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A General Type of Self-preserving Solution for the Particle-Ladden Homogeneous Isotropic Turbulent Flow

Abstract **: Self-preserving solutions of spectral equation governing the decay of turbulence energy spectrum in a particle-ladden homogeneous isotropic turbulent flow are obtained for the case of large Reynolds number. Asymptotic behaviour of such solutions for small and large values of wave number k are discussed.**

In recent times much attention has been paid to the prediction of particle-ladden turbulent flows as they occur in many technologically important areas. Research interest in these flows are generally two–fold. Islam and Mazumdar[1] considered the effect of turbulence on the particle concentration field and the modification of turbulence by the particles. We shall be concerned here with certain aspect of the problem how turbulence is modified by the particles when they are present in the flow in large enough concentration[2].

Results of investigations on the decay process of turbulence is considered interesting from both experimental and theoretical points of view. In the present paper, we shall discuss the effects of solid particles on the decay process of homogeneous and isotropic turbulence. Constructions of equations for the correlation functions and the spectral structures and their self-preserving solutions and power laws at small and large wave number are done. A basic interest would be for us to seek for well-known structure functions or to the formulation for the energy transfer spectrum. In the present approach, we shall use the very elegant method due to T. H. Ellison[3]. Moreover, it supported the well known Obukhov's assumption[4] very nicely. Many of the research workers used the term modified Obukhov's form for the above presentation of the said energy transfer spectrum.

Theoretical investigations were carried out in this area by Tchen[5], Meck and Jones[6], Reeks[7], Nir and Pismen[8] and others. Most of these works involve studies of the influence of particle inertia on the turbulent dispersion process. General confirmations of the conclusions made in these studies, were obtained from the experiments[9,10]. Reeks[11], Elghobashi and Truesdell[12], Yeung and Pope[13] and Squares and Eaton[2] have carried out numerical simulation of particle-ladden turbulent flows. It is difficult to make proper interpretations of the experimental data as they are valid only for the conditions of the experiment and can not be generalized (Elghobashi and Truesdell[14]).[1] For example, when fine droplets (or particles) of diameters ≤ 250μ are injected in a free turbulent jet, the turbulent intensity decreases, lowering the spreading rate of the half width of the jet, whereas the addition of the large particles of diameters ≥ 500μ causes an increase in the turbulent intensity. Hardalupas et al.[15] observed opposite phenomena in their experimental investigation on a particle-ladden turbulent flow.

Rao[16] studied the final periodic decay of energy spectrum of a particle-ladden homogeneous isotropic turbulence by a similarity process. In the present paper, an attempt is made to examine the similarity features of the decay of turbulence kinetic energy spectrum in a particle-ladden homogeneous isotropic turbulent flow. We shall assume that the size of the particles is sufficiently large so that the turbulence is attenuated by the dispersed phase.

Formulation of the Problem: The spectral equation governing the decay of turbulence kinetic energy in a particle-ladden homogeneous isotropic turbulent flow, is given by (Baw and Peskin[17], Tsuji[18])

$$\frac{\partial}{\partial t}E(k,t) = T(k,t) - 2\nu k^2 E(k,t) - 2\beta \frac{\nu k^2}{\frac{1}{2}+\nu k^2} E(k,t), \quad (1)$$

where k is the wave number, ν is the kinematic viscosity, τ is the characteristic time $= \frac{2}{9}\frac{\rho_s \sigma^2}{\mu}$, ρ_s is the density of the solid material , σ is the radius of the particle, μ is the co-efficient of viscosity, $\beta = \frac{\rho_p}{\rho\tau}, \rho_\rho$ is the volume concentration of the solid phase in the flow, ρ is the density of the gas, $E(k,t) = 2\pi k^2 \varphi_{ii}(k,t), \varphi_j(k,t)$ being the Fourier transform of $\overline{v_i v'_j}$, the correlation between the fluctuating gas velocity components v_i and v'_j pertaining to the points $P(\vec{X})$ and $P'(\vec{X})$ inside the flow field;

$T(k,t) = 2\pi k^2\, \Gamma_{ii}(k,t), \Gamma_{ij}(k,t)$ is the Fourier transform of $\frac{\partial}{\partial r_k}\left(\overline{v_i v_k v'_j} - \overline{v_i v'_j v'_k}\right), \vec{r} = \vec{X}' - \vec{X}$

The term on the left hand side of (1), describes the rate at which turbulence kinetic energy changes. The first term on the right hand side of (1) represents the transfer of kinetic energy at the wave number k due to turbulence self interactions. The second term describes the dissipation of turbulence kinetic energy due to the effects of molecular viscosity. The third term takes account of the effect due to the presence of particles, which are so massive that they are unaffected by the gas turbulent fluctuations[19]. In the next section we seek self-preserving solution of equation (1).

As expected we would encourage to apply for finding the self-preserving solutions of the standard spectral equations for the energy transfer spectrum utilizing the well known form of energy transfer spectrum, namely, due to above mentioned Ellison' form. Our motivation is to see that famous form as suggested by Ellison explained the decay laws appropriately at small and large wave numbers.

Self-preserving Solution for the Turbulence Energy spectrum : In order to solve equation (1), the transfer spectrum T(k, t) is to be modeled. We accept the local form for T(k, t), as suggested by T. H. Ellison[3], e.g.

$$T(k,t) = -2\gamma_3 \frac{d}{dk}\left[kE(k,t)\left\{\int_0^k k_1^2 E(k_1,t)dk_1\right\}^{1/2}\right], \quad (2)$$

where γ_3 is a non-dimensional constant.

Substituting (2) in (1) we obtain

$$\frac{\partial}{\partial t}E(k,t) = -2\gamma_3 \frac{d}{dk}\left[kE(k,t)\left\{\int_0^k k_1^2 E(k_1,t)dk_1\right\}^{1/2}\right]$$

$$-2\nu k^2 E(k,t) - 2\beta \frac{\nu k^2}{\frac{1}{\tau}+\nu k^2}E(k,t). \quad (3)$$

Based on the assumption that particles are essentially unaffected by the turbulence fluctuations, we may set

$$\frac{1}{\tau} << \nu\, k^2. \quad (4)$$

It is to be remarked that when the condition (4) is satisfied, for every high frequency fluctuations of gas, gas-solid velocity correlations are negligible as the response time of the particle is sufficiently long[19].

In view of the relation (4), the equation (3) is reducible to

$$\frac{\partial}{\partial t}E(k,t) = -2\gamma_3 \frac{d}{dk}\left[kE(k,t)\left\{\int_0^k k_1^2 E(k_1,t)dk_1\right\}^{1/2}\right]$$

$$-2\nu k^2 E(k,t) - 2\beta E(k,t). \quad (5)$$

We seek a general type of self-preserving solution of (5) in the form (Sen[20])

$$E(k,t) = \frac{1}{\alpha^2 k_0^2 t_0^2}\frac{s^2}{\tau^2} f\left(\frac{sk}{k_0}\right), \quad (6)$$

where α, k_0, t_0 are constants, $\tau = \frac{t}{t_0}$.

Substituting (6) in (5) we obtain after simplifications

$$(3c-2)f(x) + cxf'(x) = -\frac{2y_3}{\alpha}\frac{d}{dx}$$

$$\times\left\{\left[\int_0^k x_1^2 f(x_1)dx_1\right]^{1/2} xf(x)\right\} - \frac{2}{\mathrm{Re}}x^2 f(x) - 2\beta\, t(x), \quad (7)$$

where

$$x = \frac{sk}{k_0},\ \mathrm{Re} = \text{Reynolds number } = \frac{1}{\nu k_0^2 t_0}, c = \frac{\tau s_\tau}{s},$$

$$s_\tau = \frac{\partial s}{\partial \tau}$$

If we take $c = \frac{1}{2}$, the equation (7) is transformed to

$$-\frac{1}{2}f(x) + \frac{1}{2}xf'(x) = -\frac{2\gamma_3}{\alpha}\frac{d}{dx}\left\{\left[\int_0^k x_1^2 f(x_1)dx_1\right]^{1/2} xf(x)\right\}$$

$$-\frac{2}{\mathrm{Re}}x^2 f(x) - 2MA\sqrt{Rf(x)}, \quad (8)$$

where

$$R=\frac{\varepsilon t^2}{\nu}, M=\frac{\rho_p}{\rho}=\left(\frac{\rho_p}{\tau\rho}\right).\tau=\beta\tau, A=\frac{1}{\tau}\sqrt{\frac{\nu}{\varepsilon}}.$$

Clearly equation (8) may admit self-preserving solution for different choices of MA if $R=\frac{\varepsilon t^2}{\nu}$ remains constant during decay process. Usually, R = constant applies to the wave number range of the energy containing eddies and it need not apply to equilibrium range of wave number. In order to describe the self-preserving features of the turbulence energy spectrum, Heisenberg[21] assumed that the energy containing eddies would be in quasi-equilibrium, so that we may consider them as if they are in equilibrium as far as possible in view of their finite rate of decay (Hinze[22]). We accept this premise for our analysis of the present case. Let us discuss asymptotic behaviour of f(x) for the cases (i) x → 0 and (ii) x → ∞, when turbulence is characterized by very large Reynolds number Re → ∞.

Case I: Let

$f(x) \sim Bx^n$ as $x \to 0$; B is a non-zero constant and $n > 0$. (9)

Substituting (9) in equation (8), we obtain, after some calculation,

$$\left[-\frac{1}{2}+\frac{n}{2}+2MA\sqrt{R}\right]x^n+\frac{\gamma_3\sqrt{B}}{\alpha\sqrt{(n+3)}}(3n+5)\,x^{\frac{3n+3}{2}}=0. \quad (10)$$

As n>0 and x → 0, it is easily seen that the first term of (10) is significant. Equating to zero the co-efficient of x^n, we obtain

$$n = 1 - 4MA\sqrt{R}, \quad (11)$$

provided

$$MA\sqrt{R}<\frac{1}{4}. \quad (12)$$

Hence the asymptotic behaviour of $f(x)$ as $x \to 0$ is obtained as

$$f(x) \sim Bx^{1-4MA\sqrt{R}}$$

for $MA\sqrt{R}<\frac{1}{4}$.

This makes

$$E(k, t) \sim \text{constant}.\ k^{1-4MA\sqrt{R}} \quad (13)$$

for $MA\sqrt{R}<\frac{1}{4}$.

Case II: Let

$f(x) \sim B'x^{-n}$ as $x \to \infty$; B′ is a non-zero constant and $n > 0$. (14)

Taking (14) into account, equation (8) is reducible to

$$\left[-\frac{1}{2}-\frac{n}{2}+2MA\sqrt{R}\right]x^{-n}+\frac{\gamma_3\sqrt{B'}}{\alpha\sqrt{(3-n)}}(5-3n)x^{\frac{-3n+3}{2}}=0. \quad (15)$$

It can be easily seen that the second term on the left hand side of (15) is predominant if n<3. Accepting this we equate the co-efficient of $x^{\frac{3(1-n)}{2}}$ to zero and obtain $n=\frac{5}{3}$. Thus in the case when the turbulence is characterized by sufficiently large Reynolds number, the asymptotic behaviour of f(x) as x → ∞ is given by

$$f(x) \sim B'x^{-\frac{5}{3}}. \quad (16)$$

This gives

$$E(k, t) \sim \text{constant},\ k^{-\frac{5}{3}}; (k \to \infty) \quad (17)$$

Thus we have described above self-preserving solutions for the energy spectrum pertaining their asymptotic behaviour for small and large wave number. The results (13) clearly show that the self-preserving solutions are admitted for different values of MA and $R\left(=\frac{\varepsilon t^2}{\nu}\right)$ during the process of decay. For the case when the Reynolds number $R \to \infty$, the energy spectrum behaves the well-known Kolmogorov's spectrum $-\frac{5}{3}$.

Conclusion : In the last section we have worked out the physical power laws at the low wave number range and well known $-\frac{5}{3}$ power law at the higher wave number e.g., $k \to \infty$.

It is worth- mentioning that Prof. L. I. Sedov[23] in his book 'Similarity and dimensional methods is Mechanics' pointed out that the similarity method could be well applied to the field of turbulence problems in general. Such application have already been made and referred by Baw and Peskin[17]. They remarked that the turbulence energy is more dissipated in the high frequency range by the particles.

This is a qualitative explanation for the turbulence suppression by the particles.

The above discussions motivate us to work further various other turbulent aspects in particle- ladden turbulent flows. Discussions of turbulent flows were mostly qualitative and based on principles of fluid dynamics. This research situation is gradually changed. That is, measurements of turbulence in two-phase flows are being reported mainly because instruments like a laser Doppler velocity meter. Theoretical treatments have begun to be attempted as well as experimental.

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