Research Communication

Sci. and Cult. 85 (1–2) 49-53 (2019)

Light and Shadow over Earth as Seen on Real-time Navigation During Flight

***Abstract* : In modern day international and long-haul domestic flights, the flight paths are shown on screens in different scales along with information on speed of aircraft, external temperature, remaining distance to destination and its local time etc. In addition to these, map of the whole earth is periodically displayed on the screen with an overlay of lighted and shadow zones over it giving an overall idea to the viewer in which parts of the world it is day and in which ones it is evening or night at that moment. Though the sun shines on spherical earth resulting in a circular cross-section (twilight zone) separating lighted and dark areas on surface of the earth, the article explains why it appears like an undulatory curve on a flat display just as continents on the earth near the poles look inevitably distorted on a flat map.**

Many of you (like me) may have watched the flight information display during a long-haul domestic or international flight, taking time off from other entertainments provided on board. I always wondered at the shape of the boundary between the dark areas (night) and those lighted up by the sun (day) when large areas on the earth's surface was on display. One can see an undulated curve, more like a smoothed box-curve rather than a sine or cosine wave-like shape (Fig. 1).

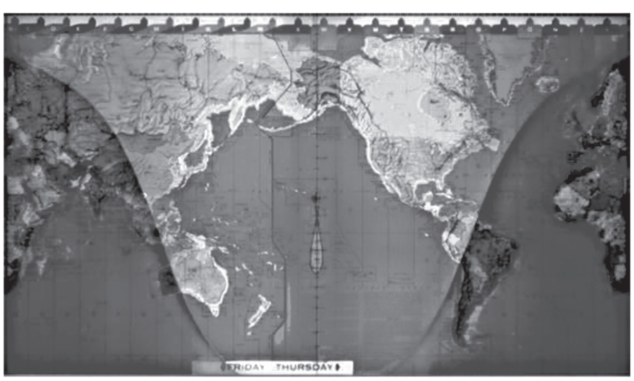

Fig. 1: Light and shadow over the earth projected on a flat surface as shown during real-time flight display.

Why does it look like this? This is the view of the earth as seen from outside or as viewed by the Sun at different times of the year – but rolled out on a flat surface instead of a spherical one. Light and shadow on spherical surface of the earth instead actually look as shown in Fig. 2. The objective of this article is to show that the circular cross-section demarcating the light and shadow – the twilight zone – look like the shape shown in Fig.1 when projected on a flat surface.

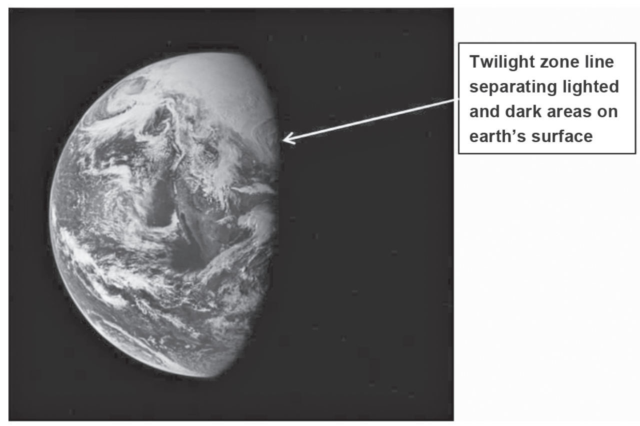

Fig. 2: Half of the globe lighted up by sun rays falling from the left leaving the other half dark

During course of the year, the amount of the surface lighted up or its orientation with respect to the earth's rotation axis vary according to the month or equivalently the position on its orbit around the sun. As is well known, the northern hemisphere has more area lighted up compared to its southern counterpart during summer, reaching its peak on the longest day (21 June, summer solstice) while darkness engulfs most of the areas on winter solstice (23 December). The southern hemisphere experiences exactly the opposite situation on the above days. In between, days and nights are almost equal on every place on earth on vernal or spring equinox on 21 March and autumnal equinox on 23 September (Fig. 3).

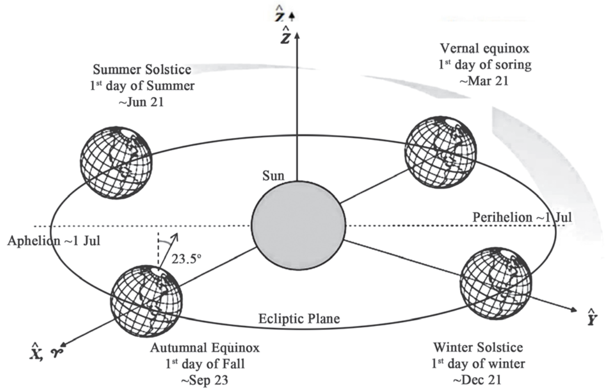

Fig. 3: Variation of seasons and day-night variation round the year

Up to now, all these are *qualitative* that we knew in our school days. We now embark on *quantitative* estimates on the extent and orientation of lighted and shadow zones on the spherical earth during the year and why it appears the way it does on a flat surface or 'map'.

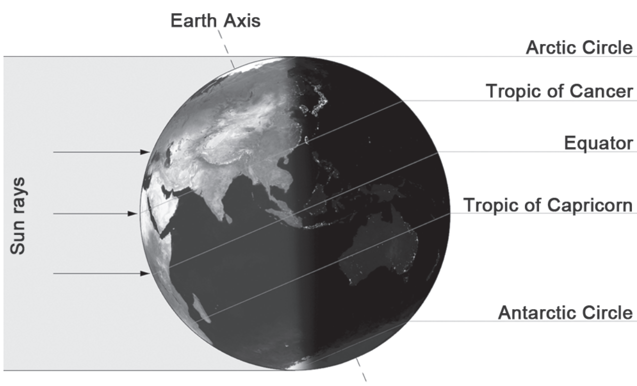

Fig. 4(a) Situation in Summer Solstice

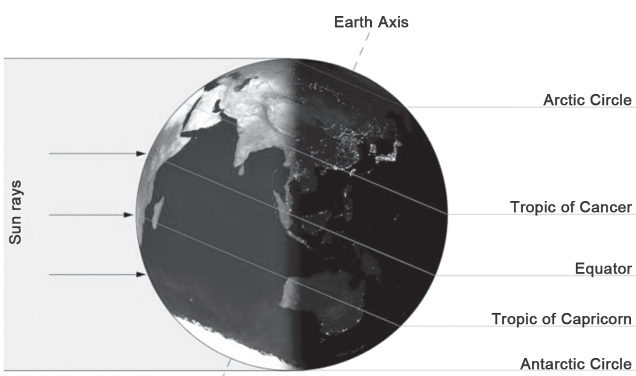

Fig. 4(b) Winter solstice in Northern hemisphere

The asymmetry between northern and southern hemisphere arises due to the tilt of earth's rotation axis by ~ 23.5 Degrees with the vertical to the plane of the orbit around the sun. We know that the longitude lines are 'great circles' – circular planes on the earth's surface passing through the centre and the poles. Similarly, as can be seen from Figs. 4(a) and 4(b), the circular cross-section demarcating the light and shadow – the twilight zone – is also a great circle (because of parallel rays from the sun) with the difference that it passes through the edges of Arctic and Antarctic circles with latitudes 66.5°N and 66.5°S respectively. The Figs 4(a) and 4(b) correspond to the summer and winter solstices in northern hemisphere. To visualize any intermediate situation, spring and autumnal equinoxes in particular, one has to imagine parallel sun rays perpendicular to the plane of Fig. 4(a) or 4(b).

It may be shown 3-dimensional coordinate geometrically (derivation given in Appendix I), by intersecting a sphere with a plane passing through the centre that the equation to above great circles is:

$$tan\phi = \tan\,(90^0 + \varepsilon)\,\cos\,(\lambda - \lambda_0) \qquad (1)$$

where ϕ is the latitude, λ the longitude of a point on the earth, λ_0, a central meridian such as the Greenwich meridian and ε is the angle between the direction of sun's rays and planes containing latitude lines. (ε ~ 0 on equinoxes since Sun rays lie in the planes of latitude circles,

$\varepsilon = 23.5^0$ (Summer Solstice), $\varepsilon = -23.5^0$ (Winter Solstice).

For other days during the year, $-23.5^o < \varepsilon < +23.5^0$)

You must have observed in school geography or map book how disproportionately big Russia, Canada and Greenland looked in a map of the world as a whole! In Fig. 1, you may even see that northern part of Greenland is truncated. This is because the surface on spherical earth is attempted to be projected on a flat surface – a map. You may see that there must be a distortion in doing that by imagining what happens while trying to flatten an orange peel – it breaks at the edges!

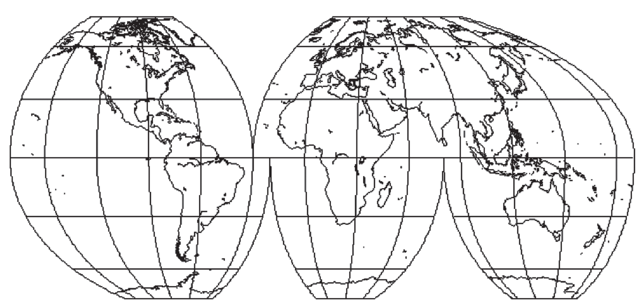

Fig.5: This representation of the world like broken orange peel is called Sinusoidal projection.

The only other way to have an unbroken continuous map is to encase the globe within a cylinder (Fig. 6(a)) or to cap it with a cone (Fig. 6(b)) which can then be rolled out into a flat surface. What is seen on the unrolled surface is the 'image' projected on the surface of cylinder or cone by an imaginary point light source at the centre of a transparent globe. It is also not difficult to imagine that this image will invariably be distorted in some way. This, in fact, is the vast subject of map projection devised by cartographers for centuries.

Depending on whether a cylinder or a cone is used to approximate areas on the globe, two major classes of map projection are (1) Cylindrical, (2) Conical Projections. The third, called Azimuthal projection, is employed near the poles by placing a plane surface near them. One can imagine that cylindrical (with its axis along that of earth's rotation) projection is good near the equator(0^o) upto about the latitudes of tropics of Cancer and Capricorn (23.5^o N/ S), while the conical projection is suitable for regions between latitudes of 20^o N/S and 75^o N/S. Both of these can be used for large extents of longitudes about a chosen central longitude or meridian. However, when it comes to depicting the whole world, cylindrical projection touching the globe at the equator (Fig. 6(a)) is the only option unless measure of areas and distances compared to those on the curved surface is of major concern. This is precisely the Mercator projection devised by the Flemish geographer and cartographer Gerardus Mercator in 1569 and has been used to display the whole world in Fig. 1. It can be seen from Figs. 6(a) and 7 why the high latitude areas get bigger and bigger and the Polar regions cannot be plotted at all! We quote the transformation formula (shown in Eq. (3) below) from geographical coordinates – longitude λ and latitude ϕ – to the rectangular grid coordinates easting (x) and northing (y) on a plane surface. This is to emphasize how the 'infinity problem' near poles (y tends to infinity at $\phi = 90^o$ in Eq. (3)) in conversion of latitude can be circumvented by a projection system called Miller with minor modification of Mercator's formula. This leads to a representation very close to that shown in Fig. 8 where the high latitude areas are 'bounded' within the rectangular frame albeit looking distorted in a compressed fashion – with distortion in shape and area of the Antarctic region.

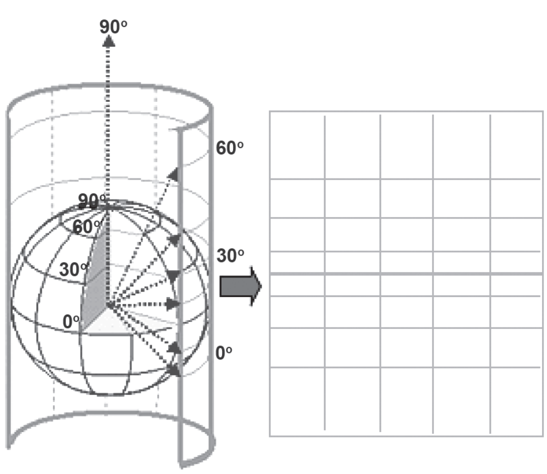

Fig.6(a): The geometry of Cylindrical projection.

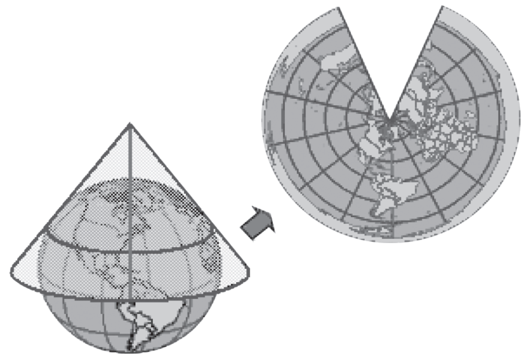

Fig. 6(b): The geometry of Conical projection.

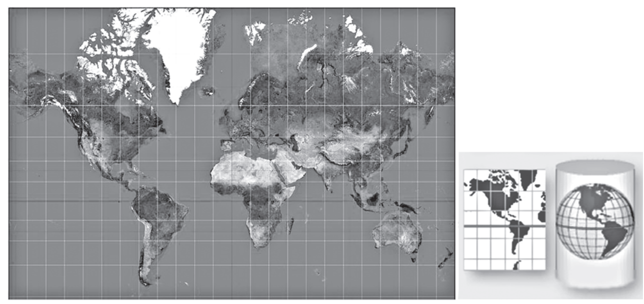

Fig.7: The geometry of Mercator projection

Formula of Mercator Projection for a sphere of radius R[1]:

$$x = R\left(\lambda - \lambda_0\right) \tag{2}$$

$$y = R \log_e \left[\tan\left(\frac{\pi}{4} + \frac{\phi}{2} \right) \right] \tag{3}$$

In case of Miller Projection, Eqn. (3) is modified as:

$$y = \frac{5R}{4} \log_e \left[\tan\left(\frac{\pi}{4} + \frac{2\phi}{5} \right) \right] \tag{4}$$

resulting in much less relative area distortion in Polar regions.

It turns out that a map in Miller Projection appear very similar to that prepared with Geographic Coordinate System (GCS) in which longitudes and latitudes are simply plotted on a flat rectangular grid without any projection formula, particularly at large scales. This can be seen by comparing Fig. 8(a) with Fig. 8(b) that has been drawn in GCS with a scale of about 1:150,000,000 showing the twilight zone near winter solstice in northern hemisphere.

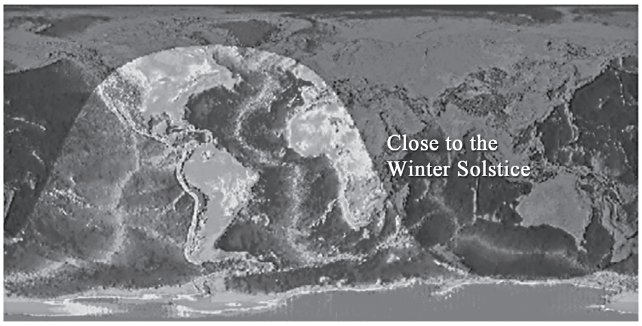

Fig.8(a): Shape of twilight curve separating lighted and dark areas across the globe near winter solstice in Northern hemisphere

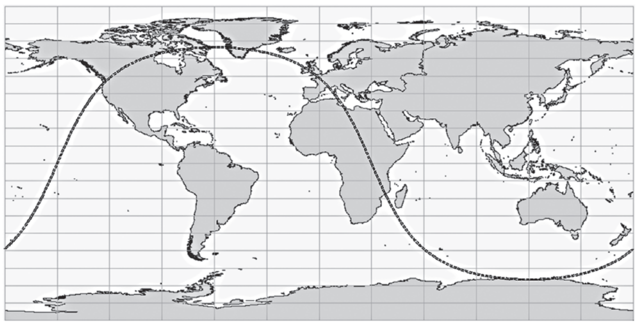

Fig.8(b): Shape of the same twilight curve drawn on ArcGIS in Geographical Coordinate System

Fig. 9(a): Shape of twilight curve separating lighted and dark areas across the globe near summer solstice in Northern hemisphere

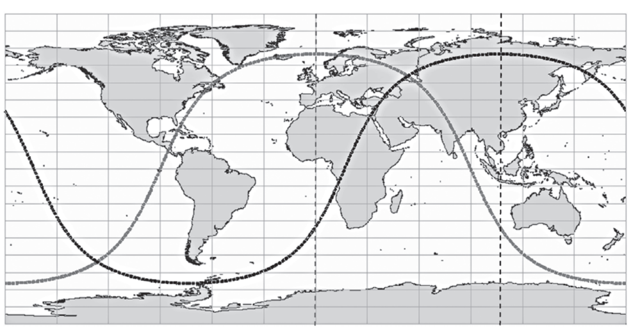

Fig. 9(b): Twilight curve near Summer solstice centred at Greenwich (red) and Jakarta (black). The black twilight curve matches the twilight zone in coloured Fig. 9(a) above

Similarly, we depict the cases of summer solstice Figs. 9(a) and 9(b). The black twilight curve in Fig. 9(b) matches with the light and shadow zone in Fig. 9(a).

Note that in Fig. 9(b) and 10(b), two twilight curves in red and black referred to two different central meridians (through Greenwich and Jakarta) are drawn to give an idea how the light and shadow zones move with time over the earth's surface during a real time flight.

Likewise, Figs. 10(a) and 10(b) depict the cases of equinoxes. The box shape of the curve follows from Eqn. (1) setting $\varepsilon \to 0$. For $\varepsilon = 0$, the equation reduces to $\cos(\lambda - \lambda_0) = 0$ that results in vertical longitude lines 180 degrees apart. The figures have been shifted with respect to each other to match the red twilight curve (through Greenwich) with that in the coloured figure 10(a).

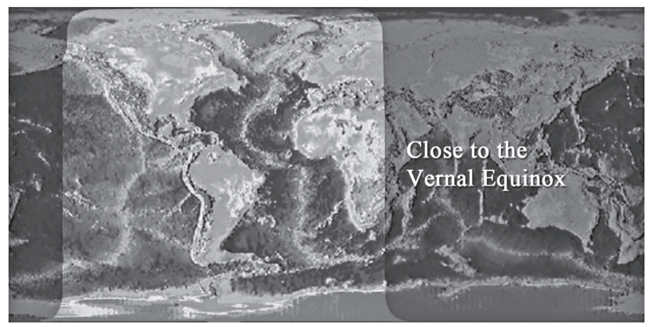

Fig. 10(a): Shape of twilight curve separating lighted and dark areas across the globe near vernal equinox. The shape of the curve will expectedly be similar near autumnal equinox.

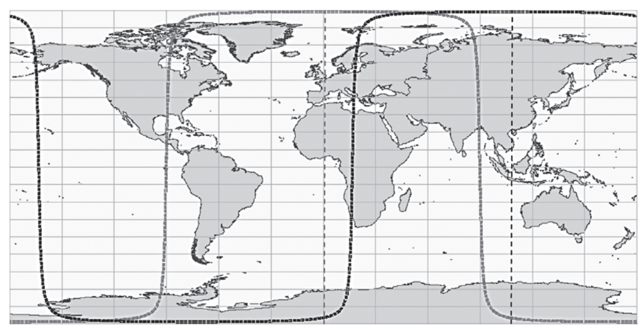

Fig. 10(b): Twilight curves near spring (21 March) and autumnal (23 September) equinoxes with the same colour codes

Figs. 10(a) and 10(b) clearly indicate day and night of equal duration everywhere on the earth's surface near vernal and autumnal equinoxes. Coloured figures 8(a), 9(a) and 10(a) are included to emphasize lighted and dark zones that could not be depicted in Figs 8(b), 9(b) and 10(b) drawn on ArcGIS.

Conclusion : It is hoped that the above discussions and illustrations meaningfully explain the appearance of dark and lighted zones on surface of the earth, as projected on a flat surface, in different times of the year as Earth rotates with its axis tilted by about 23.5 degrees with respect to the orbital plane under rays of the Sun.

Acknowledgement : The author acknowledges ESRI (Environmental Systems Research Institute) for providing data of continents in .shp file in public domain that has been used by the author to draw Figs. 8(b), 9(b), 10(b). Other figures have been taken from different sources on the internet. ❐

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Received : 28 June, 2017

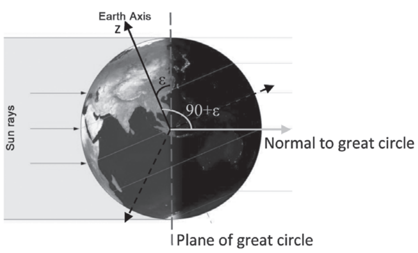
Revised : 17 August, 2017

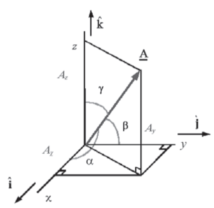
Re-revised : 11 September, 2017

1. Map Projections used by the U.S. Geological Survey, by *John P Snyder* (1982)

Appendix I

Equation to the 3 dim circular cross-section representing the twilight zone at Summer Solstice:





We choose a 3 dimensional Cartesian coordinate system whose z-axis coincides with earth's rotation axis as shown above and the origin at centre of the earth.

Equation to the sphere (the earth):

$x^2 + y^2 + z^2 = 1$ (assuming radius = 1 without loss of generality) with

$$x = \cos\phi \cos\lambda,\ y = \cos\phi \sin\lambda,\ z = \sin\phi \qquad \text{(i)}$$

where ϕ = Latitude, λ = Longitude

Equation to the intersecting plane passing through centre of the earth (resulting in the great circle separating the lighted and the dark zones):

$$lx + my + nz = 0 \qquad \text{(ii)}$$

where $l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$ are direction cosines of the normal to the great circle with angles shown above. In this case, $\gamma = 90^0 + \varepsilon$

where ε is angle of this great circle with the earth's rotation axis.

Combining (i) and (ii)m, we have

$$(l \cos\lambda + m \sin\lambda) \cos\phi + n \sin\phi = 0$$

$$(\cos\lambda_0 \cos\lambda + \sin\lambda_0 \sin\lambda)\cos\phi + \frac{n}{\sqrt{l^2 + m^2}} \sin\phi = 0,$$

with $\tan\lambda_0 = \dfrac{m}{l}$

$$\cos\left(\lambda - \lambda_0\right)\cos\phi + \frac{n}{\sqrt{1-n^2}}\sin\phi = 0$$

since $l^2 + m^2 + n^2 = 1$

$$\tan\phi = \tan(90^0 + \varepsilon)\cos(\lambda - \lambda_0)$$

setting $\cos(90^0 + \varepsilon) = -n,\ \sin(90^0 + \varepsilon) = \sqrt{1-n^2}$

which is Eqn. (1) in main body of the articles.