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Application of Leith's Model for the Energy Transfer Process in an Isotropic Gas-Solid Turbulent Flow

Abstract **: Leith initiated a diffusion approximation first to describe the non-local inertiai energy transfer between wave number components in the spectral representation of an isotropic turbulent flow. Thus it obeyed the inertial range spectrum of Kolmogorov and covered all the previous predictions of Kovasznay, Obukhov, Heisenberg and Kraichnan regarding. homogeneous isotropic turbulent flow. We finally work out the solution of the energy by series solutlon method.**

In this paper, we have discussed the procedures following a straightforward self – preserving concept which originated with Heisenberg (henceforth known as Heisenberg method and procedure) allowing a determination of the spectrum at large Reynolds numbers and the authors have tried to include those frequencies which have small wave numbers, since the distribution of energy among the largest eddies must be a geometrical and not a statistical part of the problem of spectrum and we must remark at the outset that statistical part of the problem is well understood.

According to the hypothesis of Heisenberg (1948), the equation for the dissipation of energy is

$$S_k = \left\{ \mu + pK \int_k^\infty \sqrt{\left(\frac{F(K'')}{K''^3} \right)} dK'' \right\} \int_0^k 2F\left(K'\right) K'^d \, dK'$$

Where S_k means the total loss of the energy of the spectrum that is contained between 0 and k. The physical meaning of the above equation according to Heisenberg is that for all eddies between 0 and k, the action of the smaller eddies can well be represented by an additional viscosity because of Prandit's (1945) view that the action of smaller eddies in the case of transfer of energy is similar to that of friction. This additional viscosity depends on the intensity F(k) of small eddies, that is, depends on that part of the spectrum with large k. Thus according to this hypothesis, the formula

$$\eta_k = pK \int_k^\infty \sqrt{\left(\frac{F(K'')}{K''^3} \right)} dK''$$

For the turbulent viscosity follows simply from dimensional analysis, since η_k must be the product of density, velocity and length, K being a numerical constant, the value of which is nearly, 8 from experimental data.

Finally, Heisenberg found from the equation

$$\int_0^x f(x)dx - \frac{1}{2} x f(x)$$

$$= \left\{ x + K \int_x^\infty \sqrt{\left(\frac{F(x'')}{x''^3} \right)} dx'' \right\} \int_0^x 2F(x') x'^2 dx'$$

that f(x ~ const. x for small x

and f(x) ~ const. $x^{-5/3}$ for very large x.

our line of thought is to examine the self-preserving analysis in the case where the energy transfer spectra would be that due to particle ladden case as formulated as

$$(3c-2) f(x) + cxf'(x)$$

$$= -2 \frac{d}{dx} \left(x^{\frac{13}{2}} \frac{d}{dx} \left(x^{-3} f^{\frac{3}{2}}(x) \right) \right) - \frac{2}{\mathrm{Re}\, \tau_1^{2c-1}} - 2\left(\tau_1 t_0 f(x) \right)$$

The spectral equation governing the decay of turbulence kinetic energy in a particle ladden homogeneous isotropic turbulence flow is given by (Baw and Peskin. Tsuji)

$$\frac{\partial}{\partial t} E(k,t) = T(k,t) - 2\nu k^2 E(k,t) - \frac{2\beta \nu k^2}{\frac{t}{\tau} + \nu k^2} E(k,t)$$

where k is wave number, ν is the kinematic viscosity, τ is the characteristic time $= \frac{2\rho_s}{9\mu}\sigma^2$, ρ_s being the density of the solid material, σ the radius of the particle, β the co-efficient of viscosity $= \frac{\rho_p}{\rho\tau}$, ρ_p is the volume concentration of the solid phase in the *flow*, ρ the density of the gas, $E(k,t) = 2\pi k^2 \varphi_{ii}(k,t), \varphi_{ij}(k,t)$ being the

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Fourier transform of $\overrightarrow{v_i v_j}$, the correlation between the fluctuating gas velocity components pertaining to two points $P(\vec{X})$ and $P'(\vec{X}')$ inside the flow field; $T(k,t) = 2\pi k^2 \Gamma_{ii}(k,t), \Gamma_{ij}(k,t)$ being the Fourier transform of

$$\frac{\partial}{\partial r_k}\left(\overrightarrow{v_\iota v_k v_j} = \overrightarrow{v_\iota v_j^\iota v_k^\iota}\right), \vec{r} = \overrightarrow{X'} - \vec{X}$$

The term on the LHS of (1) describes the rate at which turbulence kinetic energy changes. The first term on the RHS of (1) represents the transfer of kinetic energy at the wave number k due to the turbulence self-interactions. The second term is the dissipation of turbulence kinetic energy due to the effects of molecular viscosity. The third term takes into account the effect due to the presence of particles which are so massive that they are unaffected by the gas turbulent fluctuations (Wallace). We seek, in the next section, the self-preserving solution for the turbulence energy spectrum *E(k, t)* given by (1)

Now it is to be noted that the energy and flux functions are submitted by the simpler form:

$$\frac{\partial E}{\partial t} = \frac{\partial F}{\partial k} - 2\nu k^2 E$$

The energy spectrum E and the flux function F may be defined as

$$F = -\beta k^{\frac{13}{2}}\left(\frac{\partial}{\partial k}\right) k^{-3} E^{\frac{3}{2}}$$

so that the energy transfer spectrum T(k, t) and F(k, t) as suggested by Leith may be put in the form

$$T(k,t) = -2\gamma_L \frac{\partial}{\partial k}\left(k^{\frac{13}{2}} \frac{\partial}{\partial k}\left(k^{-3} E(k)^{\frac{3}{2}}\right)\right) \tag{2}$$

γ_L being a dimensionless constant.

Substituting (2) in (1) we get

$$\frac{\partial}{\partial t}E(k,t) = -2y_L\left(k^{\frac{13}{2}} \frac{\partial}{\partial k}\left(k^{-3}E(k)^{\frac{3}{2}}\right)\right)$$

$$-2\nu k^2 E(k,t) - 2\beta \frac{\nu k^2}{\frac{1}{\tau} + \nu k^2} E(k,t) \tag{3}$$

On the basis of the assumption that particles are essentially unaffected by the turbulent fluctuations we may take

$$\frac{1}{\tau} << \nu k^2 \tag{4}$$

It is to be noted that condition (4) being satisfied, we may assume that for very high frequency fluctuations of gas, gas-solid velocity correlations are negligible as the response time of the partcle is sufficiently long (Wallace).

In view of the relation (4), equation (3) may be taken as

$$\frac{\partial}{\partial t}E(k,t) = -2\gamma_L \frac{\partial}{\partial K}\left(k^{\frac{13}{2}} \frac{\partial}{\partial k}\left(k^{-3}E(k)^{\frac{3}{2}}\right)\right)$$

$$-2\nu k^2 E(k,t) - 2\beta E(k,t) \tag{5}$$

Let $E(k,t) = \frac{1}{y_L^2 k_0^3 t_0^2} \cdot \left(\frac{t_0}{t}\right)^{2-3c} f\left(\frac{k}{k_0}\left(\frac{t_0}{t}\right)^c\right)$ be a general type of self-preserving solution of (5), originally forwarded by Sen [1951], where k_0, t_0 are constants and c is an arbitrary constants restricted to $0 < c < \frac{2}{3}$

Let also $x = \frac{k}{k_0}\left(\frac{t}{\tau_0}\right)^c$

Then $\frac{\partial}{\partial t}E(k,t)$

$$= \frac{1}{y_L^2 k_0^3 t_0^2} \cdot (t_0)^{2-3c} \frac{\partial}{\partial t}\left(t^{3c-2} f\left(\frac{k}{k_0}\right)\left(\frac{t}{t_0}\right)^c\right)$$

$$= \frac{t_0^{2-3c}}{y_L^2 k_0^3 t_0^2} \cdot \left\{(3c-2)t^{3c-3} f\left(\frac{k}{k_0}\left(\frac{t}{t_0}\right)^c\right)\right.$$

$$\left. + t^{3c-2} f'\left(\frac{k}{k_0}\left(\frac{t}{t_0}\right)^c\right) + t^{3c-3} f\left(\frac{k}{k_0}\left(\frac{t}{t_0}\right)^c\right) \frac{kct^{c-1}}{k_0 t_0^c}\right\}$$

$$= \frac{t_0^{2-3c}}{y_L^2 k_0^3 t_0^2} \cdot \left\{(3c-2)t^{3c-3} f\left(\frac{k}{k_0}\left(\frac{t}{t_0}\right)^c\right)\right.$$

$$\left. + \frac{c \cdot t^{4c-3}}{t_0^c} \cdot \frac{k}{k_0} f'\left(\frac{k}{k_0}\left(\frac{t}{t_0}\right)^c\right)\right\}$$

$$= \frac{t_0^{2-3c}\cdot t^{3c-3}}{y_L^2 k_0^3 t_0^2}\cdot\left\{(3c-2)\,f\left(\frac{k}{k_0}\left(\frac{t}{t_0}\right)^c\right) + \frac{ck}{k_0}\left(\frac{t}{t_0}\right)^c\cdot f'\left(\frac{k}{k_0}\left(\frac{t}{t_0}\right)^c\right)\right\}$$

$$= \frac{1}{y_L^2 k_0^3 t_0^2}\left(\frac{t_0}{t}\right)^{3-3c}\cdot\left\{(3c-2)\,f\left(\frac{k}{k_0}\left(\frac{t}{t_0}\right)^c\right) + \frac{ck}{k_0}\left(\frac{t}{t_0}\right)^c\cdot f'\left(\frac{k}{k_0}\left(\frac{t}{t_0}\right)^c\right)\right\}$$

From (2), we have now

$$\frac{\partial}{\partial k}\left(k^{\frac{13}{2}}\frac{\partial}{\partial k}\left(k^{-3}E(k)^{\frac{3}{2}}\right)\right)$$

$$= \left\{\frac{\partial}{\partial k}\left(k^{\frac{13}{2}}\left(-3k^{-4}E(k)^{\frac{3}{2}} + k^{-3}\,\frac{3}{2}E(k)^{\frac{1}{2}}\frac{\partial E}{\partial k}\right)\right)\right\}$$

$$= \left\{\frac{\partial}{\partial k}\left(-3k^{\frac{5}{2}}E(k)^{\frac{3}{2}} + \frac{3}{2}k^{\frac{7}{2}}E(k)^{\frac{1}{2}}\frac{\partial E}{\partial k}\right)\right\}$$

$$= -3\frac{\partial}{\partial k}\left(k^{\frac{5}{2}}E(k)^{\frac{3}{2}}\right) + \frac{3}{2}\frac{\partial}{\partial k}\left(k^{\frac{7}{2}}E(k)^{\frac{1}{2}}\right)\frac{\partial E}{\partial k}$$

$$= -3\left\{\frac{5}{2}k^{\frac{3}{2}}E(k)^{\frac{3}{2}} + k^{\frac{5}{2}}\cdot\frac{3}{2}E(k)^{\frac{1}{2}}\frac{\partial E}{\partial k}\right\}$$

$$+\frac{3}{2}\left\{\frac{7}{2}k^{\frac{5}{2}}E(k)^{\frac{1}{2}}\frac{\partial E}{\partial k} + k^{\frac{7}{2}}\frac{1}{2}E(k)^{-\frac{1}{2}}\left(\frac{\partial E}{\partial k}\right)^2 + k^{\frac{7}{2}}E(k)^{\frac{1}{2}}\frac{\partial^2 E}{\partial k^2}\right\}$$

$$= -\frac{15}{2}k^{\frac{3}{2}}E(k)^{\frac{3}{2}} - \frac{9}{2}k^{\frac{5}{2}}E(k)^{\frac{1}{2}}\frac{\partial E}{\partial k} + \frac{21}{4}k^{\frac{5}{2}}E(k)^{\frac{1}{2}}\frac{\partial E}{\partial k} + \frac{3}{4}k^{\frac{7}{2}}E(k)^{-\frac{1}{2}}\left(\frac{\partial E}{\partial k}\right)^2 + \frac{3}{2}k^{\frac{7}{2}}E(k)^{\frac{1}{2}}\frac{\partial^2 E}{\partial k^2}$$

$$= -\frac{15}{2}k^{\frac{3}{2}}E(k)^{\frac{3}{2}} + \frac{3}{4}k^{\frac{5}{2}}E(k)^{\frac{1}{2}}\frac{\partial E}{\partial k} + \frac{3}{4}k^{\frac{7}{2}}E(k)^{-\frac{1}{2}}\left(\frac{\partial E}{\partial k}\right)^2 + \frac{3}{2}k^{\frac{7}{2}}E(k)^{\frac{1}{2}}\frac{\partial^2 E}{\partial k^2} \qquad (6)$$

Therefore, substituting

$$E(k) = \frac{1}{\gamma_L^2 k_0^3 t_0^2}\left(\frac{t_0}{t}\right)^{2-3c} f\left(\frac{k}{k_0}\left(\frac{t}{t_0}\right)^c\right)$$

in the above we get

$$\frac{\partial}{\partial k}\left(k^{\frac{13}{2}}\frac{\partial}{\partial k}\left(k^{-3}E(k)^{\frac{3}{2}}\right)\right) = -\frac{15}{2}k^{\frac{3}{2}}\frac{1}{\gamma_L^3 k_0^{\frac{9}{2}} t_0^3}\left(\frac{t_0}{t}\right)^{\frac{6-9c}{2}}$$

$$\times f^{\frac{3}{2}}\left(\frac{k}{k_0}\left(\frac{t}{t_0}\right)^c\right) + \frac{3}{4}k^{\frac{5}{2}}\frac{1}{\gamma_L k_0^{\frac{3}{2}} t_0}\left(\frac{t_0}{t}\right)^{\frac{2-3c}{2}} f^{\frac{1}{2}}\left(\frac{k}{k_0}\left(\frac{t}{t_0}\right)^c\right)$$

$$\times\frac{1}{\gamma_L^2 k_0^4 t_0^2}\left(\frac{t_0}{t}\right)^{2-4c}\frac{\partial f}{\partial x} + \frac{3}{4}k^{\frac{7}{2}}\frac{1}{\gamma_L^{-1}k_0^{-\frac{3}{2}}t_0^{-1}}\left(\frac{t_0}{t}\right)^{\frac{2-3c}{2}}$$

$$\times f^{-\frac{1}{2}}\left(\frac{k}{k_0}\left(\frac{t}{t_0}\right)^c\right)\frac{1}{\gamma_L^4 k_0^8 t_0^4}\left(\frac{t_0}{t}\right)^{2(2-4c)}\left(\frac{\partial f}{\partial x}\right)^2 + \frac{3}{4}k^{\frac{7}{2}}$$

$$\times\frac{1}{\gamma_L k_0^{\frac{3}{2}} t_0}\left(\frac{t_0}{t}\right)^{\frac{2-3c}{2}} f^{\frac{1}{2}}\left(\frac{k}{k_0}\left(\frac{t}{t_0}\right)^c\right)\frac{1}{\gamma_L^2 k_0^5 t_0^2}\left(\frac{t_0}{t}\right)^{2-3c}\frac{\partial^2 f}{\partial x^2}$$

Finally substituting all these expressions on both sides of (5), we obtain

$$(3c-2)\,f\left(\frac{k}{k_0}\left(\frac{t}{t_0}\right)^c\right) + \frac{ck}{k_0}\left(\frac{t}{t_0}\right)^c f'\left(\frac{k}{k_0}\left(\frac{t}{t_0}\right)^c\right)$$

$$= -2\left\{-\frac{15}{2}\left(\frac{k}{k_0}\left(\frac{t}{t_0}\right)^c\right)^{\frac{3}{2}} f^{\frac{3}{2}}\left(\frac{k}{k_0}\left(\frac{t}{t_0}\right)^c\right) + \frac{3}{4}\left(\frac{k}{k_0}\left(\frac{t}{t_0}\right)^c\right)^{\frac{5}{2}}\right.$$

$$\times f^{\frac{1}{2}}\left(\frac{k}{k_0}\left(\frac{t}{t_0}\right)^c\right)\frac{\partial f}{\partial x} + \frac{3}{4}\left(\frac{k}{k_0}\left(\frac{t}{t_0}\right)^c\right)^{\frac{7}{2}} f^{\frac{1}{2}}\left(\frac{k}{k_0}\left(\frac{t}{t_0}\right)^c\right)\left(\frac{\partial f}{\partial x}\right)^2$$

$$+\frac{3}{2}k^{\frac{7}{2}}\frac{1}{\gamma_L k_0^{\frac{3}{2}} t_0}\left(\frac{t_0}{t}\right)^{\frac{2-3c}{2}} f^{\frac{1}{2}}\left(\frac{k}{k_0}\left(\frac{t}{t_0}\right)^c\right)\frac{\partial^2 f}{\partial x^2}\Bigg\}$$

$$-2vtk_0^2x^2\left(\frac{t_0}{t}\right)^{2c} f(x)-2\beta tf(x)$$

as

$$\frac{\partial E}{\partial k}=\frac{\partial}{\partial k}\left(\frac{1}{\gamma_L^2 k_0^3 t_0^2}\left(\frac{t_0}{t}\right)^{2-3c} f\left(\frac{k}{k_0}\left(\frac{t}{t_0}\right)^c\right)\right)$$

$$=\frac{1}{\gamma_L^2 k_0^3 t_0^2}\left(\frac{t_0}{t}\right)^{2-3c}\frac{\partial f}{\partial x}\cdot\frac{1}{k_0}\left(\frac{t}{t_0}\right)^c, \quad x=\frac{k}{k_0}\left(\frac{t}{t_0}\right)$$

$$=\frac{1}{\gamma_L^2 k_0^4 t_0^2}\left(\frac{t_0}{t}\right)^{2-4c}\frac{\partial f}{\partial x} \text{ and } \frac{\partial^2 E}{\partial k^2}=\frac{\partial}{\partial k}\left(\frac{\partial E}{\partial k}\right)$$

$$=\frac{\partial}{\partial k}\left\{\frac{1}{\gamma_L^2 k_0^4 t_0^2}\left(\frac{t_0}{t}\right)^{2-4c}\frac{\partial f}{\partial x}\right\}$$

$$=\frac{1}{\gamma_L^2 k_0^4 t_0^2}\left(\frac{t_0}{t}\right)^{2-4c}\frac{\partial^2 f}{\partial x^2}\frac{1}{k_0}\left(\frac{t}{t_0}\right)^c$$

$$=\frac{1}{\gamma_L^2 k_0^5 t_0^2}\left(\frac{t_0}{t}\right)^{2-5c}\frac{\partial^2 f}{\partial x^2}$$

Thus

$$(3c-2)f(x)+cxf'(x)$$

$$=-2\left\{-\frac{15}{2}x^{\frac{3}{2}}f^{\frac{3}{2}}(x)+\frac{3}{4}x^{\frac{5}{2}}f^{\frac{1}{2}}(x)\frac{\partial f}{\partial x}+\frac{3}{4}x^{\frac{7}{2}}f^{-\frac{1}{2}}(x)\left(\frac{\partial f}{\partial x}\right)^2\right.$$

$$\left.+\frac{3}{2}x^{\frac{7}{2}}f^{\frac{1}{2}}(x)\frac{\partial^2 f}{\partial x^2}\right\}-2vtk_0^2x^2\left(\frac{t_0}{t}\right)^{2c} f(x)-2\beta tf(x)$$

Or

$$(3c-2)f(x)+cxf'(x)$$

$$=-2\frac{d}{dx}\left(x^{\frac{13}{2}}\frac{d}{dx}x^{-3}f^{\frac{3}{2}}(x)\right)-2vk_0^2t_0\left(\frac{t_0}{t}\right)^{2c-1}$$

$$\times x^2 f(x)-2\beta tf(x)$$

Or

$$(3c-2)f(x)+cxf'(x)$$

$$=-2\frac{d}{dx}\left(x^{\frac{13}{2}}\frac{d}{dx}x^{-3}f^{\frac{3}{2}}(x)\right)-\frac{2}{\mathrm{Re}}(\tau_1^{-1})^{2c-1}x^2f(x)-2\beta tf(x) \tag{7}$$

where $\tau_1=\dfrac{t}{t_0}$

Thus finally, we have

$$(3c-2)f(x)+cxf'(x)$$

$$=-2\frac{d}{dx}\left(x^{\frac{13}{2}}\frac{d}{dx}x^{-3}f^{\frac{3}{2}}(x)\right)-\frac{2}{\mathrm{Re}\,\tau_1^{2c-1}}-2\beta\tau_1 t_0 f(x) \tag{8}$$

as

$$\frac{d}{dx}\left(x^{\frac{13}{2}}\frac{d}{dx}\left(x^{-3}f^{\frac{3}{2}}(x)\right)\right)$$

$$=\frac{d}{dx}\left(x^{\frac{13}{2}}\left(-3x^{-4}f^{\frac{3}{2}}(x)+x^{-3}\frac{3}{2}f^{\frac{3}{2}}(x)\frac{df}{dx}\right)\right)$$

$$=-3\frac{d}{dx}\left(x^{\frac{5}{2}}f^{\frac{3}{2}}(x)\right)+\frac{3}{2}\frac{d}{dx}\left(x^{\frac{7}{2}}f^{\frac{1}{2}}(x)\frac{df}{dx}\right)$$

$$=-3\left\{\frac{5}{2}x^{\frac{3}{2}}f^{\frac{3}{2}}(x)+x^{\frac{5}{2}}f^{\frac{1}{2}}(x)\frac{df}{dx}\right\}$$

$$+\frac{3}{2}\left\{\frac{7}{2}x^{\frac{5}{2}}f^{\frac{1}{2}}(x)\frac{df}{dx}+x^{\frac{7}{2}}\frac{1}{2}f^{-\frac{1}{2}}(x)\left(\frac{df}{dx}\right)^2+x^{\frac{7}{2}}f^{\frac{1}{2}}\frac{d^2f}{dx^2}\right\}$$

$$=-\frac{15}{2}x^{\frac{3}{2}}f^{\frac{3}{2}}(x)+\frac{3}{4}x^{\frac{5}{2}}f^{\frac{1}{2}}(x)\frac{df}{dx}$$

$$+\frac{3}{4}x^{\frac{7}{2}}f^{-\frac{1}{2}}(x)\left(\frac{df}{dx}\right)^2+\frac{3}{2}x^{\frac{7}{2}}f^{-\frac{1}{2}}(x)\frac{d^2f}{dx^2}$$

Taking $c=\frac{1}{2}$ we obtain the reduced version of (8) as

$$-\frac{1}{2}f(x)+\frac{1}{2}xf'(x)=-2\frac{d}{dx}\left(x^{\frac{13}{2}}\frac{d}{dx}\left(x^{-3}f^{\frac{3}{2}}(x)\right)\right)$$

$$-\frac{2}{\mathrm{Re}}x^2 f(x)-2\beta\tau t_0 f(x) \tag{9}$$

Now (9) can be written as

$$-\frac{1}{2}f(x)+\frac{1}{2}xf'(x)=-2\frac{d}{dx}\left(x^{\frac{13}{2}}\frac{d}{dx}\left(x^{-3}f^{\frac{3}{2}}(x)\right)\right)$$

$$-\frac{2}{\mathrm{Re}}x^2 f(x)-2MA\sqrt{R}f(x) \tag{10}$$

where $A=\frac{1}{\tau}\sqrt{\frac{v}{\varepsilon}},\ R=\frac{\varepsilon t^2}{v}\quad \therefore \sqrt{R}=t\sqrt{\frac{\varepsilon}{v}}\Rightarrow t=\sqrt{\frac{Rv}{\varepsilon}}$

and $M=\frac{\rho_p}{\rho}=\frac{\rho_p}{\tau\rho}\cdot\tau=\beta\tau$ where $\beta=\frac{\rho_p}{\tau\rho}$; therefore

$$MA=\beta\tau\cdot\frac{1}{\tau}\sqrt{\frac{v}{\varepsilon}}=\beta\sqrt{\frac{v}{\varepsilon}}\Rightarrow MA\sqrt{R}=\beta\sqrt{\frac{v}{\varepsilon}}\cdot t\sqrt{\frac{\varepsilon}{v}}=\beta t$$

Asymptotic behavior of $f(x)$, the solution of the equation (10)

Case (1) when $x\to 0$

Putting $f(x)=Bx^n\quad (n>0,\ B\neq 0)$ and $Re\equiv$ very large, we get

$$-\frac{1}{2}Bx^n+\frac{1}{2}xBnx^{n-1}=-2\frac{d}{dx}\left(x^{\frac{13}{2}}\frac{d}{dx}\left(x^{-3}B^{\frac{3}{2}}x^{\frac{3n}{2}}\right)\right)$$

$$-\frac{2}{\mathrm{Re}}x^2 f(x)-2MA\sqrt{R}Bx^n \tag{11}$$

Then (11) gives

$$\left[-\frac{1}{2}+\frac{n}{2}+2MA\sqrt{R}\right]x^n+2\sqrt{B}\left(\frac{3n}{2}-3\right)\frac{d}{dx}x^{\frac{5+3n}{2}}=0$$

or $$\left[-\frac{1}{2}+\frac{n}{2}+2MA\sqrt{R}\right]x^n$$

$$+2\sqrt{B}\left(\frac{3n}{2}-3\right)\left(\frac{5+3n}{2}\right)x^{\frac{3}{2}(n+1)}=0$$

As $n>0$, and $x\to 0$, the first term of the above expression is significant. Equating the coefficient of x^n to zero, we get $n=1-4MA\sqrt{R}$ provided that $MA\sqrt{R}<\frac{1}{4}$

Hence $f(x)\sim x^{(1-4MA\sqrt{R})}$ i.e., $E(k,t)\sim k^{(1-4MA\sqrt{R})}$

Case (ii) when $x\to\infty$ **:** Let us put $x'=\frac{1}{x}$ in (11) so that $x'\to 0$ as $x\to\infty$ Let $f(x')=A'x'^{-n}$

Then from (11) we get

$$-\frac{1}{2}A'x'^{-n}+\frac{1}{2}x'^{-1}A'(-n)x'^{-n-1}\frac{dx'}{dx}$$

$$=-2\frac{d}{dx'}\left(x'^{-\frac{13}{2}}\frac{d}{dx'}\left(x'^3 A'^{\frac{3}{2}}x'^{-\frac{3n}{2}}\right)\frac{dx'}{dx}\right)\frac{dx'}{dx}-2MA'\sqrt{R}x'^{-n}$$

or

$$-\frac{1}{2}A'x'^{-n}+\frac{1}{2}x'^{-1}A'(-n)x'^{-n-1}\left(-x'^2\right)$$

$$=-2\left[\frac{d}{dx'}\left(x'^{\frac{-13}{2}}\left(\frac{d}{dx'}A'^{\frac{3}{2}}x'^{\frac{-3n}{2}+3}\right)\right)\left(-x'^2\right)\right]\left(-x'^2\right)$$

$$-2MA'\sqrt{R}x'^{-n}$$

or

$$\left(-\frac{1}{2}+\frac{1}{2}n+2M\sqrt{R}\right)A'x'^{-n}$$

$$=-2A'^{\frac{3}{2}}\left[\frac{d}{dx'}x'^{\frac{-13}{2}}\left(-\frac{3n}{2}+3\right)x'^{-\frac{3n}{2}+4}\right]x'^2$$

$$=-2A'^{\frac{3}{2}}\left(-\frac{3n}{2}+3\right)\left(\frac{d}{dx'}x'^{-\frac{3n}{2}-\frac{5}{2}}\right)x'^2$$

$$=2A'^{\frac{3}{2}}\left(-\frac{3n}{2}+3\right)\left(-\frac{3n+5}{2}\right)x'^{-\frac{3n}{2}-\frac{3}{2}}$$

or $$\left(-\frac{1}{2}+\frac{n}{2}+2M\sqrt{R}\right)-2\sqrt{A'}\left(-\frac{3n}{2}+3\right)\left(-\frac{3n}{2}-\frac{5}{2}\right)$$

$$\times x'^{-\frac{n}{2}-\frac{3}{2}}=0$$

or

$$\left(-\frac{1}{2}+\frac{n}{2}+2M\sqrt{R}\right)x'^{\frac{n}{2}+\frac{3}{2}}-2\sqrt{A'}\left(3-\frac{3n}{2}\right)\left(-\frac{3n}{2}-\frac{5}{2}\right)=0$$

Making $x'\to 0$, we get from the above

$$2\sqrt{A'}\left(\frac{3n}{2}-3\right)\left(-\frac{5}{2}-\frac{3n}{2}\right)=0$$

Either n = 2 or $n=-\frac{5}{3}$. As we have taken $c=\frac{1}{2}$, so $n=-\frac{5}{3}$.

Thus $f\left(x'\right)=A'x'^{\frac{5}{3}}$ or $f(x)\sim x^{-\frac{5}{3}};\quad x\to\infty$

Hence $E(k,t)\sim k^{-\frac{5}{3}}\quad (k\to\infty)$

We solve the differential equation

$$-\frac{1}{2}f(x)+\frac{1}{2}xf'(x)=-2\frac{d}{dx}\left(x^{\frac{13}{2}}\frac{d}{dx}\left(x^{-3}f^{\frac{3}{2}}(x)\right)\right)$$
$$-2MA\sqrt{R}f(x)$$

Subject to the initial condition $MA\sqrt{R}<\frac{1}{4}$ (12)

We choose here the value of R to be 240, a value estimated by Batchelor (1959).

Series Solution Method : Let us now follow a series solution of the problem.

Case (i); MA = 0

Let $f(x)=c_1x+c_2x^{\frac{3}{2}}+c_3x^2+c_4x^{\frac{5}{2}}+c_5x^3+c_6x^{\frac{7}{2}}$

$$+c_7x^4+c_8x^{\frac{9}{2}}+c_9x^5+.....$$

be a series solution of the equation (13) rewritten here as

$$-\frac{1}{2}f(x)+\frac{1}{2}xf'(x)=15x^{\frac{3}{2}}f^{\frac{3}{2}}(x)-\frac{3}{2}x^{\frac{5}{2}}f^{\frac{1}{2}}(x)$$
$$-\frac{3}{2}x^{\frac{7}{2}}f^{-\frac{1}{2}}(x)\left\{f'(x)\right\}^2-3x^{\frac{7}{2}}f^{\frac{1}{2}}(x)f'(x)\quad (13)$$

Then $f'(x)=c_1+\frac{3}{2}c_2x^{\frac{1}{2}}+2c_3x+\frac{5}{2}c_4x^{\frac{3}{2}}+3c_5x^2$

$$+\frac{7}{2}c_6x^{\frac{5}{2}}+4c_7x^3+\frac{9}{2}c_8x^{\frac{7}{2}}+5c_9x^4+...\quad (14)$$

Thus $f''(x)=\frac{3}{4}c_2x^{-\frac{1}{2}}+2c_3+\frac{15}{4}c_4x^{\frac{1}{2}}+6c_5x$

$$+\frac{35}{4}c_6x^{\frac{3}{2}}+12c_7x^2+\frac{63}{4}c_8x^{\frac{5}{2}}+20c_9x^3+...\quad (15)$$

And, let $f^{\frac{1}{2}}(x)=d_1x^{\frac{1}{2}}+d_2x+d_3x^{\frac{3}{2}}+d_4x^2+d_5x^{\frac{5}{2}}$

$$+d_6x^3+...\quad (16)$$

where $d_1=c_1^{\frac{1}{2}},\ d_2=\frac{c_2}{2\sqrt{c_1}},\ d_3=\frac{c_3}{2\sqrt{c_1}}-\frac{c_2^2}{8c_1^{\frac{3}{2}}},$

$$d_4=-\frac{c_2c_3}{4c_1^{\frac{3}{2}}}+\frac{c_2^3}{16c_1^{\frac{5}{2}}}-\frac{c_4}{2\sqrt{c_1}},$$

$$d_5=\frac{c_5}{2\sqrt{c_1}}-\frac{1}{4c_1^{\frac{3}{2}}}c_2c_4+\frac{3c_2^2c_3}{16c_1^{\frac{5}{2}}}-\frac{c_3^2}{8c_1^{\frac{3}{2}}},$$

$$d_6=\frac{c_6}{2\sqrt{c_1}}-\frac{1}{4c_1^{\frac{3}{2}}}\left(c_2c_5+c_3c_4\right)+\frac{3}{16c_1^{\frac{5}{2}}}\left(c_2c_3^2+c_2^2c_4\right),...$$

and hence

$$f^{\frac{3}{2}}(x)=c_1d_1x^{\frac{3}{2}}+\left(c_1d_2+c_2d_1\right)x^2+\left(c_1d_3+c_2d_2+c_3d_1\right)x^{\frac{5}{2}}$$
$$+\left(c_1d_4+c_2d_3+c_3d_2+c_4d_1\right)x^3+...\quad (17)$$

also, let

$$f^{-\frac{1}{2}}(x)=g_1x^{-\frac{1}{2}}+g_2+g_3x^{\frac{1}{2}}+g_4x+g_5x^{\frac{3}{2}}$$
$$+g_6x^2+g_7x^{\frac{5}{2}}+g_8x^2+...\quad (18)$$

where $g_1=\frac{1}{c_1^{\frac{1}{2}}},\ g_2=-\frac{1}{2c_1^{\frac{3}{2}}},\ g_3=-\frac{c_3}{2c_1^{\frac{3}{2}}}+\frac{3c_2^2}{8c_1^{\frac{5}{2}}},$

$$g_4=-\frac{c_4}{2c_1^{\frac{3}{2}}}+\frac{6c_2c_3}{8c_1^{\frac{5}{2}}}-\frac{5c_2^3}{16c_1^{\frac{7}{2}}},$$

$$g_5 = -\frac{c_5}{2c_1^{\frac{3}{2}}} + \frac{3c_3^2}{8c_1^{\frac{5}{2}}} + \frac{6c_2c_4}{8c_1^{\frac{5}{2}}} - \frac{15c_2^2c_3}{16c_1^{\frac{7}{2}}},$$

$$g_6 = -\frac{c_6}{2c_1^{\frac{3}{2}}} + \frac{6c_2c_5}{8c_1^{\frac{5}{2}}} + \frac{6c_3c_4}{8c_1^{\frac{5}{2}}} - \frac{5c_2^3}{16c_1^{\frac{7}{2}}} - \frac{15c_2^2c_4}{16c_1^{\frac{7}{2}}},$$

$$g_7 = -\frac{c_7}{2c_1^{\frac{3}{2}}} + \frac{3c_4^2}{8c_1^{\frac{5}{2}}} + \frac{6c_2c_6}{8c_1^{\frac{5}{2}}} + \frac{6c_3c_5}{8c_1^{\frac{5}{2}}} - \frac{5c_3^3}{16c_1^{\frac{7}{2}}},$$

$$g_8 = -\frac{c_8}{2c_1^{\frac{3}{2}}} + \frac{6c_3c_6}{8c_1^{\frac{5}{2}}} + \frac{6c_4c_5}{8c_1^{\frac{5}{2}}} - \frac{15c_2^2c_6}{16c_1^{\frac{7}{2}}} - \frac{15c_2c_4^2}{16c_1^{\frac{7}{2}}}$$

$$-\frac{15c_3^2c_4}{16c_1^{\frac{7}{2}}} + \frac{6c_2c_7}{8c_1^{\frac{5}{2}}},......$$

Substituting all these expressions in (13), we get $c_2 = 0 = c_3 = c_4$ and $c_5 = 12c_1^{\frac{3}{2}}$, $c_6 = \frac{3}{5}c_1^{\frac{1}{2}}....,$

Thus $f(x) = c_1x + 12c_1^{\frac{3}{2}}x^3 + \frac{3}{5}c_1^{\frac{1}{2}}x^{\frac{7}{2}} + ...$

Following Chandrashekhar and Proudman, we take $c_1 = 4$ and hence

$$f(x) = 4x - 96x^3 - 12x^{\frac{7}{2}} + ...$$

Case (ii) $M = .50$, $A = 2 \times 10^{-3}$, R = 240

proceeding exactly in the same way

we get

$$f(x) = 4x - 93.115288x^2 - 1.1709785x^{\frac{7}{2}} + ...$$

Case (iii) M = .50, A = 4×10^{-3}, R = 240.

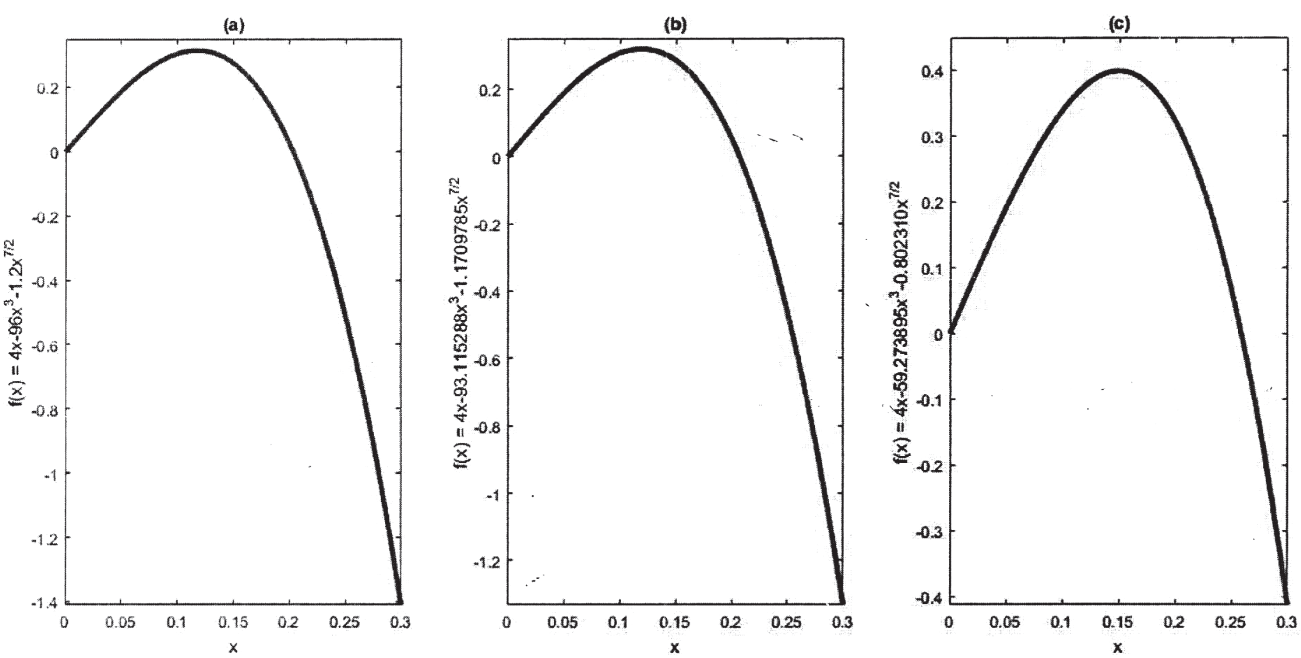
proceeding in the same manner as before we obtain

$$f(x) = 4x - 59.273895x^3 - .802310x^{\frac{7}{2}} + ...$$

From the figures shown below, it appears that as x increases from 0 to 1, $f(x)$ increases, though overall $f(x)$ decreases taking all the figures into consideration.

Conclusion **:** In this paper it has been shown that when Reynolds number is very large, that is, the term showing the dissipation of energy due to viscosity is neglected, the equation governing the decay of energy admits a class of self-preserving solution even in the case of isotropic gas-solid turbulent flow under certain condition; even asymptotically it obeys the Kolmogorov's $-\frac{5}{3}$rd law.

Following Chandrashekhar (I949) we obtained a curve $f(x)$ representing the solution of equation (8) in the text



such that it initially increases till it reaches the only maximum value and then dies down exactly as predicted.

An extensive discussion covering all aspects of the problem could almost be traced in Layek and Sunita[10] in a different perspective. ❐

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