck constant ; this energy-frequency relation came to be known as the Planck-Einstein relation.

But, for light quanta, it was really only half-way to freedom. For, in classical mechanics a particle is conceived of as a carrier of both energy and momentum. In 1908 Planck introduced the principle that flux of energy is momentum. From this principle, combined with the energy frequency relation, Einstein (1909) deduced that, in free space where the light quantum moves with the velocity of light, its momentum must be in the direction of propagation of the light. The magnitude of the momentum is given by the energy of the light quantum divided by its velocity. It is thus the Maxwell theory of electromagnetism was dented at its weakest spot. In this task Einstein was greatly assisted by his study of equilibrium between radiation and matter and the fluctuation of energy in black-body radiation. Radiation was at last conceptually endowed with a completely corpuscular structure.

The conceptions of Einstein regarding the micro-structure of light invited strong rebuttals from leading physicists like Lorentz and Planck. They rightly pointed out that a purely corpuscular theory of light failed to account for the phenomena of diffraction and coherence of wave-trains ; on the other hand, the wave theory offered a ready interpretation for these phenomena. Einstein neither denied these difficulties, nor did he kowtow before the critics. He insisted on the imperious need for introducing into light waves an essential element of discontinuity. His contentions he supported by some penetrating observations.

Soon after the publication of the so called second version of Planck's theory, the scale was heavily turned against the opponents of the hypothesis of light quanta by the intervention of the French mathematician, Jules Henri Poincaré [1854-1912], often described as "the last of the universalists".

Poincaré (1912) strove to look at the problem from an altogether new angle. He assumed the correctness of the radiation law of Planck. He then posed the question : what laws of motion must be postulated for the system in order to obtain this law ?

To this clearly posed question he answered with his characteristic clarity and completeness. According to Poincaré, no system can possibly lead to Planck's law except one in which the assumption of the quantum theory is satisfied. This is an inevitable conclusion of Planck's law.

Then, almost repeating Kirchhoff (1863) – ".... experiments which have only taught us concerning *more* and *less*, cannot strictly teach us concerning *equality*"- Poincare observed that a law found experimentally is no more than an approximation. As a natural corollary the problem arose whether one could imagine laws such that they differed from Planck's law within the errors of observation but, at the same time, would lead to a continuous system of dynamica1 laws. The answer to this is again provided by Poincaré in the definitive negative. It is shown that departures, small or even finite, from Planck's law in no way dispose of the necessity of discontinuity. It is conclusively and definitely demonstrated that the mere fact that the total radiation at a finite temperature is finite requires that the ultimate motion should be in some way discontinuous.

Meanwhile, Einstein submitted his hypothesis of light quanta to the tests of life. In the process, the hypothesis explained simply and naturally a number of phenomena which completely baffled the wave theory. In 1905 Einstein, in the paper already referred to, successfully provided an explanation of what is known as photoelectric effect ; this is an effect associated with a peculiarity observed in the processes of energy exchanges between matter and electromagnetic radiation. It is again Einstein (1907) who gave further evidence for the existence of light quanta through his theory of specific heat ; inter alia some very disquieting paradoxes were removed. The doctrine of quanta was thus successfully extended to the molecular theory of solid bodies. All these served to point out that the quantum effects were not a specific property of radiation only ; they were also a general feature of physical systems.

Emboldened by the successes Einstein strove hard to create a new foundation of physics compatible with the new knowledge thus far gained. But "all my attempts, however, to adapt the theoretical foundation of physics to this [new type of] knowledge failed completely. It was as if the ground had been pulled out from under one, with no firm foundation to be seen anywhere upon which one could have built" (1949). Very soon the Danish physicist, Niels Henrik David Bohr [1885-1962], was to succeed in creating the structure, which eluded Einstein, on the rickety and contradictory foundation then prevailing.

"– and appears to me as a miracle
even today"–Einstein (1949).

With a unique instinct and a strong and subtle logical intuition Bohr (1913) explored the possibility of the atomic picture of the New-zealand-born physicist, Ernest Rutherford [1871-1937]. According to this model, the atom, its etiological origin notwithstanding, is divisible, The positively charged part of the atom is confined into an exceedingly small space, the so called nucleus; the mass of the atom is assumed be principally concentrated in the nucleus. Electrons, which are extremely light compared to the nucleus, circle round the nucleus like planets orbiting round the sun. With the help of bold hypotheses Bohr succeeded in bringing the model under the purview of the quantum theory. As a result, essentially new and fundamental features which violently clashed with classical concepts were incorporated.

An electron could no longer revolve around the nucleus in all possible paths permitted by the laws of mechanics ; they were allowed only certain discrete orbits determined by the quantum theory.

The orbits thus allowed are known as the stationary states ; they are, in a sense, also stable states of motion. In order to endow the atom with size and stability, Bohr was bold enough to postulate–contrary to Maxwell's theory, contrary to the experiments of Hertz, contrary to everything canonized by the classical theory–that no radiation can take place as long as the electron remains in any stationary orbit. Once the "classical" radiation of the atom is done away with, a new hypothesis is called into play to account for the observed emission. According to this hypothesis, an electron can emit (absorb) energy by jumping from a stationary state of higher (lower) energy to one of lower (higher) energy; the frequency of the emitted or absorbed energy is given by the difference in the energy values, divided by the elementary quantum of action. This is referred to as the Bohr frequency condition. Evidently, in the type of transitions referred to, only a single light quantum is involved.

With an uncanny insight, Bohr could anticipate that the new concepts would eventually lead to a new point of view. Taking the hint of Planck (1906)—"the classical theory can simply be characterized by the fact that the quantum of action becomes infinitesimally small"-he wanted to keep the links with the classical theory as clearly and closely as possible. He therefore expressed the essence of his approach in the so called principle of correspondence.

" . . .to make what is said on
what said before. . ."-Galileo

According to the early formulation of Bohr, in the transition between two stationary orbits, the frequency of the light emitted approaches more and more the value classical theory would associate with the resulting common period. The mechanism of the emission in the quantum theory, is without doubt, totally different from the classical one. Nevertheless, the results obtained in the quantum theory approximate more and more to the classical ones, the closer the conditions are approached in which the latter are known to be valid. The Bohr theory, in this sense, corresponds to the results of the classical theory. The correspondence principle eventually turned out to be a remarkably versatile and productive conceptual device for the rapid progress of the quantum theory, in as much as it successfully provided deep insight into the dialectics of the contradictions between the classical concepts and the emerging quantum ideas.

The work of Bohr imparted fresh impetus to the rapid development of the quantum theory. The riddle of atomic stability was solved. The spectral series ceased to be mysterious. The main features of the periodic system no longer defied explanation. With Bohr the quantum theory transcended the bounds of such particularisms as the linear oscillators of Planck.

Under the circumstances the suggestion naturally arose to derive afresh the radiation law by appealing, in the place of the Planck oscillator, to the Bohr atom as the elementary literature "as induced or stimulated" absorbing and emitting structure. It is again Einstein who solved the problem in 1916-17. In a sense the method of Einstein parallels the one employed by Wien to derive his law of radiation.

In discussing anew this problem of statistical equilibrium, Einstein strove to determine the rates of all competing elementary processes which led to the equilibrium in question. This is a rather difficult, though familiar, approach in statistical physics ; for evidently, what is required, in addition to the counting of the equally probable cases, is a real knowledge of the mechanism involved.

Einstein now considers transitions between two given stationary states of an atom. He first introduces a simple hypothesis for the process of spontaneous emission, in which the atom passes, without the influence of external agencies, from a higher state to a lower one. The frequency of all such transitions is assumed to follow the same statistical law as that which governs the disintegration of radioactive bodies.

A different law however regulates the processes called into existence by the influence of the external radiation. Under the impact of the external radiation, an atom could pass to a higher state by absorption of energy.

If only these two processes were allowed to balance one another, the theory would not lead to the Planck formula. So Einstein was compelled to introduce, for the first time in the quantum theory, a third process, namely an influence of the radiation on the emission process. In this process the atom is induced, as a result of the phase-relation between the field of the external radiation and the atom, to lose energy through the action of the impinging radiation. As a result the atom passes to a state of lower ,energy. This process is therefore referred to in the emission.

From the consideration of the energy-equilibrium between incoming and outgoing radiation at a given temperature, Einstein deduced the Planck formula without the necessity of evaluating the various transition probabilities. In the derivation he had to compare his results with the Rayleigh and Wien formulae. Inter alia, it was shown that so far as transitions stimulated by radiation are concerned, there is a symmetrical probability. That is to say, the stimulated emission probability is equal to the probability of absorption. The spontaneous emission probability is simply related to one of the remaining. two probabilities. These results are fundamental in the theory . of the exchanges between matter and radiation. They have been extensively used in the development of quantum theory.

. . . . in an amazingly simple and general way" – Einstein

During the first two decades of the century numerous attempts have been made to found the correct radiation law on a logically consistent basis, i.e. either on the corpuscular structure or on the basis of the wave theory. The otherwise surprisingly simple and elegant manner in which the radiation law is derived by Einstein from a minimum of hypotheses of a general character, is marred by the intrusion of the Rayleigh and Wien laws in the final stage of the calculation.

The undulatory and corpuscular descriptions of light thus remained inextricably intertwined in any otherwise acceptable derivation of the Planck law. The task of unravelling this tangled skin was accomplished with amazing simplicity by the Indian physicist, Satyendranath Bose [1894-1924].

Bose brushed aside whatever was wrong in the ideas of his predecessors. He seized upon what was right ; he enriched them precisely with that which was lacking in order to make them fruitful. In the process he discovered the statistics for light quanta or photons, whose real significance could be realized only after the foundation of quantum mechanics was laid.

" . . .Sounds the death-knell of the wave theory"– Sommerfeld

Bose took the corpuscular theory of light quanta seriously ; he totally ignored the wave mechanical concept. And he had added reason for doing so. Hardly a year earlier (1923), one of the great discoveries was made by the American physicist, Arthur Holly Compton [1892-1962]. Compton investigated scattering of x-rays. It was so far known that the wavelength of the radiation did not change in such scattering. Compton however observed, in addition, a new type of scattering in which the wave length of the scattered rays increased as compared with the initial wave length. This phenomenon came to be known as the Compton scattering or the Compton effect.

The theory of this effect was given by Compton (1923) and almost simultaneously by Debye (1923) ; it is based on the Einstein picture of photons. In this theory one interprets the photon scattering on a free electron as the collision of elastic spheres : that is to say, in the elementary process both energy and momenta change.

The new discovery put an end to the controversy regarding the reality of photons as contrasted with the spherical waves of light, about which we have spoken earlier. For in the explanation offered by Compton, the incident and scattered quanta were supposed to have definite directions of propagation. As a result one of the most convincing proof of the existence, of light quanta was demonstrated. According to Compton (1924) : "In a recent letter to me Sommerfeld has expressed the opinion that this discovery of the change of wave length of radiation, due to scattering sounds the death-knell of the wave theory."

Bose .therefore sought to derive the radiation law solely on basis of the corpuscular picture. At the very outset he combined the Einstein relation for a light quantum with the assumption of Planck (1911/13) that the elementary region of phase-space, the six dimentional space of coordinates and momenta of a photon, is given by the cube of the elementary quantum of action. He thus obtained the number of cells per unit volume of space. In the process he obtained the famous factor that occurred in Planck's derivation of the relation between the average energy of an oscillator and the average radiation energy ; it should be recalled that this relation was derived on the basis of the electromagnetic theory. Having thus eliminated the need of the Maxwell theory, he proceeded to treat the photons, like the particles of a gas, with the method of statistical mechanics.

The phase space is now partitioned into cells of elementary volume. Taking the polarization into consideration Bose obtained the total number of cells. He then calculated the number of ways in which the number of photons in a given frequency range were allocated among the cells belonging to this range. For this purpose the cells were labeled according to the number of photons belonging to them.

In order to obtain the correct thermodynamic probability, an essentially new assumption entered the theory. If one considers a quantum state, then all values for the number of particles in that state are taken to be equally probable ; the probability of any distribution specified by the number of cells belonging to a given frequency range and containing a given number of photons is measured by the number of different ways in which it can be realized.

This is the reason why Bose did not have to bother himself with distributing the individual photons over a set of states ; instead he counted the number of states which contain a given number of particles. When Bose supplemented this combinatory process with the physical condition obtaining, he was led, on completing standard calculations, to the law he was after.

It is evident that, in seeking to solve the problem as a statistical one, Bose departed from the conventional method of treating a gas of material particles. Subsequent work showed that, had he not done so, he would not have arrived at the desired law.

The rather novel combinatorial process emphasizes that an essentially new feature in the procedure followed by Bose is the suggestion that the particles, i.e. photons, are indistinguishable.

The contribution of Bose is a fitting finale to an arduous and tortuous trek undertaken by some pioneers in physics. Forthright and precise, it is conspicuous by the generality and economy of the method expected of a crowning achievement. It was published in the German periodical Zeitschrift fur Physik in 1924.

The paper was motivated in the following words :

"Since the publication (of Planck's law) in the year 1901, many methods of derivation of this law have been proposed. It is acknowledged that the fundamental assumptions of the quantum theory are incompatible with the laws of classical electrodynamics . . . . A remarkably elegant derivation was furnished by Einstein. . . .

However, he has to take the help of Wien's displacement law and Bohr's correspondence principle. Wien's law is based on the classical theory and the correspondence principle assumes that the quantum theory is in tune with classical theory in certain critical cases.

In all cases it appears to me that the derivations have not been sufficiently justified logically. As opposed to these the light quantum hypothesis combined with the statistical mechnics (as Planck adapted it to the requirement of quantum theory) appears to me to be sufficient for the deduction of the law independent of the classical theory . . . .".

With these observations Bose buckled down to the task set by himself.

"An important step forward . ." Einstein

But the evident significance of the paper lay in the fact that it was translated by Einstein. At the end was appended Einstein's own opinion about the merit of the work in the following lines : "Bose's derivation of Planck's formula signifies, in my opinion, an important step forward. The method used here gives also the quantum theory of an ideal gas, as I shall show elsewhere".

This is how Einstein responded to the request which Bose made in his letter dated June 4, 1924 : "I am anxious to know what you think of it.. . . I do not know sufficient German to translate the paper. If you think the paper worth publication, I shall be grateful if you arrange for its publication in Zeitschrift fur Physik" (1924a).

Einstein was, to be sure, greatly inspired by Bose's work. It is certainly not the derivation of the famous formula which drew his attention. He was attracted by the novelty of the approach. So inspired was he by the new method that very soon he succeeded in demonstrating its effectiveness. He sought to extend Bose's idea to a gas of monatomic particles. It may be recalled here that photons differ in some respects from the gas particles of Einstein ; for example, the energy of a photon is simply equal to the momentum multiplied by its velocity, whereas a different relation holds for the gas particles with finite rest mass. In addition, in Bose's problem the total energy is fixed, though the number of photons is indefinite ; in the case of Einstein the total number of particles is definite. The general plan of the investigation, however, is not significantly affected by these differences. Thus Einstein (1924/25) successfully constructed the so called quantum theory of monatomic gases. This is the reason why people used to refer to Bose's method as "Bose-Einstein Statistics." Presently it is referred to simply as "Bose Statistics".

The application of Bose's method to the study of a monatomic ideal gas led to some significant differences in the gas laws. The method must have impressed him most when Einstein attacked the problem of mean-square energy or density fluctuation of such a gas. Almost fifteen years earlier Einstein (1909) had applied similar considerations to an assembly of light quanta in an enclosure which was at a given temperature. As a result, it was already known that this energy fluctuation of the electromagnetic radiation consisted of two characteristic parts. The first part owed its existence to the corpuscular nature of the light quinta. The other one was attributed, in the light of Lorentz's work in 1916, to the interference of wave-trains supposed to be crossing the enclosure in every direction. The part exhibiting the corpuscular character wonld not appear if the non-quantum electromagnetic theory of light had been used. Here it should be recalled that the energy fluctuations of a gas treated according to the conventional statistical methods were analogous to the first part, interference part being absent. This is how the similarity of photons and material particles were emphasized.

When, armed with the new statistical method, Einstein attempted his old game in the case of material particles, he must have been extemely happy to find fhe result. It was a welcome surprise to Einstein that both types of terms appeared for the material particles as well. Under ordinary circumstances the appearance of a term, which in the case of radiation was due to the interference of waves, would have been a sufficient cause for rejecting the method. But placed as he was and great as he was, Einstein discovered the strength of the method here where other physicists, would, most probably have found its weakness. For, the great physicist had already been introduced, thanks to the French physicist, Paul Langevin, to the revolutionary ideas proposed (1923/24) by Prince Louis-Victor de Broglie, [1892-].

"more than a formal analogy" -Einstein

Thus, as a consequence of the discovery of the new statistics, Einstein could derive a result which went a long way in influencing the emergence of wave mechanics. In 1949 the noted Austrian-born physicist, Wolfgang Pauli [1901-58] recalled that it was Einstein who, in the autumn of 1924, proposed to search for interference and diffraction phenomena with molecular beams. The papers of Einstein together with those of de Broglie, stimulated the Austrian-born Schroedinger [1887-1961] to develop his wave mechanics. Thus Bose's work was largely responsible for involving Einstein in the foundation of wave mechanics. The intervention of Einstein hastened the emergence of wave mechanics, just as an accoucheur assists in alleviating the anxiety and agony associated with pangs of birth.

In his diffidence about the appropriateness of his approach to the problem and without the invigorating influence of a live, not to speak of competitive, science around, the young discoverer of the photon statistics could not pay any attention to the physical significance of the new combinatory process proposed by him. As such, neither in his work nor in the letters (1924) to Einstein could he make any explicit reference to the new feature of his method. This feature first came to be discussed at length in the papers of Einstein.

One of the peculiar consequences of the new statistical method, as noted first by Einstein and subsequently by Planck (1925), is that the most probable distribution of a large number of "identical"–in to-day's parlance, indistinguishable-particles over a large number of identical phase cells is not uniform. Expressed in a different language, two members of an ideal gas–that is to say, a gas whose constituents are strong enough for exchange of energy but weak enough for all other purposes-of Bose-Einstein particles have a tendency to clump in the same phase cell more often than is in accord with the property of random action or statistical independence with which a classical particle is endowed. In classical statistical mechanics if the occurrence of an event–that is to say, a result of an observation-depends in no way upon the occurrence or otherwise of another event, then the two events are statistically independent. According to Einstein, a "completely mysterious" "mutual interaction" prevents identical particles from behaving as statistically independent entities : some as yet unknown interaction interferes with the unrestricted freedom of the particles to enter phase cells at random, a freedom which is a stringent requirement for classical statistics. The probability of a particle is influenced, in this case, by the number of particles already occupying the cell. It is this effect which leads to a non-random multiplicity of the Bose-Einstein particles in individual phase cells. This is how the clustering effect, characteristic of the new statistics, is produced.

It appears from Einstein's paper (1925a) that Ehrenfest and some colleagues criticized the new statistical method on this score : they pointed out that the direct use of the new formula for counting distribution treated particles as non-independent entities and thus devoid of random action. Einstein conceded that the criticism was "absolutely correct".

But he went on to emphasize that if the Bose counting was not employed then one would be led not to Planck's law but to the radiation law of Wien.

This conclusion, however, could not come as a real surprise to the critics. For, as early as 1911, a number of workers, not excluding Ehrenfest, had come to a similar conclusion when they contrasted the Planck counting with the one which regarded light quanta as independent entities. In particular, the Polish mathematician, L. Natanson, had arrived at a remarkable conclusion in 1911. He was able to show that the Einstein hypothesis of non-interacting light quanta did not lead to the radiation law of Planck but to Wien's law. It is in this analysis of the statistical procedure of Planck that the problem of distinguishability of elementary entities was raised for the first time.

In seeking to smother the criticism for the time being, Einstein was compelled to conclude : "The formula [i.e. Bose counting], therefore, indirectly expresses a certain hypothesis about a mutual interaction which remains completely mysterious for the present. . . .".

However, in the process of arriving at this conclusion, Einstein made some remarkable and illuminating observations on the nature of a gas of identical particles. He pointed out how the counting of complexions was affected : a micro-state characterized by the first particle here and the second particle there should be exactly the same as that determined by "the mere permutation of the identical" particles, that is to say, if the second particle were here and the first particle were there. These states should not be distinguished ; on the contrary they together should be counted as "one single" state.

It is in this sense that the last vestige of individuality of identical particles is lost. These particles would then be endowed with a statistical preference for having the same velocity. This preference would finally lead, at an extremely low but well defined temperature, to a kind of change of state of aggregation not previously visualized : the particles would "condense" into the lowest quantum state, that is to say, the state of vanishing momentum. No detailed proof of this was given by Einstein. Neither did this observation receive any serious attention at that time.

This is how Einstein generalized and enriched the ideas of Bose. This is how the great physicist, with an unerring insight and rich experience, made a detailed but pointed reference to the strange and radical properties of an assembly of identical particles, which were implied by the new statistical method. Sooner than expected, Bose's idea received fresh but powerful support from an altogether different direction. In 1925 Pauli, proposed a new law relating to electron states in order to explain some spectroscopic data. This is known in the literature as the Pauli exclusion principle. As a consequence, the Italian physicist, Enrico Fermi [1901-54], was able to propose immediately a statistics now named after him.

We should now recall some wellknown facts about mechanics, both quantum and classical, to realize the depth of departure from the classical concepts that was suggested by Bose's approach. It is worth' mentioning the fact that when Bose's paper was published quantum mechanics had yet to see the light of the day.

In mechanics the state of a particle is described by a set of dynamical variables. In classical mechanics nothing prevents one from determining completely the values of these variables and hence the state. In this sense, in classical mechanics, the concept of an individual particle is well-defined. On the other hand, in quantum mechanics, the state of a particle is described by the same set of dynamical variables but with an entirely different content. The source of this new content lies in treating the dynamical variables as-random variables. To be precise: one no longer tries to calculate the values of the variables. Instead, one seeks to determine the laws of distribution obeyed by these variables.

It is from this point of view that one may assert that it is quantum mechanics alone which is capable of interpreting Bose's work logically and scientifically. Conversely, it would not be unfair to say that Bose's work, evaluated from a historical perspective, is one of the fore-runners of the present-day quantum theory.

It is indeed hard to justify otherwise Bose's counting of equally probable states. For it is in quantum mechanics that one describes a state of particles not by noting their individual positions and momenta, but by a symmetric wave function containing the co-ordinates as arguments. The reason for this is indeed not far to seek. It is to be found in the uncertainty principle enunciated by the German physicist, Werner Heisenberg [1901-], in 1927. This principle is at the foundation of the quantum mechanics. Thanks to the principle that the hope of distinguishing between different like particles with the help of the continuity of their motion in space-time is fast fading out. This is why one speaks of a symmetrical wave function representing one and only one state. Accordingly one has to count such a state only once. It is thus also possible to encompass the Fermi approach, where the wave function is skew-symmetric.

" . . . .a general relation to which all special hypotheses will lead. . . ."– Bose

Bose's paper. on Planck's law was quickly followed by another on thermodynamic equilibrium of a radiation field in the presence of matter. An examination of this new work will not fail to reveal that the paper on Planck's law is an integral part of the more general investigation contained in the second Paper. In this Paper Bose was concerned with the general problem of equilibrium of matter in interaction with radiation. This problem he sought to “I solve the method of statistical mechanics, independent of any hypothesis about the mechanism of the competing elementary processes. In the words of Base : “In place erecting suitable hypotheses of elementary processes which give Planck’s law, we seek to derive the relation to which all hypotheses will lead, if Planck’s law for radiation field and Maxwell’s law for material particles are valid.” As a result, he obtained a general relation valid for all special hypotheses. That is to say, from the general relation found one could derive the results obtained previously by Pauli (1923) and Einstein and Ehrenfest (1923).

In the concluding part, however, Bose questioned the necessity of the hypothesis of stimulated emission enunciated earlier by Einstein. He suggested that this hypothesis be dropped altogether. This led Bose to redefine the remaining transition probabilities and thus into conflict with Einstein’s hypothesis mentioned earlier.

However, Einstein had this paper translated into German. It was published in 1924, again with a note by Einstein. But this time it was primarily a note of dissent.

In this note the commentator made it clear how far he agreed with Bose : he did not disagree with the results obtained. He also clearly stated where he could not agree with Bose : the hypothesis of Bose relating to the transition probabilities was unacceptable. He however did not fail to point out why he differed : Bose’s hypothesis contradicted the correspondence principle and it led to a prediction which was not observed in nature.

Precious little has so far been published about the origin of the two papers just referred to. It is therefore deemed desirable that the events leading to the papers should be recorded here.

When Bose left for Dacca his acquaintance with statistical mechanics and gas theories was rather superficial to his familiarity with the theories of electro-magnetism and relativity. It was sometime after March, 1924, that had a meeting with Saha. In the course of the discussion Saha referred to the papers by Pauli (1923) and Einstein and Ehrenfest (1923) published in the then recent issues of Zeitschrift fur Physik. (In fact, Saha is supposed to have left the papars with Bose). He complained about some strange relation in the Pauli paper and asked Bose to examine it. What is the relation that Saha had talked about ?

In the first half of 1923 the works of Compton and Debye on the X-ray scattering off an electron considerably agitated the minds of the then leading physicists. It was, however, young Pauli who took up the problem of finding a quantum-theoretic mechanism for the interaction of radiation with free electrons. He subjected the interaction to the requirement that electrons with the Maxwellian distribution of velocities were in equilibrium with radiation; it was, of course, assumed that the spectral distribution of radiation obeyed Planck’s law. Pauli thus obtained an expression for the probability of a Compton interaction between a photon and an electron. The expression, however, consisted of two parts. One part depended on the radiation density of the primary frequency alone, while the other depended also on the radiation density of the frequency which arose through the Compton process. It was this second term which was intriguing and puzzling from the philosophical point of view : the existence of this term sought to imply that the probability of something happening depended upon something that had yet to happen ! The paper by Einstein and Ehrenfest was a generalization of Pauli’s work.

It is to this “crazy idea”– this is how Bose often referred to the work of Pauli–that Saha drew the attention of Bose. It is thus that young Bose was inducted into the brilliant contributions of Debye (1910) and Einstein (1917). It was thus that he was led into the ever green wonderland of radiation and statistical physics. From the works of the creators of the quantum theory he learned about the true status of the Planck formula. From them he also learned how to return to the fray again and once again. Thus the two papers of Bose grew out of one single problem ; till the end of his life Bose considered them as integral parts of one and the same work.

To appreciate the contributions by the Indian scientists in the early decades of this century it is worth the condition under which a physicist had to work sixty years ago. It is indeed difficult for a young scientist– and by a young one we are afraid we have to mean one under sixty–to realize the appalling condition that a researcher had to face in calcutta even in the second decade of this century.

When Bose and his classmate, Meghnad Saha [1893-1956], who became a physicist with rare width of vision, ventured to embark upon the study of what then constituted the mainstreams of research in physics they had to look frantically even for books, not to speak of equipments. strangely enough, from a quarter least expected. When they had almost lost their heart having failed in their attempts, they came to learn that there, in the Bengal Engineering College at Sibpur, was a strange character. His name was P. J. Brühl. He was born German. He was a doctorate in botany but held the post of professorship in physics. Professor Brühl not only generously supplied the young researchers with many advanced text books but, in a sense, also inspired them by the story of his own struggle for existence. After he had received his doctorate in botany. Brühl was afflicted with tuberculosis at a rather young age. The doctor advised him to move to a sunny temperate climate. Which is why he came over to India and took the teaching job near Calcutta. Since out-door activity was virtually ruled out he switched over to physics and built up a reputable laboratory at the college.

The books which Saha and Bose received were all written in German ; they included some books by Max Planck. However, unlike Saha, Bose did not know the German language at that time, So he had to learn the language in order to get an opening into the world of modern physics. But life was, however, a bit simplified by a suitable division of labour : it was agreed that, while Saha would specialize in thermodynamics and statistical mechanics Bose would concentrate on the theories of electromagnetism and relativity. □