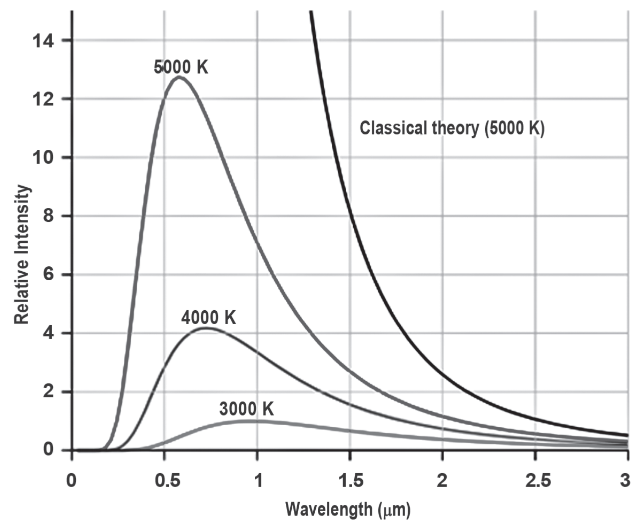
S.N.BOSE AND THE DEVELOPMENT OF QUANTUM MECHANICS

RABIN BANERJEE*

In 1924 Satyendra Nath Bose wrote three papers in quick succession and sent them to Albert Einstein for possible publication. Einstein translated two of these in German and arranged for their publication in Zeitschift für Physik. While the first paper received a very favourable response from him, the second one had a note of dissent. The third paper was never published and, to all intents and purposes, seems lost, although some elements of it may be gleaned by following the correspondence between Bose and Einstein. In his first two papers Bose developed a theory of a gas of photons that led to the development of quantum mechanics as we know it today and eventually to quantum field theory. In this article we would like to trace this development.



Breakdown of Classical Physics

The canonized laws of classical physics developed by Galileo and Newton had successfully explained all natural phenomena for a period of three hundred (1600-1900) years. All this dramatically changed when detailed investigations into blackbody radiation were carried out. Experiments gave a dumbbell-shaped curve where the peak shifted towards the shorter wavelength for increasing temperature.

Using Maxwell's equation, however, one obtained a radically different result. The curve never dropped down and simply peaked towards infinity.

This phenomenon was referred in the press as 'ultraviolet catastrophe'. It is obviously catastrophic since energy is conserved and cannot grow up to infinity. Leaving aside mathematical analysis, it is easy to understand the nature of the classical curve from physical arguments. Take a small spread, say 5 mm in the red and blue regions. Since blue has a smaller wavelength than red, more of blue light can be squeezed in the region as compared to red light. Hence the blue end of the spectrum will have more energy than the red end. Hence there will be a continuous rise of the energy density towards the short wavelength region which explains the nature of the curve. However, experiment does not support this curve leading to a serious crack in the classical edifice of physics.

While the failure of classical physics to explain the blackbody radiation curve was felt unanimously, there was another phenomenon discussed by J. Willard Gibbs, where a similar feature was revealed, but it was largely unnoticed.

This was discovered in the 1860s and is called the Gibbs paradox. To see this consider a room filled with air which is in a state of equilibrium, implying a static unchanging configuration, inspite of the fact that the atoms are colliding with one another at short distance. If the room is now slowly divided into the equal halves, this equilibrium state should remain unaffected. In that case entropy of one

* S.N. Bose National Centre for Basic Sciences, Block-JD, Sector 3, Salt Lake, Kolkata-700106, India,
e-mail : rabin@bose.res.in

half of the room must be half of the total entropy of the full room because entropy is an extensive variable. According to Gibbs if this were not true, then spurious pressure and temperature disequilibrium would occur by the act of simply partitioning the room. However, classical theory fails to give this result.

These were two outstanding issues that challenged classical physics at the turn of the 19th century. The work of S.N.Bose gave a comprehensive solution to both, besides giving birth to the new quantum mechanics.

Max Planck and the Birth of Old Quantum Theory

It is desirable to give a brief review of the circumstances that eventually culminated in Bose's work. As Abraham Pais mentions in his biographical book on Einstein, Subtle is the Lord, "Bose's paper was the last of the four revolutionary papers on old quantum mechanics, other three being by Planck, Einstein and Bohr".

Max Planck was a forty year old physicist at the University of Berlin who was an authority on thermodynamics. He was aware of the problem of blackbody radiation and attempted to resolve it. He divided the emitted light at a given frequency into packets, or quanta, giving them an energy proportional to the frequency,

$$E = hf$$

where the proportionality constant is now referred as the famous Planck's constant.

The physical reasoning behind this choice is simple but deep and effective. High frequency (or low wavelength) will cost more energy. Thus, at any given temperature for the blackbody, there will just not be enough energy for emission beyond a specific frequency. The ultraviolet catastrophe is avoided and, instead of blowing up, the curve would bend down in the low wavelength region. Using this relation between energy and frequency and making certain adjustments to the prevailing ideas of Maxwell, Boltzmann and Gibbs, he was able to produce a formula that exactly reproduced the black body radiation curve. With this formula, Planck opened the door to the quantum world, now known as the old quantum theory.

There were two fundamentally new concepts introduced by Planck. According to Maxwell's classical electromagnetic theory, energy is continuously emitted and can have any continuous value that depends on the amplitude of the electric and magnetic fields in the wave distribution of light. The frequency of the light does not play any role. In Planck's formulation, on the other hand, energy is emitted in discrete packets whose value depends on the frequency of the light.

Einstein extended Planck's concepts. The discrete nature, according to Einstein, was a fundamental property of light itself not connected just to the emission and absorption processes. With this an explanation for the photoelectric effect was possible. Finally, Bohr used these ideas to introduce stationary orbits for the hydrogen atom. Not only the lack of stability of the atom as envisaged by Rutherford was resolved, the theoretical predictions agreed perfectly with the experimental results for the spectrum of the hydrogen atom.

The three fundamental papers mentioned by Abraham Pais, those by Planck, Einstein and Bohr, have been discussed. This brings us to the last of the four fundamental papers, that by S.N.Bose.

S.N.Bose and Planck's Law

Despite the tremendous successes of the old quantum theory, there was one particular flaw that could not be eliminated. The point was that Planck's derivation of his black-body formula was not built solely on quantum concepts. There were two pieces in his formula. The second, exponential type piece, was obtained on quantum principles. But the first factor that multiplied the second one needed classical concepts. In this way light was treated both as a particle (for the second piece) and as a wave (for the first piece), simultaneously. This inherent anomaly could not be rectified, despite repeated attempts by all leading physicists including Einstein himself.

Bose who was teaching thermodynamics and statistical mechanics at the physics department of Dacca (Dhaka) University, did not fail to recognise this shortcoming. In the set of papers mentioned at the beginning of this article, he proceeded to give a derivation of Planck's law based solely on quantum concepts. In his first paper, he gave a static derivation while in the second paper, he gave a dynamic derivation of this law.

Bose divided the total phase space volume in cells or boxes of size h^3, where h is the Planck constant. Then he found the number of possible distributions of the light quanta (photons) of a macroscopically defined radiation among these cells. This gives the entropy which was then used to compute the energy distribution using standard techniques. The photons were treated as indistinguishable and hence devoid of any individuality. Thus, states were not characterized by saying which photons had which energy. Rather, this was done by the arrangement of photons in the cells. This new way of treating a gas of photons was a revolutionary concept and led to the blackbody radiation law. No classical concept was invoked at any stage of the calculation.

Bose Statistics and Indistinguishability

After the appearance of the paper by Bose, what attracted the attention of the scientific community was not the final result (after all it was a known result given by Planck) but the particular way of attaining it. While the fact that a completely quantum derivation of the Planck's law was in itself a great achievement, the idea contained in it was even greater and immediately led to generalisation in several directions.

The new counting method of Bose, based on treating photons as indistinguishable, involved a new (Quantum) statistics, called Bose statistics in honour of its originator. Indistinguishability and the way it was introduced and exploited by Bose was the big new idea that shaped the subsequent development of quantum mechanics and quantum field theory.

It ought to be mentioned that the notion of indistinguishability to resolve issues where classical physics failed had been suggested before. But this was mostly by way of ad hoc assumptions and their implications were completely missed. It was left to Bose to introduce this idea in a logical and self-consistent manner that could be immediately applied and extended to other situations. It is noteworthy that the word 'indistinguishable' does not even appear in Bose's papers. However, it was a natural consequence of his approach and not an assumption. To see this, we recall his approach. The photons were put inside cells of volume h^3 that composed the phase space. Since h is the fundamental scale, objects inside this fundamental cell cannot be probed. This is the theoretical restriction imposed by quantum mechanics. So the photons cannot be seen or distinguished, hence they are indistinguishable. Also, when particles are indistinguishable, they are no longer independent. This feature is also contained in Bose's analysis because, in his next step, he computed the number of states by counting the number of possible arrangements of say N photons in M cells. No specific mention of which photon had which energy(or frequency) was done so that the independence or individuality of photons was simply non-existent. As we now discuss these ideas had a big impact.

Indistinguishability and Gibbs Paradox

The work of S.N.Bose immediately resolved the paradox found by Gibbs, mentioned at the beginning of this article. In the usual approach it was found that entropy is not a strictly additive quantity. Mixing two volumes of an ideal gas at same temperature and pressure, the final entropy is found to be greater than the sum of the two. This was resolved in a highly ad hoc and unsatisfactory way by dividing the partition function by the factorial of the number of particles, while the validity of the Maxwell-Boltzmann statistics was never questioned. In Bose's approach, the particles are all identical so that the factorial term is introduced naturally. The Gibbs paradox gets resolved.

Indistinguishability and Clustering

A non-trivial consequence of indistinguishability is the tendency of particles to cluster together. We can see this by considering a couple of specific examples.

Take a coin and toss it. Then the probability of getting a head is $\frac{1}{2}$ and a tail is also $\frac{1}{2}$. Now take two coins, say A and B. Then the probability of getting two heads is $\frac{1}{4}$ provided A and B are distinguishable. Indeed a total of four options is possible. Likewise, the probability of getting two tails is also $\frac{1}{4}$. Hence the probability of getting either two heads or two tails is $\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$. Now consider A and B to be indistinguishable. Here there are three possibilities, both yield head, both yield tail and one yields a head and the other one, a tail. Hence the probability of getting either two heads or two tails is $\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$. Since $\frac{2}{3}$ is greater than $\frac{1}{2}$ we conclude that the probability of getting the same things together is greater when the coins are indistinguishable.

A similar example, couched in a different language, is more dramatic. Consider two boxes A and B. Also, we have two balls, one red and one blue. We ask the question, in how many ways can the balls be arranged in the boxes? There are four ways to do this. Box A has both balls, box B has none. Box B has both balls, box A has none. Box A has the red ball and box B has the blue ball. Finally, box A has the blue ball and box B has the red one. Then the probability of getting together the maximum number of balls (in this case 2) in this arrangement is $\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$. Now repeat the process where both balls are identical, say red. Then the corresponding probability in this case is $\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$. Thus we find that indistinguishability favours clustering.

This example is of prime importance. It shows the breakdown of standard formulation of statistical mechanics. The probabilistic interpretation that had held sway for all these years had to be modified. Consider an enclosure filled with gas, partitioned by boxes. Then if we pump in a single gas molecule, it has the same probability in entering any of the boxes. However, in the present scenario, it will have a greater probability of entering a box where there are more gas molecules. The situation may be explained by an analogy. Consider a fair or an exhibition with many shops. As the persons enter they have an equal probability of visiting any shop. This is the classical picture. In Bose's picture, persons entering the fair with the gradual passage of time will have a greater probability of entering the shops already having more number of visitors. Of course, here we are considering that all persons are indistinguishable.

As is well known, immediately after translating Bose's paper, Einstein wrote a set of papers where hr extended Bose's work to a gas of material atoms, say helium. Two modifications were necessary from a gas of photons. One was the non zero mass of the atoms while the other was the particle number conservation leading to non zero chemical potential. With these changes, Einstein showed that for such a dilute gas (of extremely low density) cooled to temperatures approaching absolute zero, a large fraction of it condenses into a single quantum state. The new state of matter is the Bose-Einstein condensate. There are nice reviews of it in the present volume and so we will not digress on it. However, it should be realised that the physical origin of this phenomenon is hidden in the property of indistinguishability which, as we saw, favours clustering (or condensations).

Indistinguishability,Wave-Particle Duality and the Birth of Modern Quantum Mechanics

From a static picture of Bose's new counting method we have envisaged the effect of clustering. It is worthwhile to visualize a dynamic picture. The previous features are reinforced with new implications and consequences.

At the time Einstein received Bose's papers, he also received the doctoral dissertation of louis de Broglie to act as an examiner. Louis de Broglie had written a remarkable thesis where he introduced the wave characteristics of material particles through his famous formula,

$$\lambda = \frac{h}{p}$$

where λ is the wavelength associated with a particle of momentum p. The presence of h indicates the quantum nature of the equation. Louis de Broglie's work was complementary to the ideas of Planck, Einstein and others where a particle concept was given for the electromagnetic wave. Here a wave concept was provided for particles. However, despite being impressed, Einstein was a bit uneasy and apprehensive. He was not a great champion of the photon concept even through he had worked on it and got the Nobel prize. But Bose's paper changed everything. A heuristic presentation follows.

We have seen from a static derivation how indistinguishability supports clustering. Einstein had also used the same idea of indistinguishability in getting the Bose-Einstein condensate. It is now simple to see the connection between de Broglie's relation and clustering. At low temperature (which implies low momentum), the wavelength increases. For a certain critical temperature this value becomes greater than the size of the particle. Once this happens the waves associated with a particle latch on to the waves of another particle so that the two particles lose their independent identity and can be regarded as a single entity. Apart from low temperature, the gas must be dilute otherwise classical collisions would overshadow and destroy the quantum effects. It follows therefore from Bose's ideas that material particles must show a wave like phenomenon. Indeed Einstein even presented a paper (8th Jan, 1925) at the Prussian academy of sciences to this effect. Now he was convinced of de Broglie's idea of waves associated with ponderable matter and even nominated him for the Nobel prize which was duly awarded.

Meanwhile, Heisenberg had developed a mathematical formalism for discussing time evolution in quantum mechanics. Since it involved non-commuting objects (matrices) physicists were quite reluctant to either use it or appreciate it. After all, everybody likes an equation of motion like Newton's equation to describe time evolution.

At this point, Schrodinger, in going through de Broglie's work, realised that the mathematical structure of Heisenberg could be recast as a familiar equation of physics describing wave disturbances. But he had his doubts. These were eventually cleared by his exchange of letters with Einstein. Indeed, in one of his letters to Schrodinger, the graphic example of placing two balls in two boxes discussed earlier was used by Einstein to convey the significance of indistinguishability and wave-particle duality. As Schrodinger admits, it was this understanding of Bose's work that eventually cleared his doubts. The rest was technique. He just derived the quantum version of Newton's equation by introducing a wave function. In this way, the ideas of Bose eventually led to the formulation of the Schrodinger equation and, with it, modern quantum mechanics came into existence.

Indistinguishability, Exchange Symmetry and Atomic Structure

Indistinguishability leads to a symmetry of identical particles in quantum mechanics that is of supreme importance in understanding the physical world. Consider a system of two particles - say helium atom that has two orbiting electrons. The quantum mechanical wave function for this two particle system at time t is given by $\psi(x_1, x_2, t)$ where x_1 and x_2 are the coordinates of the particles. Then the probability of finding the first particle at x_1 and the second at x_2 at the time t is given by $|\psi(x_1, x_2, t)|^2$. Now exchange the position of the particles so that the first particle is at x_2 and the second at x_1. Under this exchange, the laws of physics must remain unchanged since the particles are indistinguishable. This exchange symmetry therefore implies,

$$|\psi(x_1, x_2, t)|^2 = |\psi(x_2, x_1, t)|^2$$

which has two solutions,

$$\psi(x_1, x_2, t) = \psi(x_2, x_1, t)$$

$$\psi(x_1, x_2, t) = -\psi(x_2, x_1, t)$$

Quantum mechanics admits both possibilities but with profound differences. The first case corresponds to a symmetric wave function. Particles with this property obey Bose statistics and are called bosons. Likewise, the second case refers to an antisymmetric wave function and particles obeying this are called fermions. Their statistics were found soon after the work by Bose. It is called Fermi-Dirac statistics in honour of its founders. It is useful to mention that, like Bose statistics, indistinguishability is again the key point in Fermi-Dirac statistics.

Crucial inferences can be drawn if we set $x_1 = x_2$. In the first case

$$\psi(x_1, x_1, t) = \psi(x_1, x_1, t)$$

which is trivially valid. It implies that two bosons can be localised at the same point in space. Indeed by taking many bosons localised in the same region of space described by one big wave function, it is possible to prove that the most probable place for all these bosons is piled on top of one another. In other words, all these indistinguishable bosons can be easily put into a quantum state with all the particles having exactly the same value of the momentum. Then we say that the bosons condense into compact or 'coherent' states which is the Bose-Einstein condensation. There are many variants of this coherence or condensation. For instance, lasers produce photons (an example of bosons) coherently packed, moving in the same quantum state of motion. Another instance is the formation of Cooper pairs to prodce superconductivity.

For the second case, putting $x_1 = x_2$ yields,

$$\psi(x_1, x_1, t) = -\psi(x_1, x_1, t)$$

This means that no two identical fermions can occupy the same point in space. This property is called Pauli's exclusion principle. Pauli proved it originally for electrons. His proof used the basic rotational symmetries of the laws of physics for a spin $\frac{1}{2}$ particle. Here we see it emerge from the idea of indistinguishability. Also, it is more general and valid for any fermion instead of just the spin $\frac{1}{2}$ electrons.

The exclusion property is largely responsible for the stability of matter. It gives a logical explanation of the periodic table originally constructed by Mendeleev on an empirical basis. If the electrons did not satisfy the exclusion principle, all of them would collapse into the ground state so that everything would behave like hydrogen gas.

In a letter to the organisers celebrating '50 years of Bose statistics', Paul Dirac wrote famously that Bose's work was important for the 'development of modern atomic theory' and followed this up by saying that 'your country can be proud of it'.

Bose's Second Paper and Beyond

Bose's 2nd paper was also translated by Einstein and published in the same journal Zeit.f.phys, as the first. Presumably, due to Einstein's note of dissent, this paper did not receive any attention. Nevertheless, according to E.C.G Sudarshan, this was also 'a great paper'.

We shall briefly summarise the salient points of that paper. Contrary to a static derivation of the Planck law given in the first paper, the second one gives a dynamic derivation of an equilibrium configuration as the most probable configuration. In this configuration, transitions into and out of each state compensate one another. Using this approach, Bose rederived the distribution law. It may be mentioned that Einstein had reproduced Planck's law by taking spontaneous and stimulated emission on one hand while stimulated absorption on the other. But this was done for the two-level Bohr atom with monochromatic radiation. Bose generalized this to arbitrary atoms with arbitrary energy levels taking into account radiation of all wavelength. This was obviously a highly nontrivial generalization and this is the reason Einstein recommended its publication. Moreover, Bose also corrected the deficiency of Boltzmann's collision formula.

The reason that Einstein gave a note of dissent is perhaps contained in the fact that Bose disagreed with Einstein on the issue of spontaneous emission. According to Bose, this emission was not an intrinsic property of an isolated atom, completely independent of the radiation field. Planck's law was just the statistical property of a photon gas and it was unnecessary to bring in the complications of spontaneous/stimulated emission etc. Einstein, who greatly liked the spontaneous emission process and its ramifications would clearly disapprove of such notions.

Conclusion

The work by S.N.Bose clarified several aspects of the light quantum hypothesis advocated by Planck and Einstein. His set of papers provide the link between the old quantum theory and the modern one as we know it now. Bose ushered in the crucial idea of indistinguishability that played a vital role in the subsequent development of quantum mechanics that was carried out by Schrodinger, Heisenberg, Pauli, Dirac, Jordan, Wigner just to name a few. The statistical property of photons ensured that these were just the levels of the radiation field. Creation and destruction of a photon simply implied the movement of this field from one level to another. The concept of waves and particles unite and wave-particle duality is just a natural consequence. The very essence of the quantization of the electromagnetic field was contained in the set of papers by Bose. In this sense, he may rightly be called the founder of quantum field theory. ❐

References

1. S.N. Bose, The Man and His Work (Part 1 and Part 2), editors C.K.Majumdar, P.Ghose, E.Chatterjee, S.Bandyopadhyay and S.Chatterjee (Principal editor); 1994, S.N.Bose National Centre for Basic Sciences.
2. S.N.Bose, His Life and Times, S elected works (with commentary), editor, Kameshwar C. Wali, 2000, World Scientic.