ON THE THEATRE ATTENDANCE MODEL AND ADVANCE RESERVATION SYSTEMS

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In this Research Communication I provide a description of the Theatre Attendance (TA) problem[1], which can be seen as a generalization of the El Farol Bar problem and its related games, such as the Kolkata Paise Restaurant problem. Then I describe one of the possible real life applications of the TA model, related to the modeling of customers' behavior in advance reservation systems.

Introduction

I propose a model[1] that constitutes a generalization of the El Farol Bar problem[2], in which people decide independently, in each time period, whether to go to a bar, and if more than k people (with $k < n$) attend the bar, i.e., if it is too crowded, then it is better to stay at home. This game has been extensively analyzed by employing a variety of different approaches: Arthur (1994) assumes that each agent employs forecasting models to predict the value of attendances in the next period; other authors make use of cognitive models relying, for instance, on the mechanism of reinforcement learning[3]; other papers focus on a game-theoretic analysis[4]. Several contributions introduce and investigate some classes of games that constitute abstractions and generalizations of the El Farol Bar problem. Among them, it is worth mentioning the Minority Games, that are binary games in which players have to make a choice between two alternatives and those on minority side win[5]. A signicant extension of the Minority Game is the Kolkata Paise Restaurant problem: in each period, N agents choose between n restaurants, which are ranked according to their quality levels, but each restaurant has the capacity to serve food only to one customer per period[6-8]. In the mentioned games and, to the best of my knowledge, in the other models related to the El Farol Bar problem, each agent can choose among a finite number of alternatives and can consequently achieve a finite number of outcomes. Conversely, in the Theatre Attendance (TA) problem, the action space of each agent is a compact real set and the payoff function (i.e., the function that species the utility that an agent gets given his action and the other players' ones) is a piecewise continuous real-valued function. In the remainder of the paper, I briefly show the basic set-up of the TA model (Section II), then I show how a similar model can be used in order to analyze the customers' behavior in advance booking systems (Section III). Section IV concludes.

The Theatre Attendance Model

In the TA problem, in each period, each one of $n \in \mathbb{N}$ agents decides the arrival time at a theatre with $k \in \mathbb{N}$; $k < n$, seats. The agents want to minimize the waiting time before the show begins but consider as valuable to assist to the show comfortably seated. Assuming that the theatre opens at time $t = 0$ and the show begins at time $t = 1$, each agent has to choose the arrival time $t \in [0, 1]$ in order to maximize an utility function of the form:

$$u(t; t^*) = \begin{cases} f(t) & \text{if } t \le t^* \\ g(t) & \text{if } t > t^* \end{cases}$$

where t^* denotes the arrival time of the k-th individual (the last one who finds a seat), $f(t) : [0, 1] \to \mathbb{R}$ (resp.

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$g(t) : [0, 1] \to \mathbb{R}$) is the utility level gained if the considered agent arrives at time $t \leq t^*$ ($t > t^*$), i.e., if he finds (does not find) a seat. Both the functions $f(t)$ and $g(t)$ are decreasing in the waiting time (given by $1 - t$), hence $f'(t) > 0$, $g'(t) > 0$. I also consider: $\forall t^* \in (0, 1)$: $g(t^*) = f(t^*) - \beta$, where $\beta > 0$ is a parameter (whose value can vary across the agents) representing the utility of having a seat. The considered utility function is thus an increasing function of t with a jump discontinuity in t^*.

It is clear that, since only those arriving first find a seat, the utility gained by each agent depends on the others' choices, as it happens in the El Farol Bar problem and the related games. Given the complexity of the problem, it is not possible to find a deductive rational solution. In particular, at least as far as the static game is concerned, it is easy to show that neither pure strategies nor mixed strategies Nash Equilibria exist: from a mathematical perspective, this is due to the lack of continuity of the pay-off function. Intuitively, for each player, given any ($n - 1$)-tuple of (pure or mixed) strategies of the other players, it is always convenient to deviate from its specified strategy, by choosing the arrival time in order to be (in expectation, in the mixed strategies case) the last individual who finds a seat, since in this case he would minimize waiting time and stay comfortable: therefore any specified -tuple of strategies cannot constitute a Nash Equilibrium of the static TA game. This conclusion is surely valid in the most interesting case in which the utility function, as the TA model[1], is specied in such a way that for each agent it is better to be the last one to find a seat than to arrive at time $t = 1$.

Therefore, in the TA model[1], as in most of the contributions about the El Farol Bar problem and the related games, I follow a bounded rationality approach: I propose a new inductive reasoning model for a simplified version of the problem, in which $u(t; t^*)$ is specified as a piecewise linear function. In particular, I specify an updating rule, i.e., a function such that, given the arrival time in the previous period, the fact that the considered agent has or has not found a seat and the utility level he has achieved, specifies his arrival time in the current period. This behavioral rule can be seen as a sort of deterministic version of the reinforcement learning model, applied in[3] to the El Farol Bar problem.

Customers' Behavior in Advance Booking Systems

In this section, after a short review of the related literature, I will briefly illustrate how the basic set-up of the TA model can be applied to build a stylized agents' decision model in an advance booking game.

Related Literature : From the firms' viewpoint, the opportunity of introducing an advance booking system, granting discounts to the customers who book in advance, has been deeply investigated in a variety of contributions[9,10]. A different research area concerns queuing systems that support advance reservations: most contributions focus on performance evaluation and algorithmic aspects of different advance reservation systems[11], while other papers analyze customers behavior in queues in a game theoretical framework[12]. Due to some similarities with the TA model and its application to advance reservation systems that I am going to introduce, it is worth mentioning the works of[13,14] the former models a queuing system that caters to a finite number of customers, which are served on a first-come first-served basis and, as in the TA model, each agent chooses the arrival time and wants to minimize the waiting time, but is also interested in early service completion; the latter, which focuses on services like cloud computing or networks (more generically, the time-slotted systems), analyzes, by means of Poisson games, the strategic behavior of agents who decide whether to make an advance reservation of server resources in future time slots.

A Simple Model : In advance booking systems, for instance of hotel rooms or trains, it is quite common to observe that bigger discounts are offered either to the individuals who book a lot of time in advance or to the first ones to book. This consideration should lead people to book as soon as possible, but actually there are some costs associated to the decision of booking in advance, which are reasonably increasing with respect to the interval of time between the booking and the service date. For instance, when you book in advance a train or a hotel room, you often do not yet have a clear plan of your holiday/business trip, and, after the booking, you lose the opportunity to find a better solution (given that, very often, the booking cancellation costs are high), more suitable for your actual needs: hence, by booking in advance, you incur in sort of costs that we can call less-flexibility and/or sub-optimal solution costs. As far as the specic case of advance booking of hotel rooms is concerned, a customers' decision model is proposed by[15] and then extended, in[16], in order to take into account the impact of the time before the date of stay on the various decision model variables.

We can model this customers' decision problem as follows. Consider $n \in \mathbb{N}$ individuals who are interested in in booking a given hotel room or train for a certain date $t = 1$ and assume that the advance booking is possible starting from time $t = 0$. The utility of each agent is a real valued function of: (i) the evaluation of the considered service, given -for simplicity- by a specied value $\omega > 0$; (ii) the time instant $t \in [0, 1]$ in which the considered agent books the room or the train; (iii) the price he pays, $p(t)$, which depends on when the advance booking occurs. For simplicity, consider the following specification for the described utility function:

$$u(t) = \omega - \left[f(t) + p(t) \right]$$

where $f(t) : [0, 1] \rightarrow$ □ represents the advance booking costs described above, therefore: $f'(t) < 0$. As far as the service provider's pricing policy is concerned, it is reasonable to consider the following step function, with $H \in$ □ different prices $\{p_i\}_{i \in \{1,\ldots,H\}}$, that depend on the moment in which the advance booking occurs:

$$p(t) = \begin{cases} p_1 & \text{if } 0 \le t \le t_1^* \\ p_2 & \text{if } t_1^* < t \le t_2^* \\ & \ldots\ldots \\ p_H & \text{if } t_{H-1}^* < t \le 1 \end{cases}$$

where $p_1 < p_2 < \ldots < p_H$. If the $\{t_i^*\}_{i \in \{1,\ldots,H-1\}}$ values are independent on other agents' booking times and are known to each agent, then each one chooses to book at the instant $t \in \{t_i^*\}_{i \in \{1,\ldots,H-1\}}$ which maximizes his utility: it would make no sense to book at a time t such that $t_i^* < t < t_{i+1}^*$ (for any $i \in \{1, \ldots, H-1\}$), since, by booking in t_{i+1}^*, he would get the same price but incurring in less advance booking costs.

The problem becomes much more interesting when the $\{t_i^*\}_{i \in \{1,\ldots,H-1\}}$ values depend on other agents' choices, as it happens if the service provider sets a pricing policy which species that the first k_1 individuals who book pay p_1, the following k_2 pay p_2, and so on: in this case, we are in a framework very similar to the TA problem.

In this setting, it is worth introducing other realistic features. For instance, we can consider that: (i) an individual does not book at all when $u < 0$ (or $u < \bar{u}$, where $\bar{u} \ge 0$ is an outside option utility, given by the opportunity to buy a different service, maybe from a different service provider); (ii) the agents do not know whether the service capacity, i.e., the number of hotel rooms or train seats, is lower than n; or, better to say, do not know whether the number of people interested in that service is so high that only the first ones who book can buy it.

Concluding Remarks

In this Research Communication, I have reviewed the Theatre Attendance model, "based on the idea that people choose their arrival time at a theatre, taking into account the comfort of a seat and the waiting time before the theatrical show"[1], and I have shown that an analogous timing choice problem in an agents' interdependence setting arises for customers in advance reservation systems. To analyze this model and its implications for firms who design advance reservation systems and decide whether and to which extent to offer discounts for advance booking, possibly competing with each other, is left for future research.

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References

1. M. Bisceglia, A Theatre Attendance Model, *International Journal of Applied Behavioral Economics* **6(3)**, 23, (2017)
2. B. W. Arthur, Inductive Reasoning and bounded rationality, *The American Economic Review,* **84(2)**, 406, (1994)
3. R. Franke, Reinforcement Learning in the El Farol Model, *Journal of Economic Behavior & Organization,* **51(3)**, 367, (2003)
4. E. Zambrano, The interplay between analytics and computation in the study of congestion externalities: The case of the El Farol Problem, *Journal of Public Economic Theory,* **6(2)**, 375, (2004)
5. D. Challet, Y-C. Zhang, Emergence of Cooperation and Organization in an Evolutionary Game, *Physica A*, **246**, 407, 1997
6. B. K. Chakrabarti, Kolkata Restaurant Problem as a Generalised El Farol Bar problem, in Econophysics of Markets and Business Networks, Eds., B. K. Chakrabarti, A. Chatterjee, Springer, (2007)
7. A. S. Chakrabarti, B. K. Chakrabarti, A. Chatterjee, M. Mitra, The Kolkata Paise Restaurant problem and resource utilization, *Physica A*, **388**, 2420, (2009)
8. B. K. Chakrabarti, A. Chatterjee, A. Ghosh, S. Mukherjee, B. Tamir, Econophysics of the Kolkata Restaurant Problem and Related Games: Classical and Quantum Strategies for Multi-agent, Multi-choice Repetitive Games. Springer, (2017)
9. L. R. Weatherford, P. E. Pfeifer, The economic value of using advance booking of orders, *Omega*, **22(1)**, 105, (1994)
10. C. S. Tang, K. Rajaram, A. Alptekinolu, J. Ou, The benets of advance booking discount programs: Model and analysis, Management Science, **50(4)**, 465-478, (2004)

11. N. Charbonneau, V. M. Vokkarane, A survey of advance reservation routing and wavelength assignment in wavelength-routed WDM networks, *IEEE Communications Surveys & Tutorials,* **14(4)**, 1037, (2012)
12. R. Hassin, M. Haviv, To queue or not to queue: Equilibrium behavior in queueing systems, Vol. 59, Springer Science & Business Media, (2003)
13. R. Jain, S. Juneja, N. Shimkin, The concert queueing game: to wait or to be late, *Discrete Event Dynamic Systems*, **21(1)**, 103, (2011)
14. E. Simhon, D. Starobinski, Game-theoretic analysis of advance reservation services, Paper presented at the *48th Annual Conference on Information Sciences and Systems*, (2014)
15. Z. Schwartz, Advanced booking and revenue management: Room rates and the consumers strategic zones, *International Journal of Hospitality Management,* **25(3)**, 447, (2006)
16. Z. Schwartz, Time, price, and advanced booking of hotel rooms, *International Journal of Hospitality & Tourism Administration,* **9(2)**, 128, (2000)